Electrical Engineering EE3TR4

Day Class
Duration of Examination: 3 Hours
McMaster University Final Examination

Instructor: Dr. J. Reilly
April, 2013

This examination paper includes 5 pages and 6 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(b) There are 6 questions. A full paper consists of all six.

(c) You must show your work for full marks.

(d) All major questions are of equal weight (10 marks).

(e) **Make sure you read the entire exam over in its entirety before you start!**

(f) The tables of Fourier transforms, trigonometric identities and the Q(·) function at the back of this exam may be useful.

(g) Good luck on this exam and have a great summer!

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1. Explain the purposes of the transmit and receiver filters in a digital communication system. Also explain characteristics of the responses of these filters if the overall system is to have optimal performance.

2. A sample $x(t)$ of a random process may be described according to the following equation

$$x(t) = \sigma w(t) + A\cos(2\pi 10t + \Theta) \tag{1}$$

where $\sigma = 1$ volt, $A = 2$ volts, and Θ is a random variable uniformly distributed over $[0, 2\pi]$. Calculate the autocorrelation function $R_x(\tau)$ and power spectral density function $S_x(f)$. Assume the effective bandwidth of the noise process is 100 Hz. Include all relevant numerical values in your response.

3. We have an available bandwidth of 10 MHz over which we wish to transmit a digital bitstream at 8 Mbits/sec.

a) Draw the spectrum of the signal-only component of the received signal which appears immediately before the sampler, for this specific case, that gives rise to zero inter-symbol interference (ISI). Indicate values of any relevant parameters. (4 marks)

b) If the value $N_o/2 = 7.8125 \times 10^{-9}$Watts/Hz, and the received signal has a level of 1 V at the input to the sampler, what is the bit error rate (BER)? You may use the following form for $Q(a)$, which is valid for $a > 3$.

$$Q(a) \simeq \frac{1}{\sqrt{2\pi}\, a} \exp\left\{-\frac{a^2}{2}\right\}.$$

If you are used to working with the erfc$(\cdot)$ function instead of $Q(\cdot)$, then $Q(a) = \frac{1}{2}\text{erfc}(\frac{a}{\sqrt{2}})$. (3 marks)

c) How would you increase the bit rate of the system to 16 Mbits/sec? What would happen to the BER in this case, given that the noise and signal energies were unchanged? Explain your answer. (3 marks)

4. Consider the DSB/SC modulation system shown below. The message waveform $m(t)$ is shown in the figure. It is a 1 KHz square wave of amplitude 1 V as shown. *i)* Draw the waveforms and corresponding spectra at points A,B and C, for the case when $c(t) = \cos(2\pi f_c t)$. (6

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marks) *ii*) Repeat part (*i*) at points B and C when $c(t) = \sin(2\pi f_c t)$. (4 marks) Show all values in each case.

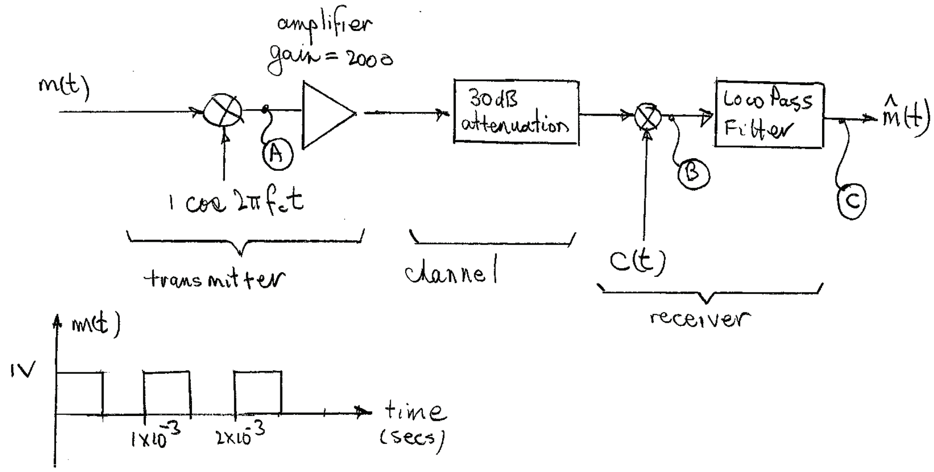

Figure 1: DSB/SC modulation system for question 4.

5. a) Find the time–domain signal $g(t)$ corresponding to the spectrum $G(f)$ whose magnitude and phase responses are shown in the figure below.

b) The signal $g(t)$ from part (a) is sampled at a rate f_s =2KHz. Draw the resulting sampled time waveform and corresponding spectrum. In this case, assume the phase response is zero for all frequencies.

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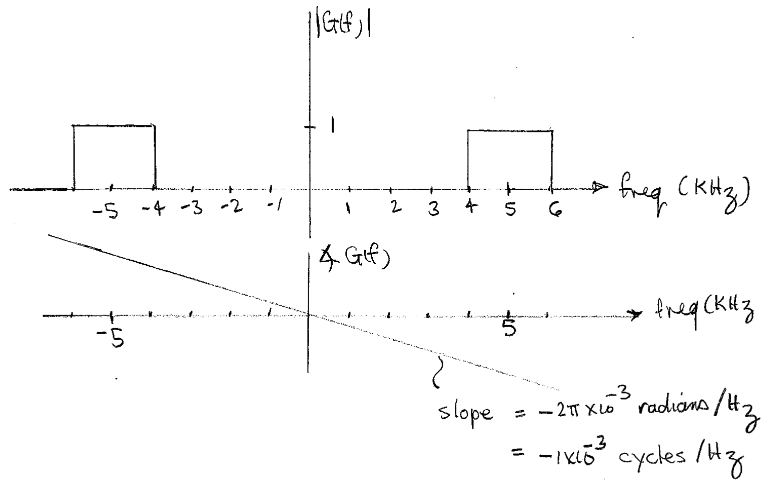

Figure 2: Magnitude and Phase functions of $G(f)$

6. A zero–mean white noise process with variance 1 $(\text{volt})^2$ is applied to the input of a discrete-time filter whose impulse response is shown in the figure below. The pulses have unity amplitude. Find the autocorrelation function and power spectral density of the filter output.

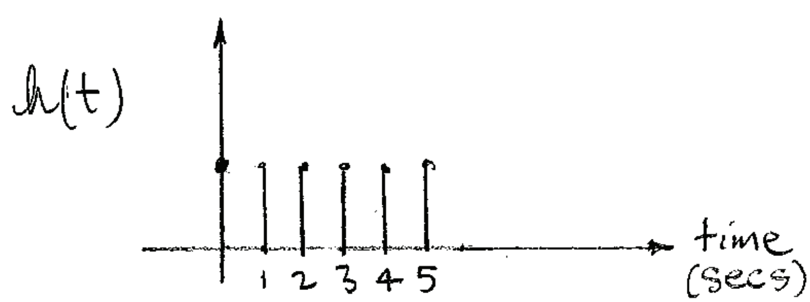

Figure 3: Impulse response of filter for Question 6.

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Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}\big(\text{fT}\big)$
$\text{sinc}\big(2Wt\big)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos(\theta) = \frac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]$$
$$\sin(\theta) = \frac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]$$
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \frac{1}{2}\big[1 + \cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\big[\cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\big[\cos(\alpha - \beta) + \cos(\alpha + \beta)$$
$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\big[\sin(\alpha - \beta) + \sin(\alpha + \beta)$$

The End.