Electrical Engineering EE3TR4

Midterm test: 1.5 Hours

Instructor: Dr. J. Reilly
February, 2015

This examination paper includes 3 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted.**

(b) There are 3 questions. Attempt all three.

(c) You must show your work for full marks.

(d) **Make sure you read the entire paper over in its entirety before you start!**

(e) The tables of Fourier transforms and trigonometric identities at the back of this exam may be useful.

1. A message signal $m(t) = A_m \cos(2\pi f_m t)$.

a. Sketch the AM–modulated wave for 125% modulation assuming $A_m = A_c = 1$. Also sketch the corresponding spectrum. Show all relevant values. (3 marks)

b. Repeat part (a) for double sideband suppressed carrier (DSB/SB) modulation. (2 marks)

c. Explain how you can recover the message signal in part (a) without distortion. (5 marks)

Continued on Page 2

2. a. Sketch the spectrum of a 2 KHz periodic square wave with 50% duty cycle. (2 marks)

   b. Same as part (a) except with a 25% duty cycle. (3 marks)

   c. The magnitude–squared response of a Butterworth filter is given by the expression

$$|H(f)|^2 = \frac{1}{1+(f/f_c)^{2n}} \tag{1}$$

   where n is the order of the filter and f_c is the cutoff frequency. The signal of part (a) is applied to the input of the Butterworth filter to produce a sinusoid at 2 KHz at the output. What order of filter is required so that all unwanted harmonics are suppressed by at least 40 dB relative to the fundamental frequency at 2 KHz? Assume $f_c = 2$ KHz. (5 marks)

3. a. Draw the spectrum corresponding to the NON-periodic rectangular pulse shown in Fig 1. (2 marks)

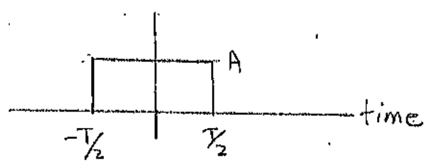


Figure 1: Pulse for Q. 3(a)

   b. Draw the spectrum of the triangular pulse shown in Fig 2. (5 marks)

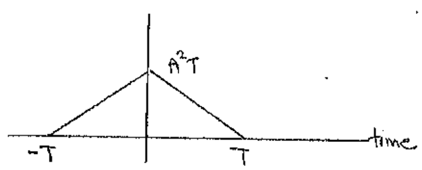


Figure 2: Pulse for Q. 3(b)

   c. Draw the spectrum of the pulse of part (a) after it has been multiplied by the sinusoid $\cos(2\pi f_c t)$. Show all relevant values in all parts (a), (b) and (c). (3 marks)

Continued on Page 3

Fourier Transform Pairs

Time Function	Fourier Transform
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T\mathrm{sinc}(fT)$
$\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f-f_c)$
$\exp(-at)u(t),\quad a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|),\quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right]$

A periodic rectangular wave in the time domain with amplitude A, period T and "ON" time T_o has Fourier coefficients c_n given by the expression $c_n = \frac{AT_o}{T}\mathrm{sinc}\left(n\frac{T_o}{T}\right)$, where $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

Trigonometric Identities

$$\cos(\theta) = \tfrac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$$
$$\sin(\theta) = \tfrac{1}{2j}\left[\exp(j\theta)-\exp(-j\theta)\right]$$
$$\sin^2(\theta)+\cos^2(\theta) = 1$$
$$\cos^2(\theta)-\sin^2(\theta) = \cos(2\theta)$$
$$\cos^2(\theta) = \tfrac{1}{2}\left[1+\cos(2\theta)\right]$$
$$2\sin(\theta)\cos(\theta) = \sin(2\theta)$$
$$\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right.$$
$$\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right.$$
$$\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right.$$

The End.

3TR4 Midterm 2015 Solutions Jim Reilly

Q1

Here $m(t) = \cos 2\pi f_m t$ since $A_m = 1$. The expression for the general AM wave is

$$s(t) = A_c\left[1 + k_a m(t)\right]\cos 2\pi f_c t$$

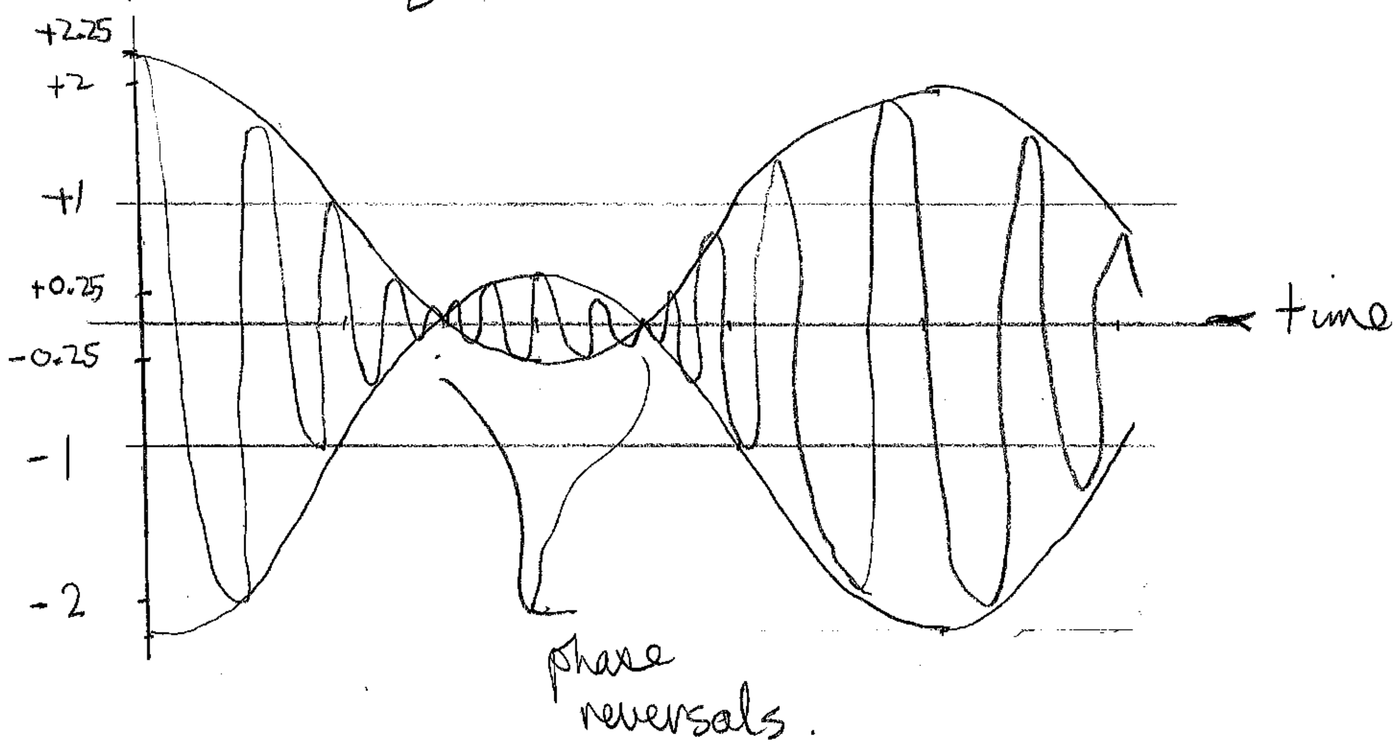
The value for k_a for 125% modulation is found by solving

$$\max\left|k_a m(t)\right| = 1.25$$

$\therefore$ $k_a = 1.25$. Since $A_c = 1$, $s(t)$ for this case is

(a)

$$s(t) = \left[1 + 1.25\cos 2\pi f_m t\right]\cos 2\pi f_c t$$



spectrum of $s(t)$:

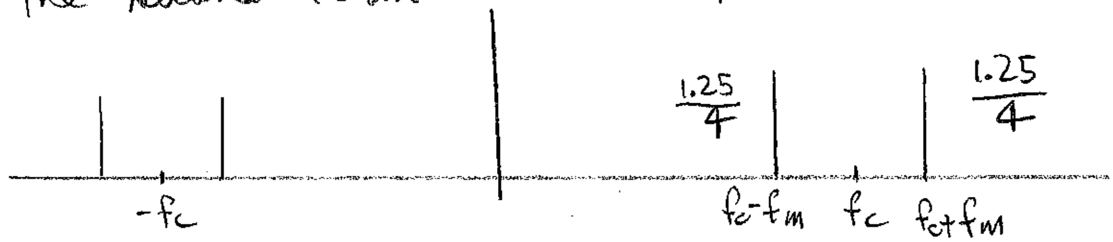
$$s(t) = \cos 2\pi f_c t + 1.25 \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

The spectrum for the first term (carrier) consists of δ- functions at $\pm f_c$ with weight $\frac{1}{2}$.

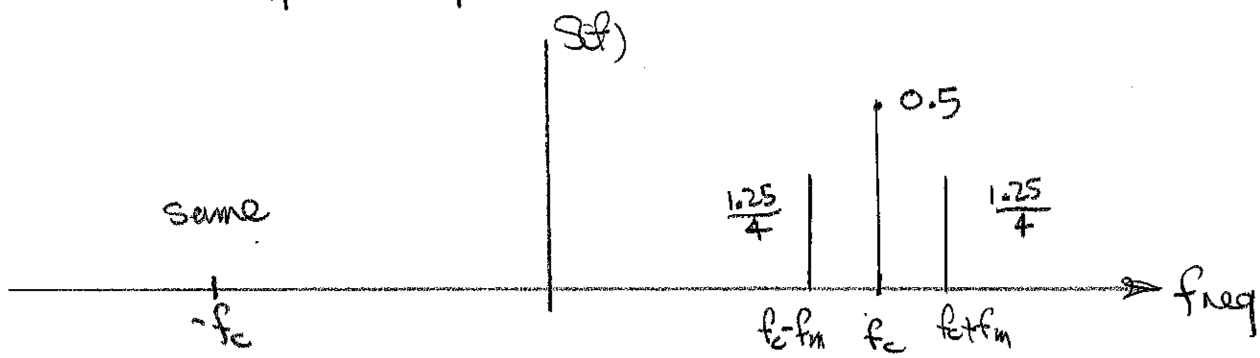
For the second term, we use the trig identity:

$$\cos A \cos B = \tfrac{1}{2}\cos(A+B) + \tfrac{1}{2}\cos(A-B)$$

$\therefore$ the second term has the spectrum



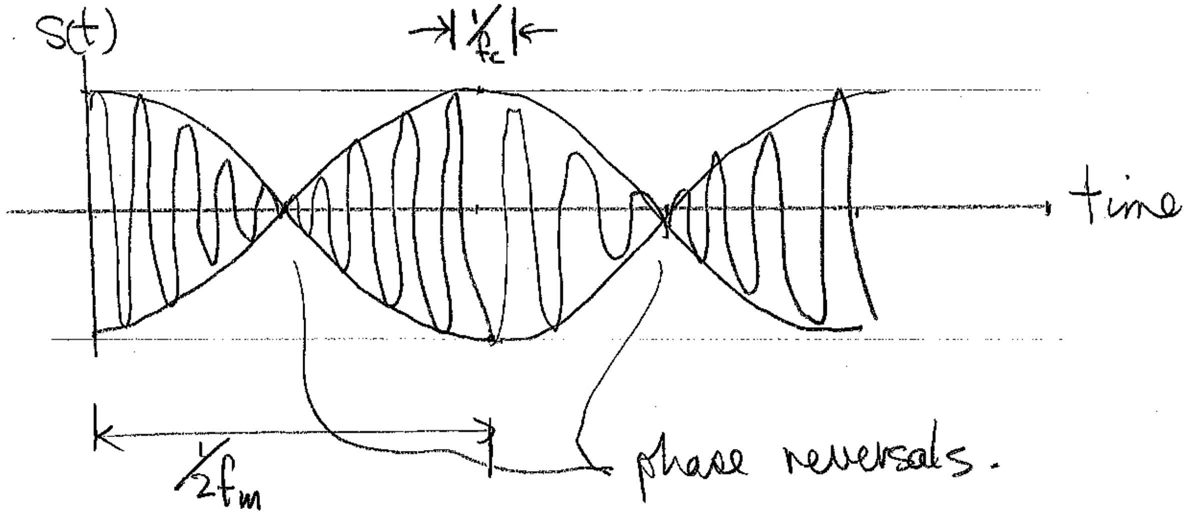
$\therefore$ the complete spectrum $S(f)$ is

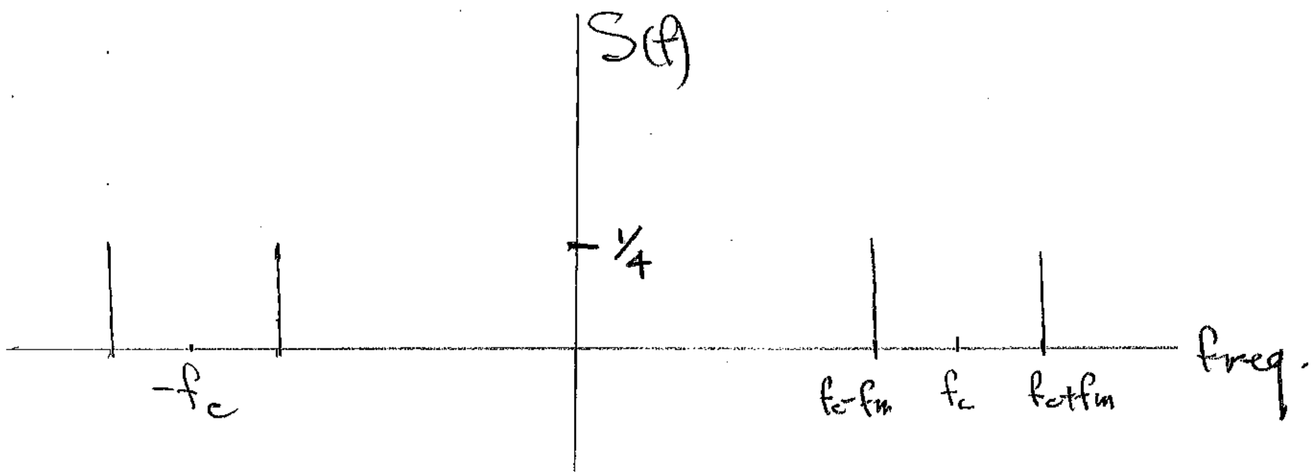


1b. DSB/SC.

$$s(t) = \cos(2\pi f_m t) \cdot \cos(2\pi f_c t)$$

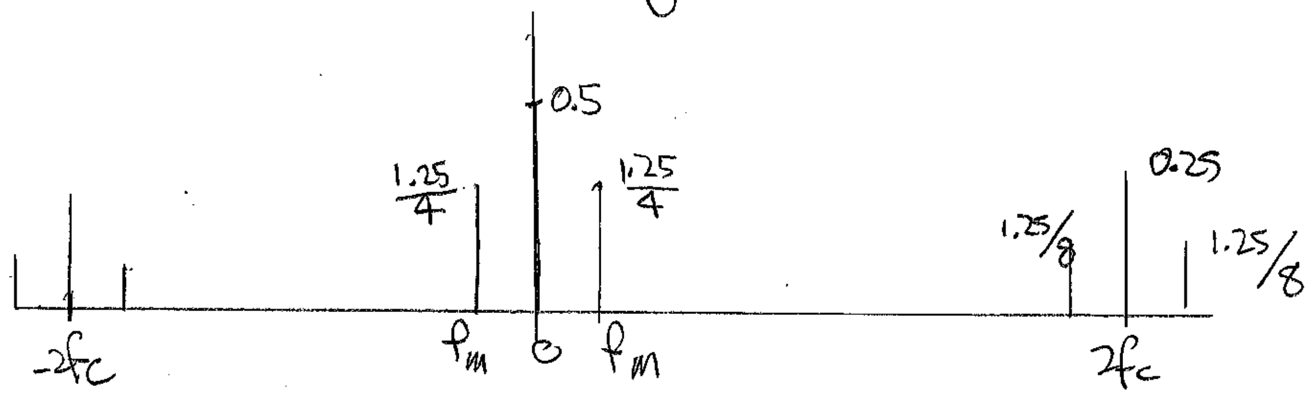
$$= \frac{1}{2}\cos\big(2\pi(f_c+f_m)t\big) + \frac{1}{2}\cos\big(2\pi(f_c-f_m)t\big)$$





1c. If any signal is multiplied by $\cos 2\pi f_c t$, then its spectrum is shifted up and down by f_c Hz, and weighted by a factor of $\frac{1}{2}$.

∴ After multiplying the AM signal from part (a) by $\cos 2\pi f_c t$, the resulting spectrum becomes



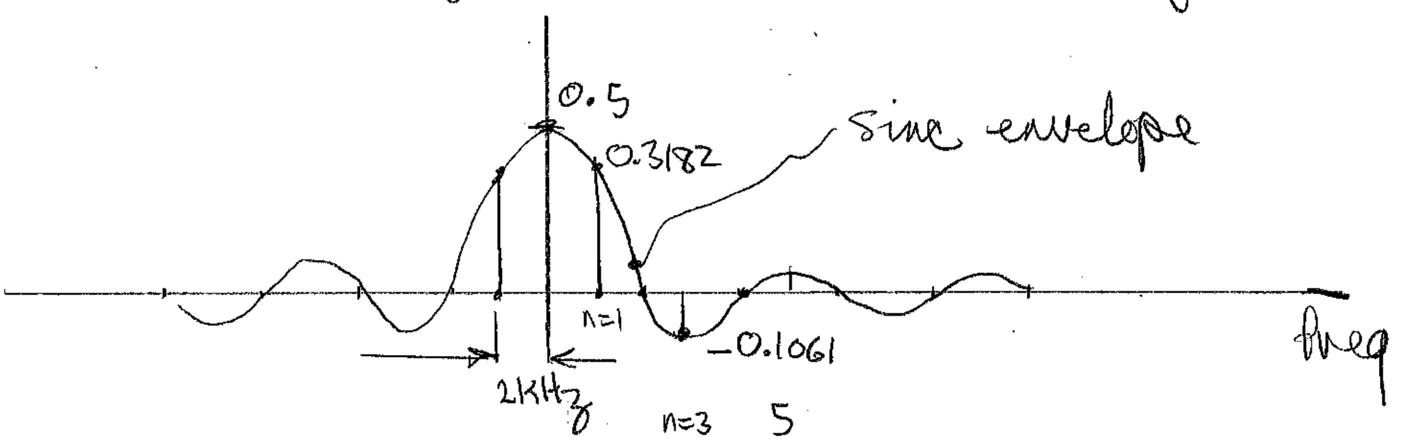
∴ after passing through a low-pass filter to isolate the baseband portion, and removing the DC component (by passing through a series capacitor), we recover the message m(t), within a scale factor.

Q2.

(a). The coefficients of the spectral components of the periodic square wave are given in the formula sheet at the end of the paper.

In this case, the duty cycle $\frac{T_0}{T} = 0.5$.

$\therefore$ $c_n = \frac{A}{2} \operatorname{sinc}(n/2)$. The spectral components are spaced by the fundamental frequency, 2KHz.

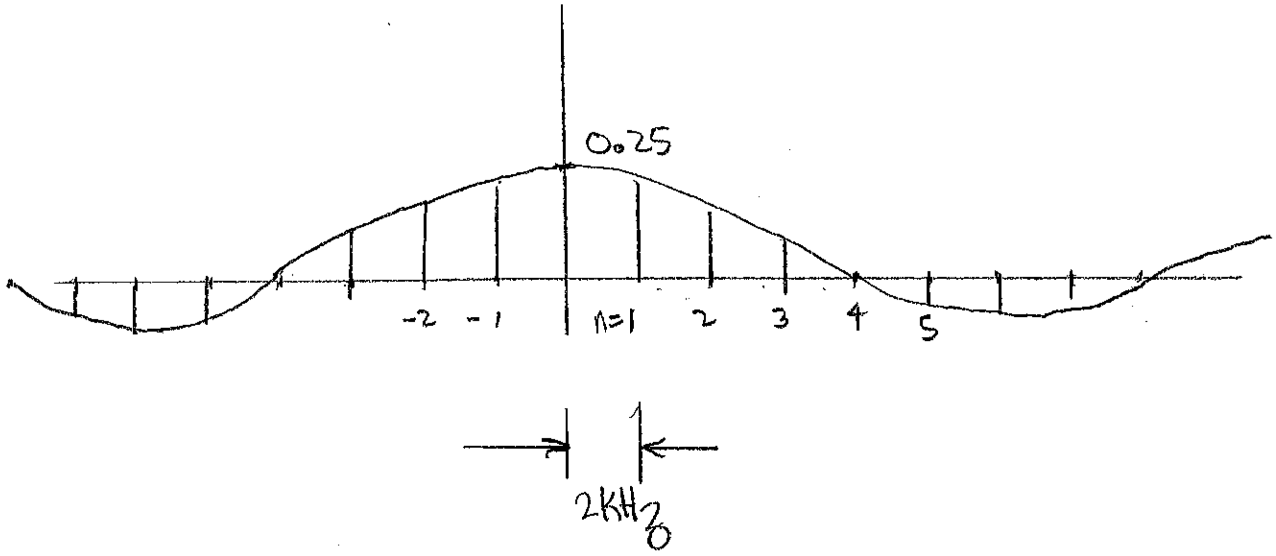


We assume the peak-to-peak amplitude of the square wave is one.

It is acceptable to assume the DC component is zero in which case the spike at 0 Hz is absent.

(b) The only difference with part (a) is that here, the duty cycle is 25%. The fundamental frequency remains the same.

$$C_n = \frac{A}{4}\,\mathrm{sinc}\left(\frac{n}{4}\right)$$



C. In this case, the first unwanted harmonic is at $n=3$ or 3X the fundamental freq f_o. For this case we have $f_c = f_o$ as given in the question.

The 3rd harmonic is already $20\log_{10}\frac{0.1061}{0.3183} = -9.5$ dB attenuated with respect to the fundamental due to the sinc envelope. The cutoff of the filter $f_c = f_o = 2$ kHz. At the cutoff freq, the filter attenuates the desired fundamental component by

$$10\log_{10} \frac{1}{1+(1)^{2n}} = 10\log_{10}\frac{1}{2} = -3\text{ dB}$$

regardless of n.

The required attenuation at $3f_0$ is 40 dB.

∴ the additional attenuation required by the filter

is $-40 + 9.5 - 3 = -33.5$ dB.

due to sinc env.

the desired component is suppressed 3 dB by the filter.

To determine the required filter order n, we solve

$$\frac{1}{1+\left(\frac{3f_0}{f_0}\right)^{2n}} = \frac{1}{1+(3)^{2n}} \equiv -33.5\text{ dB}.$$

$$1+(3)^{2n} \cong 3^{2n} \equiv 33.5\text{ dB}^{*} \equiv 2.2387\times 10^{3} \qquad (1)$$

* Note that the $\frac{1}{1+(3)^{2n}}$ term is mag^2 response which directly affects power at the output. Therefore the corresponding ratio is determined as

$$10^{33.5/10} = 2.2387\times 10^{3}$$ (Note we divide exponent by 10 rather than 20).

To solve (1) directly we can take logs on both sides:

$$\log_{10} 3^{2n} = 3.35$$

$$2n(\log_{10} 3) = 3.35$$

$$2n\,(0.4771) = 3.35$$

$$\therefore \quad n = 3.5108 \quad (*),$$

Notes:

1. Filter order must be an integer value, so we would round n up to 4.

2. It would be acceptable NOT to include the 3dB loss of the fundamental component due to attenuation of the filter at the cutoff freq. f_c.

   In this case (1) becomes

$$3^{2n} \equiv 36.5\,\text{dB} \equiv 4.4668 \times 10^{3}$$

$$\rightarrow \quad n = 3.82$$

This does not affect the value of n.

(notes, cont'd)

3. We may want to consider the effect of component tolerances. Errors in the values of the components of the active filter will affect the value of the cutoff freq. and hence the attenuation at $3f_0$.

   To accommodate this effect, we could raise the value of n to 5.

   (no marks taken off if this effect is not considered).

<u>Q3-</u> (a) The answer follows directly from the results from the provided Tables:

$$\mathrm{rect}\left(\frac{t}{T}\right) \rightleftharpoons T\,\mathrm{sinc}(fT)$$

so our function is $A\,\mathrm{rect}\,\frac{t}{T}$

$\therefore$ its spectrum is $AT\,\mathrm{sinc}(fT)$.

(continuous)

b. The triangular wave is the convolution of the rectangular wave in part (a) with itself. $\therefore$ since conv in time $\rightleftharpoons$ mult in freq, then the spectrum for part b. is $(AT)^2 \operatorname{sinc}^2(fT)$.

c. Since mult in time by $\cos 2\pi f_c t$ shifts the spectrum up and down by f_c Hz (weighted by ½), the spectrum for part (c) is $\frac{1}{2} AT \operatorname{sinc}((f-f_c)T) + \frac{1}{2} AT \operatorname{sinc}((f+f_c)T)$