Electrical Engineering EE3TR4

Midterm test: 1.5 Hours | Instructor: Dr. J. Reilly
February, 2016

This examination paper includes 4 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of an invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(b) There are 3 questions. Attempt all three. Show your work for full marks.

(c) **Make sure you read the entire paper over in its entirety before you start!**

(d) The tables of Fourier transforms and trigonometric identities at the back of this exam may be useful.

1. a. We have 4 modulated signals that we are transmitting over a common channel. The signals each have 100 KHz bandwidth and are arranged contiguously as shown in Fig. 1. Explain an effective method for selecting the desired channel at the receiver. Use diagrams or schematics in your explanation. 8 marks

b. Explain why the selected signal cannot be converted (i.e., mixed, or multiplied) directly to baseband. 2 marks

Continued on Page 2

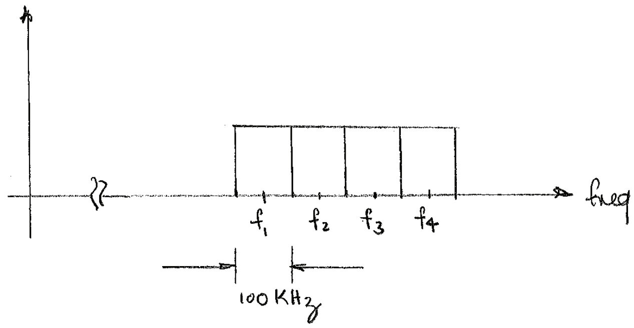

Figure 1: Signal configuration for Q1. Only positive frequency values are shown.

2. We have an impulse train in the time domain as shown in Fig 2(a). Its Fourier transform is also shown in the figure in part (b). We wish to convert the impulse train in time to a sine wave at the fundamental frequency $1/T$, by passing the impulse train through a low–pass filter, similar to what we did in Lab 1. Design a Butterworth low–pass filter so that any unwanted harmonics are suppressed by at least 20 dB. The magnitude–squared Butterworth response is given by the expression

$$|H(f)|^2 = \frac{1}{1 + \left(\frac{f}{f_o}\right)^{2n}} \tag{1}$$

where f_o is the cutoff frequency and n is the filter order. The value T in the Figure is 0.5 msec. *Hint:* By "design", we mean specify f_o and n.

Continued on Page 3

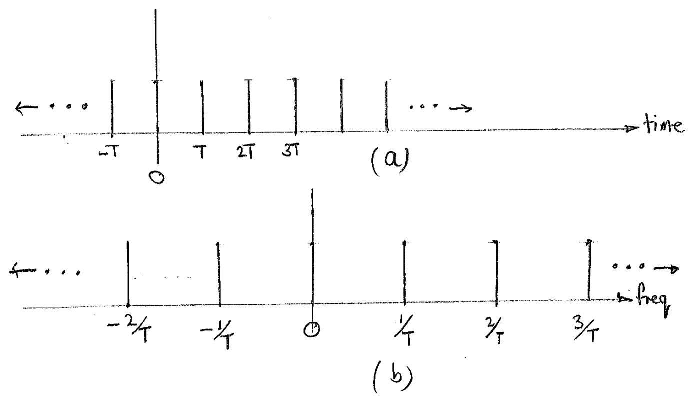

Figure 2: (a) Impulse train in the time domain and (b) its Fourier transform. Each impulse in frequency has a uniform weight.

3. Consider the scheme drawn in Figure 3. The noise source generates a zero mean Gaussian random process with standard deviation $\sigma = 1\text{V}$. (The standard deviation is equivalent to the root mean–squared (*rms*) voltage). Sketch the probability density function of the random process $x(t)$ when the DC source is set to 1V for the following two cases: *i)* the voltage gain G of the amplifier is 1, and *ii)* $G = 2$. *Note*: Only the qualitative aspects of the curve are necessary – you do not have to show numerical values.

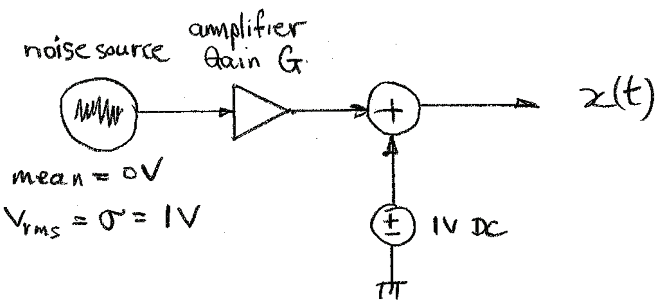

Figure 3: Schematic for Q3.

Continued on Page 4

Fourier Transform Pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T\text{sinc}\big(fT\big)$
$\text{sinc}\big(2Wt\big)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

A periodic rectangular wave in the time domain with amplitude A, period T and "ON" time T_o has Fourier coefficients c_n given by the expression $c_n = \frac{AT_o}{T}\text{sinc}\left(n\frac{T_o}{T}\right)$, where $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.

Trigonometric Identities

$$
\begin{aligned}
&\cos(\theta) = \tfrac{1}{2}\big[\exp(j\theta) + \exp(-j\theta)\big]\\
&\sin(\theta) = \tfrac{1}{2j}\big[\exp(j\theta) - \exp(-j\theta)\big]\\
&\sin^2(\theta) + \cos^2(\theta) = 1\\
&\cos^2(\theta) - \sin^2(\theta) = \cos(2\theta)\\
&\cos^2(\theta) = \tfrac{1}{2}\big[1 + \cos(2\theta)\big]\\
&2\sin(\theta)\cos(\theta) = \sin(2\theta)\\
&\sin(\alpha)\sin(\beta) = \tfrac{1}{2}\big[\cos(\alpha-\beta) - \cos(\alpha+\beta)\\
&\cos(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\cos(\alpha-\beta) + \cos(\alpha+\beta)\\
&\sin(\alpha)\cos(\beta) = \tfrac{1}{2}\big[\sin(\alpha-\beta) + \sin(\alpha+\beta)\\
&\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = \cos(\alpha+\beta)
\end{aligned}
$$

The End.