Electrical Engineering EE3TR4

Duration of test: 1.5 Hours

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This test paper includes 2 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this test. **No other aids are permitted**.

(b) There are 3 major questions. A full paper consists of all three.

(c) Marks for each question are indicated in parenthesis.

(d) **Make sure you read the test over in its entirety before you start!**

1. Consider the AM modulated waveform given by

$$s(t) = A_c \left[1 + 2\cos 2\pi f_m t\right] \cos 2\pi f_c t.$$

i) Sketch the spectrum of $s(t)$ showing all relevant values. (2)

ii) Sketch $s(t)$ showing all relevant values. (2)

iii) Sketch the output after passing $s(t)$ through an ideal envelope detector. Does this scheme recover the message without distortion? (3)

iv) Sketch the output after passing $s(t)$ through a DSB/SC demodulator. (i.e., $s(t)$ is multiplied by $\cos 2\pi f_c t$ and passed through an ideal low-pass filter with an appropriate cutoff frequency. Does this scheme recover the message without distortion? (3)

2. A square wave periodic signal with amplitude 1 and frequency 1 KHz is passed through a filter whose frequency response $H(f)$ is given by

$$H(f) = \frac{1}{1 + \frac{j2\pi f}{a}}$$

where the constant $a = 2\pi \times 1000$. The output of this first filter is then fed into a second filter which is an ideal lowpass filter whose cutoff frequency is 2 KHz. Give a closed-form expression for the output of the second filter, giving values to all parameters. ("Closed–form" means give a mathematical expression describing the output). (10 marks)

3. Answer each of the following sub-questions : (2 marks each)

i) What is the Hilbert transform of $\cos 2\pi f_m t$?

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ii) Consider question 1 above. Suppose k_a has the value 1 instead of 2. Will the DSB/SC demodulation scheme recover the message without distortion? Explain.

iii) Give an expression for a single sideband (SSB) modulated signal when the message is a cosine at frequency f_m and the carrier is a cosine at f_c. Use the lower sideband.

iv) The signal $x(t) = 2W\text{sinc}2Wt$, where W is a constant, is multiplied by $c(t) = cos2\pi f_o t$ to give $y(t)$. Sketch $Y(f)$.

v) Give an expression for the complex envelope corresponding to the SSB signal in part iii) above.

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}\big(\text{fT}\big)$
$\text{sinc}\big(2Wt\big)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f - f_c) + \delta(f + f_c)\big]$

Trigonometric Identities

$$\cos A \cos B - \sin A \sin B = \cos(A + B)$$
$$\cos^2 A = \frac{1}{2}\left[1 + \cos 2A\right]$$
$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

The End.