Electrical Engineering EE3TR4

Duration of test: 1.5 Hours

Instructor: Dr. J. Reilly
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This test paper includes 2 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this test. **No other aids are permitted**.
(b) There are 3 major questions. A full paper consists of all three.
(c) Marks for each question are indicated in parenthesis.
(d) **Make sure you read the test over in its entirety before you start!**
(e) Make sure you work in pen, and do not use white-out, if you would like your paper to be considered for remarking.

1. An ideal 1000Hz square wave with amplitude one and a duty cycle of 50% (i.e., it is on for 50% of the time) is applied at the input of a linear time–invariant filter whose transfer function $H(j2\pi f)$ is given by

$$H(j2\pi f) = \frac{2}{\left(\frac{j2\pi f}{2\pi 1000}\right)^2 + \left(\frac{j4\pi f}{2\pi 1000}\right) - 1} \tag{1}$$

a. Sketch the spectrum of the input signal showing all relevant values. (4 marks)
b. Calculate the amplitude and phase of the spectral component at the output of the filter at frequency 1000 Hz. (4 marks)
c. repeat part **b.** above, except for the spectral component at frequency 1500 Hz. (2 marks)

2. Sketch the Fourier transforms (i.e., the spectra) of the time domain signals in the accompanying Figures 1 and 2. Note that the signal in Figure 2 is periodic. Show all relevant values. (5 marks each).

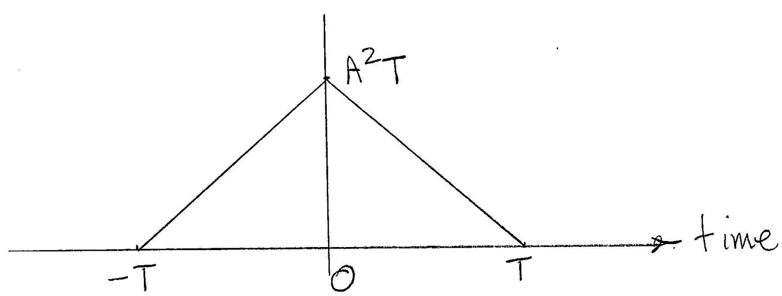

Figure 1: Spectrum 1 for Question 2.

Continued on Page 2

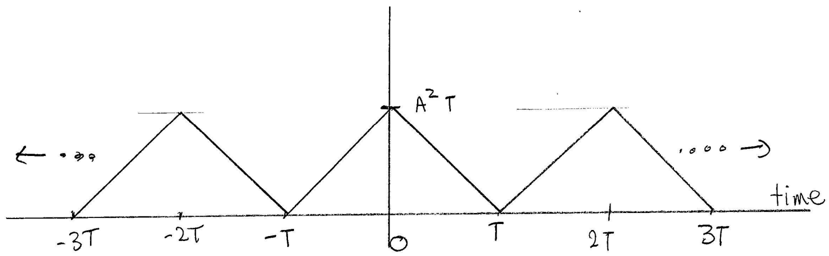

Figure 2: Spectrum 2 for Question 2.

3. a. Consider an AM modulated wave $s(t)$ whose message signal is given as $m(t) = A_m \cos(2\pi f_m t)$. Sketch the spectrum $S(f)$ of the resulting wave $s(t)$, for 125% modulation showing all relevant values, using a carrier amplitude value $A_c = 2$. Percent modulation is defined as $\max |k_a m(t)| \times 100\%$. (5 marks)

b. This message cannot be recovered without distortion using an envelope detector. Carefully describe (a perhaps somewhat more expensive) method for demodulating the signal that is distortion free. *Hint* Do not use an envelope detector. (5 marks)

Fourier Transform Pairs

Time Function	**Fourier Transform**
$\text{rect}\left(\frac{t}{T}\right)$	$\text{Tsinc}(\text{fT})$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f - f_c)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\|t\|), \quad a > 0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$

The Fourier components of a periodic square wave of amplitude A, fundamental frequency f_o, and "on" time T have amplitudes given by $\frac{A}{2}\text{sinc}(n f_o T)$.

Trigonometric Identities

$$\cos A \cos B - \sin A \sin B = \cos(A + B)$$
$$\cos^2 A = \frac{1}{2}\left[1 + \cos 2A\right]$$
$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

The End.