Electrical Engineering EE3TR4

Midterm test: 1.5 Hours

Instructor: Dr. J. Reilly
February, 2013

This examination paper includes 4 pages and 3 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions

(a) The McMaster Standard Calculator (Casio FX991) is the only calculator approved for this exam. **No other aids are permitted**.

(b) There are 3 questions. Attempt all three.

(c) You must show your work for full marks.

(d) Make sure you read the entire paper over in its entirety before you start!

(e) If you want your paper to be considered for remarking, then avoid pencil and white-out.

(f) The tables of Fourier transforms and trigonometric identities at the back of this exam may be useful.

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1. The message signal shown in Figure 1 is amplitude modulated onto a carrier.

a. draw the corresponding modulated waveform $s(t)$ with 100% modulation. Assume the carrier amplitude $A_c = 1$. (2 marks)

b. draw the magnitude spectrum $S(f)$ of the modulated waveform above. Show all relevant values. (3 marks)

c. repeat parts **a.** and **b.** using 50% modulation. (5 marks)

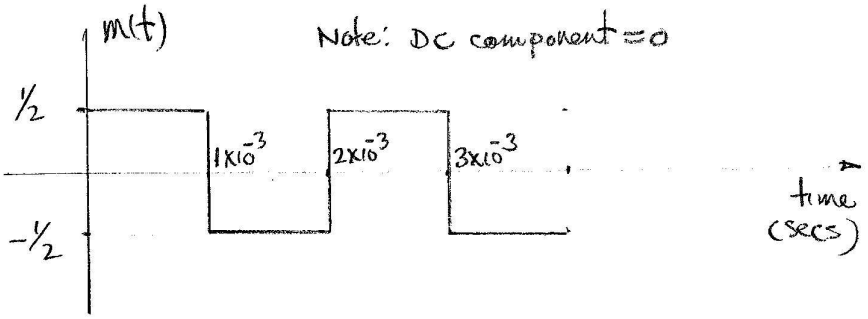

Figure 1: Message signal for Question 1.

2. Consider the configuration shown in Figure 2 below, with inputs to the upper and lower branches being $\cos(2\pi f_m t)$ and $\sin(2\pi f_m t)$ respectively. (each following sub-question is 2 marks)

a. Give a mathematical expression in its simplest form for the modulated waveform $s(t)$ at the modulator output.

b. Draw the corresponding spectrum $S(f)$.

c. What type of modulation does this system represent?

d. What is the complex envelope corresponding to the input signal?

e. Sketch the waveform at point A in the figure.

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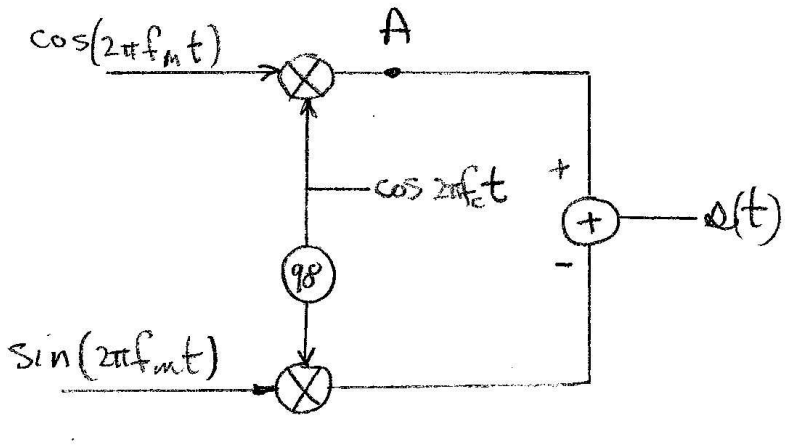

Figure 2: Modulation configuration for Question 2.

3. The signal $x(t) = 2\cos[2\pi(1.5)t] + \cos[2\pi(0.5)t]$ is applied to the input of a filter, with frequency response $H(s)$ given by

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \tag{1}$$

where $s = \frac{if}{f_o}$, where $f_o = 1$ Hz. Write an expression for the output $y(t)$ of the filter.

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Fourier Transform Pairs

Time Function	Fourier Transform
$\mathrm{rect}\left(\frac{t}{T}\right)$	$\mathrm{Tsinc}\big(\mathrm{fT}\big)$
$\mathrm{sinc}\big(2Wt\big)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
$\exp(2\pi f_c t)$	$\delta(f-f_c)$
$\exp(-at)u(t),\ \ a>0$	$\frac{1}{a+j2\pi f}$
$\exp(-a\lvert t\rvert),\ \ a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\delta(t)$	1
1	$\delta(f)$
$\cos(2\pi f_c t)$	$\frac{1}{2}\big[\delta(f-f_c)+\delta(f+f_c)\big]$

Trigonometric Identities

$$\cos(\theta)=\tfrac{1}{2}\big[\exp(j\theta)+\exp(-j\theta)\big]$$
$$\sin(\theta)=\tfrac{1}{2j}\big[\exp(j\theta)-\exp(-j\theta)\big]$$
$$\sin^2(\theta)+\cos^2(\theta)=1$$
$$\cos^2(\theta)-\sin^2(\theta)=\cos(2\theta)$$
$$\cos^2(\theta)=\tfrac{1}{2}\big[1+\cos(2\theta)\big]$$
$$2\sin(\theta)\cos(\theta)=\sin(2\theta)$$
$$\sin(\alpha)\sin(\beta)=\tfrac{1}{2}\big[\cos(\alpha-\beta)-\cos(\alpha+\beta)$$
$$\cos(\alpha)\cos(\beta)=\tfrac{1}{2}\big[\cos(\alpha-\beta)+\cos(\alpha+\beta)$$
$$\sin(\alpha)\cos(\beta)=\tfrac{1}{2}\big[\sin(\alpha-\beta)+\sin(\alpha+\beta)$$

The End.