<u>Explanation of Lab 1.</u> J Reilly Jan. 2019

The complex form of the Fourier series is discussed

Let $g_p(t)$ be a real periodic signal with period T and let $f_o = 1/T$ be the fundamental freq.

Then Fourier theory says that $g_p(t)$ can be expressed as

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j 2\pi n f_o t) \qquad (1)$$

where $$c_n = \frac{1}{T}\int_{-T/2}^{T/2} g_p(t) \exp(-j 2\pi n f_o t)\, dt$$

In the case when $g_p(t)$ is pure real, then it is straightforward to show that

$$c_{-n} = c_n^* .$$

let $c_n = r_n e^{j\theta_n}$ (polar form)

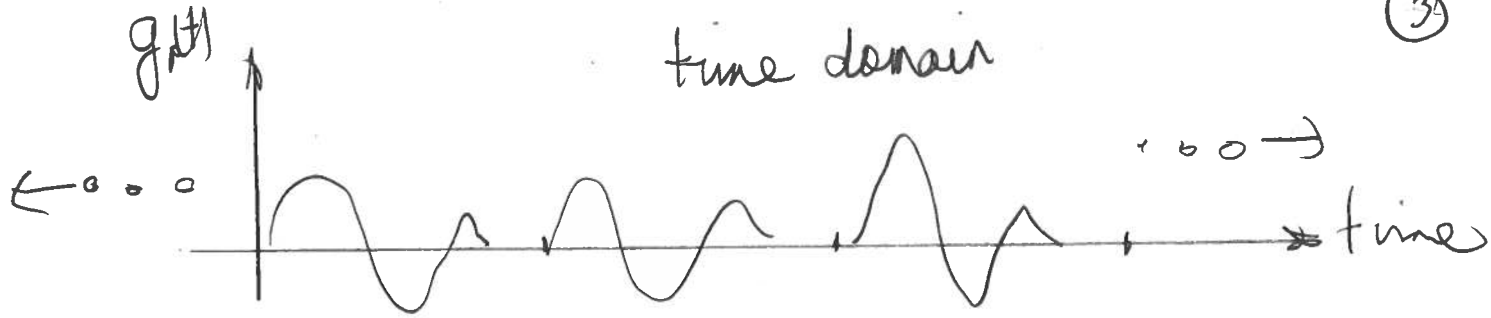
then $c_{-n} = r_n e^{-j\theta_n}$ (2)

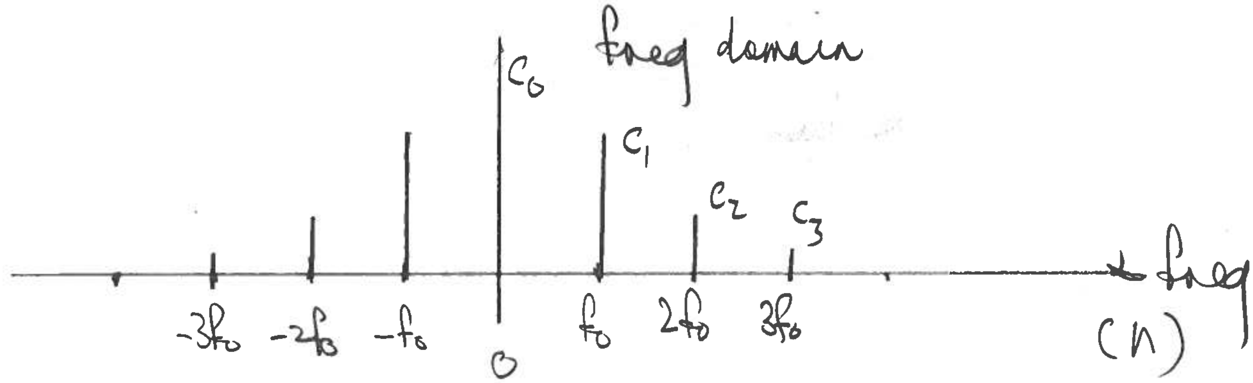
Using (2) in (1) and separating out the $n=0$ (DC) term, we can write

$$g_p(t) = c_0 + 2\sum_{n=1}^{\infty} \frac{r_n}{2}\Big[\exp(j2\pi n f_0 t + \theta_n) + \exp(-j2\pi n f_0 t + \theta_n)\Big]$$

$$= c_0 + 2\sum_{n=1}^{\infty} r_n \cos(2\pi n f_0 t + \theta_n) \quad (3)$$

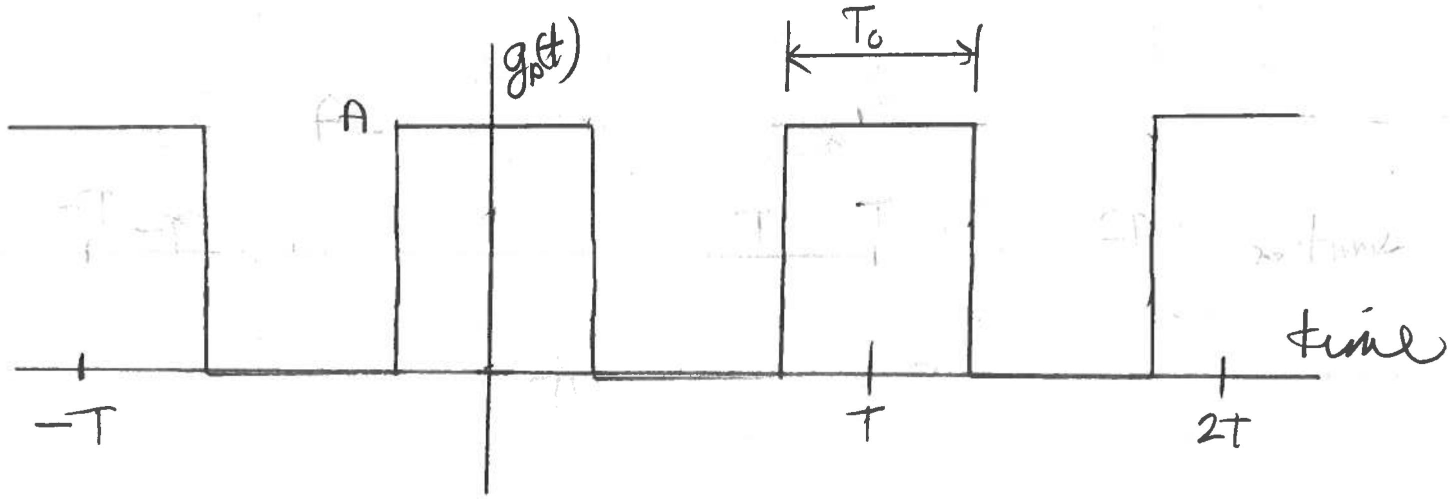
Thus, a real periodic signal may be expressed as a superposition of harmonically related sinusoids, at multiples of f_0.





The freq domain is discrete – spectrum exists only at multiples of f_0. Each freq. component is weighted by the complex value c_n.

<u>Example:</u> $g_p(t)$ is a periodic square wave (period T)



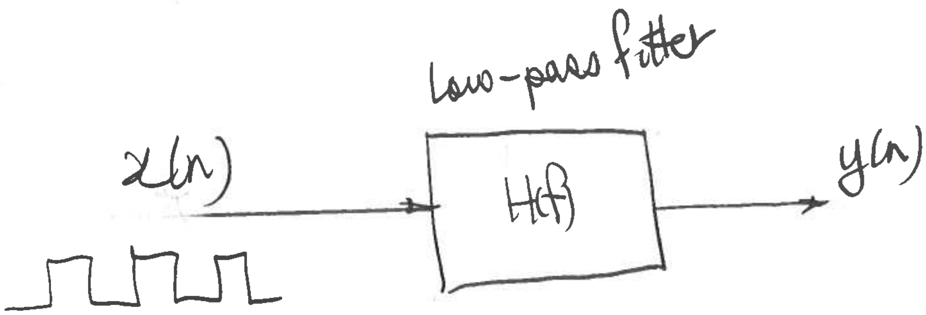
Let $\frac{T_0}{T} \triangleq \delta = 1/2$ (duty cycle) in this case.

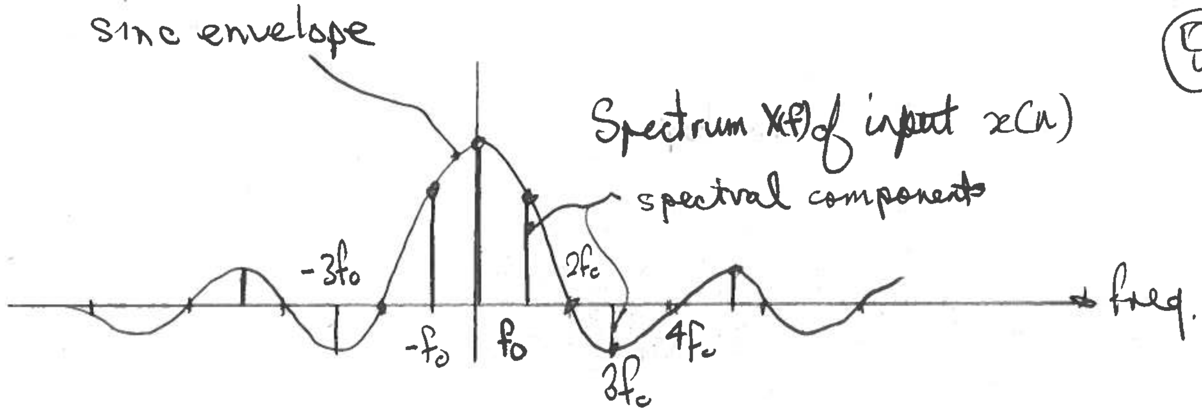
It is straightforward to show that the c_n in (1) in this case evaluates to

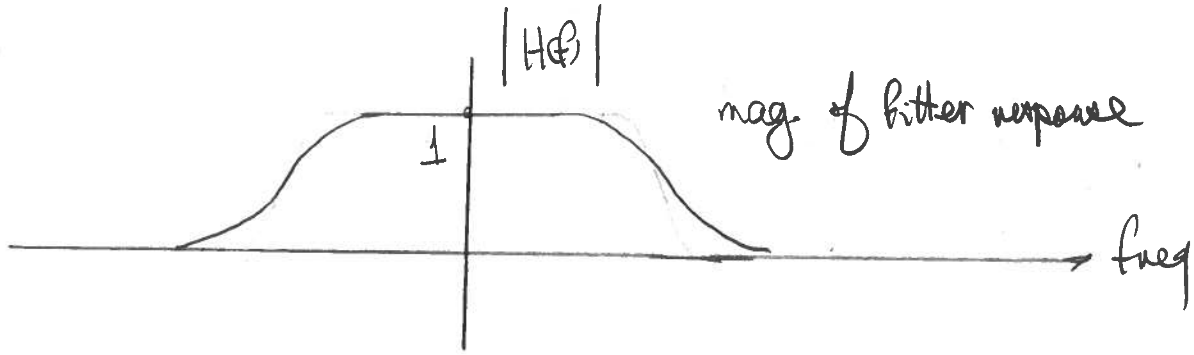
$$c_n = A\delta \, \mathrm{sinc}(\delta n) = \frac{A}{2} \mathrm{sinc}(n/2)$$

which are pure real in this case $(\theta_n = 0)$.

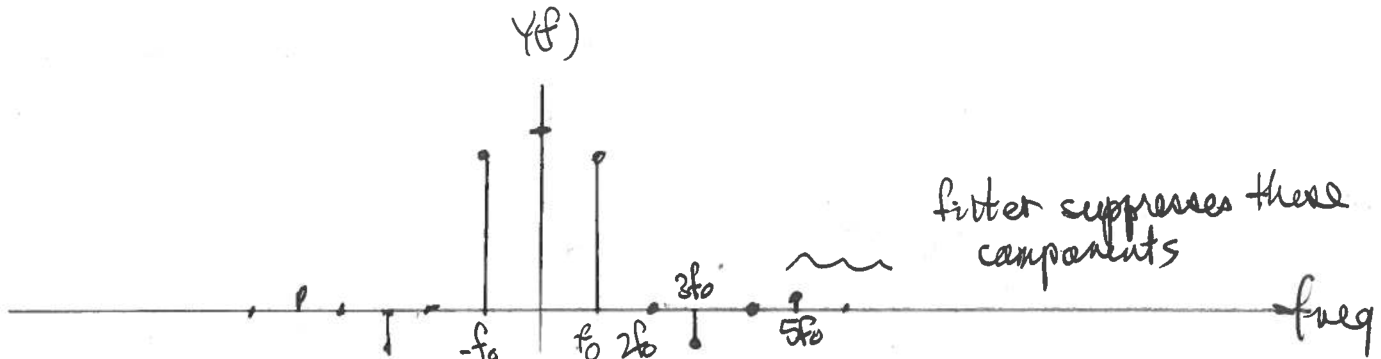
What happens when the square wave is put through a filter







The output (discrete) spectrum will be the point-by-point mult. of X(f) & H(f) above:



all components beyond the 3rd harmonic are attenuated by the filter.

Not shown is the phase shift. Each freq component is shifted in phase by the phase response of the filter ie phase shift $\theta_n = \measuredangle H(nf_0)$.

Once $Y(f)$ is known, then we write each freq. component $Y(nf_0)$ of $Y(f)$ in polar form as:

$$Y(nf_0) = r_n e^{j\theta_n}$$

These r_n and θ_n values are substituted into (3) to give the periodic output waveform $y(t)$.