(Q2) Find the minimum value of

$$f(x,y) = (x-3)^2 + (y-2)^2$$

Starting at $x=1$ and $y=1$, using the steepest descent method with a stopping criterion of $\varepsilon_s = 1\%$.

Solution: $\frac{\partial f}{\partial x} = 2(x-3)$, $\frac{\partial f}{\partial y} = 2(y-2)$

$$\Rightarrow \vec{\nabla f} = 2(x-3)\vec{i} + 2(y-2)\vec{j}$$

$$(x_0,y_0): \quad g(\alpha) = f\left(x_0 + \left.\frac{\partial f}{\partial x}\right|_{(x_0,y_0)} \cdot \alpha ,\ y_0 + \left.\frac{\partial f}{\partial y}\right|_{(x_0,y_0)} \cdot \alpha\right)$$

$$= f\left(x_0 + 2(x_0-3)\cdot\alpha ,\ y_0 + 2(y_0-2)\alpha\right)$$

$$= (x_0 + 2(x_0-3)\alpha - 3)^2 + (y_0 + 2(y_0-2)\alpha - 2)^2 , \quad x_0 = 1,\ y_0 = 1$$

$$= (-2-4\alpha)^2 + (-1-2\alpha)^2$$

$$\Rightarrow g'(\alpha) = 2(-4)(-2-4\alpha) + 2(-2)(-1-2\alpha) = 0$$

$$\Rightarrow -8(-2-4\alpha) = 4(-1-2\alpha) \Rightarrow \boxed{\alpha = -0.5}$$

$$\alpha^* = \alpha = -0.5$$

$$\Rightarrow x_1 = x_0 + 2(x_0-3)\alpha^* = 1 - 4(-0.5) = 1+2 = 3$$

$$y_1 = y_0 + 2(y_0-2)\alpha^* = 1 + 2(1-2)(-0.5) = 1+1 = 2$$

$\Rightarrow x_1 = 3$, $y_1 = 2$, and we iterate using these new values for x and y ....