A Linear Programming Approach for Optimal Contrast-Tone Mapping

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***Abstract*—This paper proposes a novel algorithmic approach of image enhancement via optimal contrast-tone mapping. In a fundamental departure from the current practice of histogram equalization for contrast enhancement, the proposed approach maximizes expected contrast gain subject to an upper limit on tone distortion and optionally to other constraints that suppress artifacts. The underlying contrast-tone optimization problem can be solved efficiently by linear programming. This new constrained optimization approach for image enhancement is general, and the user can add and fine tune the constraints to achieve desired visual effects. Experimental results demonstrate clearly superior performance of the new approach over histogram equalization and its variants.**

***Index Terms*—Contrast enhancement, dynamic range, histogram equalization, linear programming, tone reproduction.**

I. Introduction

In MOST image and video applications it is human viewers that make the ultimate judgement of visual quality. They typically associate high image contrast with good image quality. Indeed, a noticeable progress in image display and generation (both acquisition and synthetic rendering) technologies is the increase of dynamic range and associated image enhancement techniques [1].

The contrast of a raw image can be far less than ideal, due to various causes such as poor illumination conditions, low quality inexpensive imaging sensors, user operation errors, media deterioration (e.g., old faded prints and films), etc. For improved human interpretation of image semantics and higher perceptual quality, contrast enhancement is often performed and it has been an active research topic since early days of digital image processing, consumer electronics and computer vision.

Contrast enhancement techniques can be classified into two approaches: context-sensitive (point-wise operators) and context-free (point operators). In context-sensitive approach the contrast is defined in terms of the rate of change in intensity between neighboring pixels. The contrast is increased by directly altering the local waveform on a pixel by pixel basis. For instance, edge enhancement and high-boost filtering belong to the context-sensitive approach. Although intuitively appealing, the context-sensitive techniques are prone to artifacts such as ringing and magnified noises, and they cannot preserve the rank consistency of the altered intensity levels. The context-free contrast enhancement approach, on the other hand, does not adjust the local waveform on a pixel by pixel basis. Instead, the class of context-free contrast enhancement techniques adopt a statistical approach. They manipulate the histogram of the input image to separate the gray levels of higher probability further apart from the neighboring gray levels. In other words, the context-free techniques aim to increase the average difference between any two altered input gray levels. Compared with its context-sensitive counterpart, the context-free approach does not suffer from the ringing artifacts and it can preserve the relative ordering of altered gray levels.

Despite more than half a century of research on contrast enhancement, most published techniques are largely ad hoc. Due to the lack of a rigorous analytical approach to contrast enhancement, histogram equalization seems to be a folklore synonym for contrast enhancement in the literature and in textbooks of image processing and computer vision. The justification of histogram equalization as a contrast enhancement technique is heuristic, catering to an intuition. Low contrast corresponds to a biased histogram and, thus, can be rectified by reallocating underused dynamic range of the output device to more probable pixel values. Although this intuition is backed up by empirical observations in many cases, the relationship between histogram and contrast has not been precisely quantified.

No mathematical basis exists for the uniformity or near uniformity of the processed histogram to be an objective of contrast enhancement in general sense. On the contrary, histogram equalization can be detrimental to image interpretation if carried out mechanically without care. In lack of proper constraints histogram equalization can over shoot the gradient amplitude in some narrow intensity range(s) and flatten subtle smooth shades in other ranges. It can bring unacceptable distortions to image statistics such as average intensity, energy, and covariances, generating unnatural and incoherent 2-D waveforms. To alleviate these shortcomings, a number of different techniques were proposed to modify the histogram equalization algorithm [2]–[7]. This line of investigations was initiated by Pisano *et al.* in their work of contrast-limited adaptive histogram equalization (CLAHE) [8]. Somewhat ironically, these authors had to limit contrast while pursuing contrast enhancement. Recently, Arici *et al.* proposed to generate an intermediate histogram $\mathbf{h}$ in between the original input histogram $\mathbf{h}_i$ and the uniform histogram $\mathbf{u}$ and then performs histogram equalization

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of $\mathbf{h}$. The in-between histogram h is computed by minimizing a weighted distance $\|\mathbf{h} - \mathbf{h}_i\| + \lambda\|\mathbf{h} - \mathbf{u}\|$. The authors showed that undesirable side effects of histogramequalization can be suppressed via choosing the Lagrangian multiplier λ. This latest paper also gave a good synopses of existing contrast enhancement techniques. We refer the reader to [9] for a survey of previous works, instead of reparaphrasing them here.

Compared with the aforementioned works on histogram-based contrast enhancement techniques, this paper presents a more rigorous study of the problem. We reexamine contrast enhancement in a new perspective of optimal allocation of output dynamic range constrained by tune continuity. This brings about a more principled approach of image enhancement. Our critique of the current practice is that directly manipulating histograms for contrast enhancement was ill conceived. The histogram is an unwieldy, obscure proxy for contrast. The wide use of histogram equalization as a means of context-free contrast enhancement is apparently due to the lack of a proper mathematical formulation of the problem. To fill this void we define an expected (context-free) contrast gain of a transfer function. This relative measure of contrast takes on its base value of one if the input image is left unchanged (i.e., identity transfer function), and increases if a skewed histogram is made more uniform. However, perceptual image quality is more than the single aspect of high contrast. If the output dynamic range is less than that of the human visual system, which is the case for most display and printing technologies, context-free contrast enhancement will inevitably distort subtle tones. To balance between tone subtlety and contrast enhancement we introduce a counter measure of tone distortion. Based upon the said measures of contrast gain and tone distortion, we formulate the problem of optimal contrast-tone mapping (OCTM) that aims to achieve high contrast and subtle tone reproduction at the same time, and propose a linear programming strategy to solve the underlying constrained optimization problem. In the OCTM formulation, the optimal transfer function for images of uniform histogram is the identify function. Although an image of uniform histogram cannot be further enhanced, histogram equalization does not produce OCTM solutions in general for arbitrary input histograms. Instead, the proposed linear programming-based OCTM algorithm can optimize the transfer function such that sharp contrast and subtle tone are best balanced according to application requirements and user preferences. The OCTM technique offers a greater and more precise control of visual effects than existing techniques of contrast enhancement. Common side effects of contrast enhancement, such as contours, shift of average intensity, over exaggerated gradient, etc., can be effectively suppressed by imposing appropriate constraints in the linear programming framework.

In addition, in the OCTM framework, L input gray levels can be mapped to an arbitrary number Ł of output gray levels, allowing Ł to be equal, less or greater than L. The OCTM technique is, therefore, suited to output conventional images on high dynamic range displays or high dynamic range images on conventional displays, with perceptual quality optimized for device characteristics and image contents. As such, OCTM can be useful tool in high dynamic range imaging. Moreover, OCTM can be unified with Gamma correction.

Analogously to global and local histogram equalization, OCTM can be performed based upon either global or local statistics. However, in order to make our technical developments in what follows concrete and focused, we will only discuss the problem of contrast enhancement over an entire image instead of adapting to local statistics of different subimages. All the results and observations can be readily extended to locally adaptive contrast enhancement.

The remainder of the paper is organized as follows. In the next section, we introduce some new definitions related to the intuitive notions of contrast and tone, and propose the OCTM approach of image enhancement. In Section III, we develop a linear programming algorithm to solve the OCTM problem. In Section IV, we discuss how to fine tune output images according to application requirements or users' preferences within the proposed linear programming framework. Experimental results are reported in Section V, and they demonstrate the versatility and superior visual quality of the new contrast enhancement technique.

II. Contrast and Tone

Consider a gray scale image I of b bits with a histogram $\mathbf{h}$ of K nonzero entries, $x_0 < x_1 < \cdots < x_{K-1}, 0 < K \leq L = 2^b$. Let p_k be the probability of gray level x_k, $0 \leq k < K$. We define the expected context-free contrast of I by

$$C(\mathbf{p}) = p_0(x_1 - x_0) + \sum_{1 \leq k < K} p_k(x_k - x_{k-1}). \tag{1}$$

By the definition, the maximum contrast $C_{\max} = L - 1$ and it is achieved by a binary black-and-white image $x_0 = 0, x_1 = L - 1$; the minimum contrast $C_{\min} = 0$ when the image is a constant. As long as the histogram of I is full without holes, i.e., $K = L, x_k - x_{k-1} = 1, 0 \leq k < L, C(\mathbf{p}) = 1$ regardless the intensity distribution $(p_0, p_1, \ldots, p_{L-1})$. Likewise, if $x_k - x_{k-1} = d > 1$, $0 \leq k < K < L$, then $C(\mathbf{p}) = d$.

Contrast enhancement is to increase the difference between two adjacent gray levels and it is achieved by a remapping of input gray levels to output gray levels. Such a remapping is also necessary when reproducing a digital image of L gray levels by a device of Ł gray levels, $L \neq$ Ł. This process is an integer-to-integer transfer function

$$T : \{0, 1, \ldots, L-1\} \rightarrow \{0, 1, \ldots, \text{Ł} - 1\}. \tag{2}$$

In order not to violate physical and psychovisual common sense, the transfer function T should be monotonically nondecreasing such that T does not reverse the order of intensities.[1] In other words, we must have $T(j) \geq T(i)$ if $j > i$ and, hence, any transfer function T has the form

$$\begin{gathered} T(i) = \sum_{0 \leq j \leq i} s_j, \; 0 \leq i < L \\ s_j \in \{0, 1, \ldots, \text{Ł} - 1\} \\ \sum_{0 \leq j < L} s_j < \text{Ł}. \end{gathered} \tag{3}$$

[1]This restriction may be relaxed in locally adaptive contrast enhancement. But in each locality the monotonicity should still be imposed.

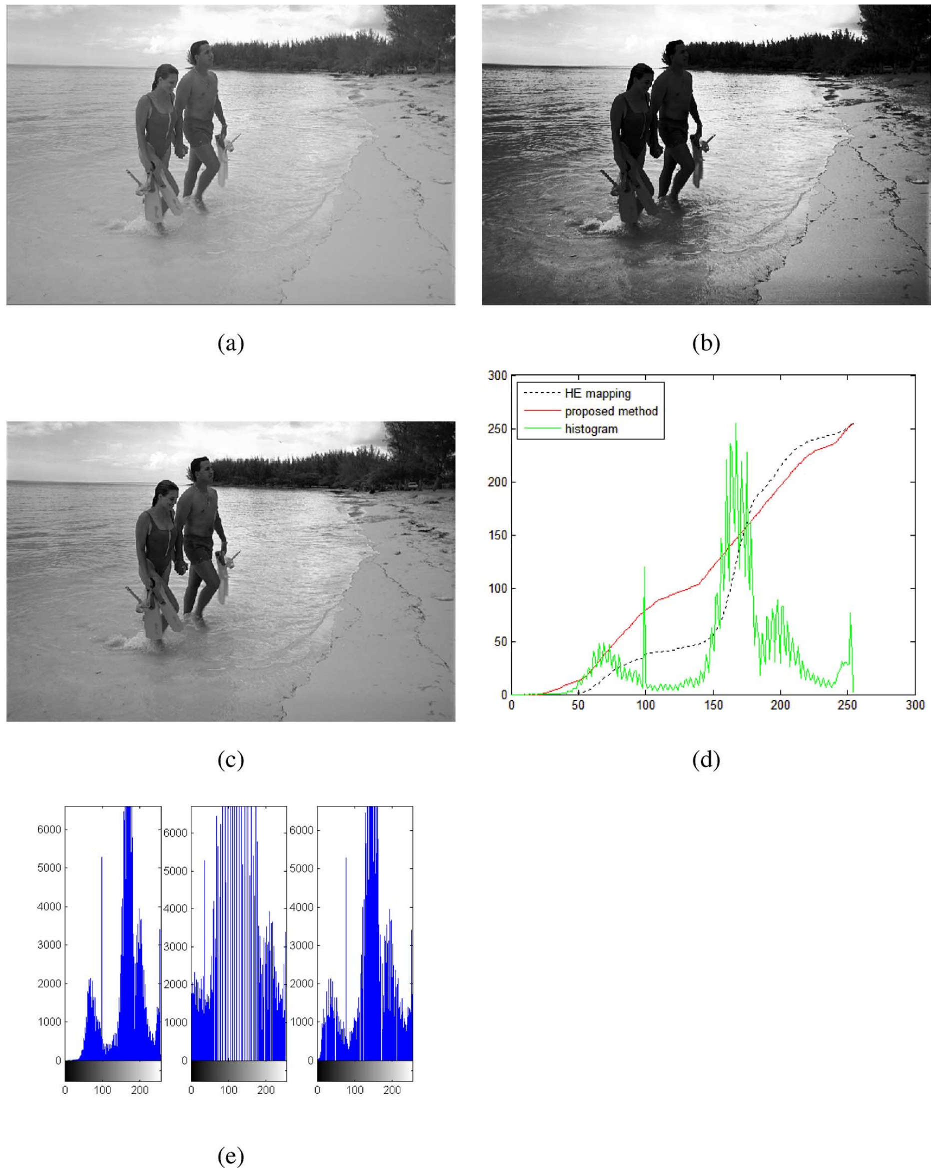


Fig. 1. (a) Original. (b) Output of histogram equalization. (c) Output of the proposed OCTM method. (d) Transfer functions and the original histogram. (e) Histograms of the original image (left), the output image of histogram equalization (middle), and the output image of OCTM.

where s_j is the increment in output intensity versus a unit step up in input level j (i.e., $x_j - x_{j-1} = 1$), and the last inequality ensures the output dynamic range not exceeded by $T(i)$.

In (3), s_j can be interpreted as context-free contrast at level j, which is the rate of change in output intensity without considering the pixel context. Note that a transfer function is completely determined by the vector $\mathbf{s} = (s_0, s_1, \ldots, s_{L-1})$, namely the set of contrasts at all L input gray levels. Having associated the transfer function T with context-free contrasts s_j's at different levels, we induce from (3) and definition (1) a natural measure of expected contrast gain made by T

$$G(\mathbf{s}) = \sum_{0 \le j < L} p_j s_j \tag{4}$$

where p_j is the probability that a pixel in I has input gray level j.

The previous measure conveys the colloquial meaning of contrast enhancement. To see this let us examine some special cases.

Proposition 1: The maximum contract gain $G(\mathbf{s})$ is achieved by $s_k = \text{Ł}-1$ such that $p_k = \max\{p_i | 0 \le i < L\}$, and $s_j = 0$, $j \neq k$.

Proof: Assume for a contradiction that $s_j = n > 0$, $j \neq k$, would achieve higher contrast gain. Due to the constraint $\sum_{0 \le j < L} s_j < \text{Ł}$, s_k equals at most $\text{Ł} - 1 - n$. But $p_j n + p_k(\text{Ł} - 1 - n) \le p_k(\text{Ł} - 1)$, refuting the previous assumption. ■

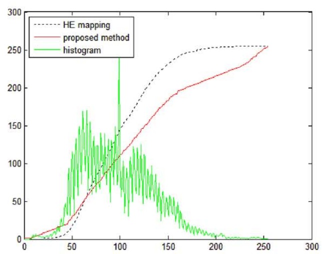
Proposition 1 reflects our intuition that the highest contrast is achieved when T achieves a single step (thresholding) black to white transition, converting the input image from gray scale to binary. The binary threshold is set at level k such that $p_k = \max\{p_i | 0 \le i < L\}$ to maximize contrast gain.

One can preserve the average intensity while maximizing the contrast gain. The average-preserving maximum contrast gain is achieved by $s_k = \text{Ł}-1$, $s_j = 0, j \neq k$, such that $\sum_{0 \le j < k} p_j \approx \sum_{k \le j < L} p_j$. Namely, $T(i)$ is the binary thresholding function at the average gray level.

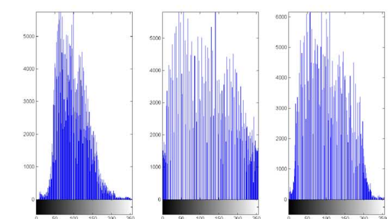
(a)

(b)

(c)

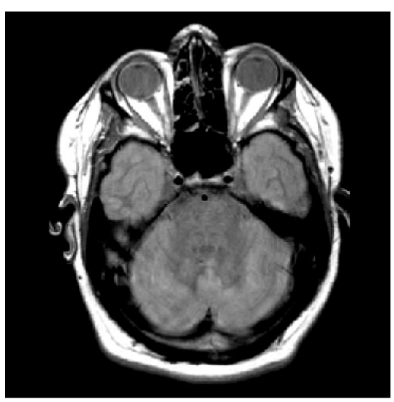


(d)

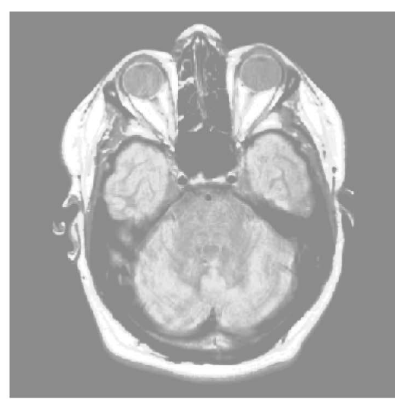


(e)

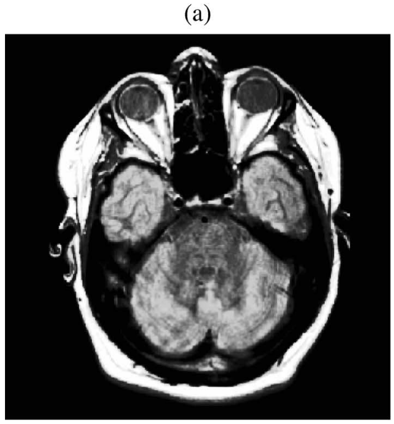
Fig. 2. (a) Original. (b) Output of histogram equalization. (c) Output of the proposed OCTM method. (d) Transfer functions and the original histogram. (e) Histograms of the original image (left), the output image of histogram equalization (middle), and the output image of OCTM.



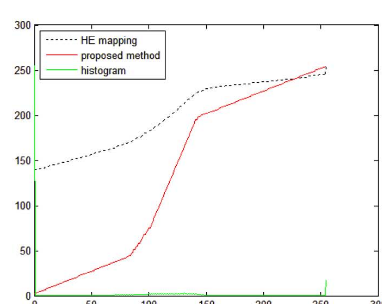
(a)



(b)



(c)



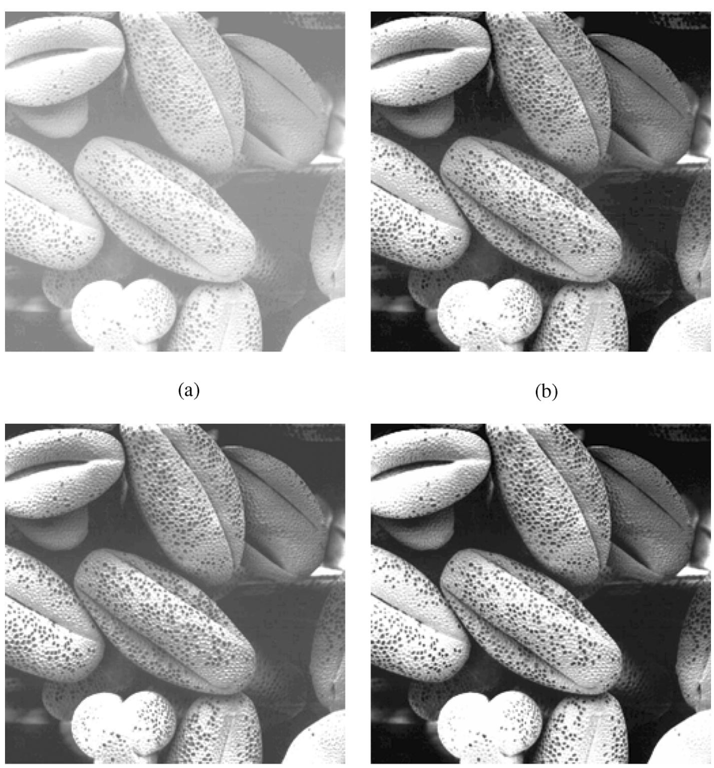
(d)

Fig. 3. (a) Original. (b) Output of histogram equalization. (c) Output of the proposed OCTM method. (d) Transfer functions and the original histogram.

If $L =$ Ł (i.e., when the input and output dynamic ranges are the same), the identity transfer function $T(i) = i$, namely, $s_i = 1, 0 \leq i < L$, makes $G(\mathbf{1}) = 1$ regardless the gray level distribution of the input image. In our definition, the unit con-

(a) (b) (c) (d)

Fig. 4. (a) Original image before Gamma correction. (b) After Gamma correction. (c) Gamma correction followed by histogram equalization. (d) Joint Gamma correction and contrast-tone optimization by the proposed OCTM method.



(a) (b) (c) (d)

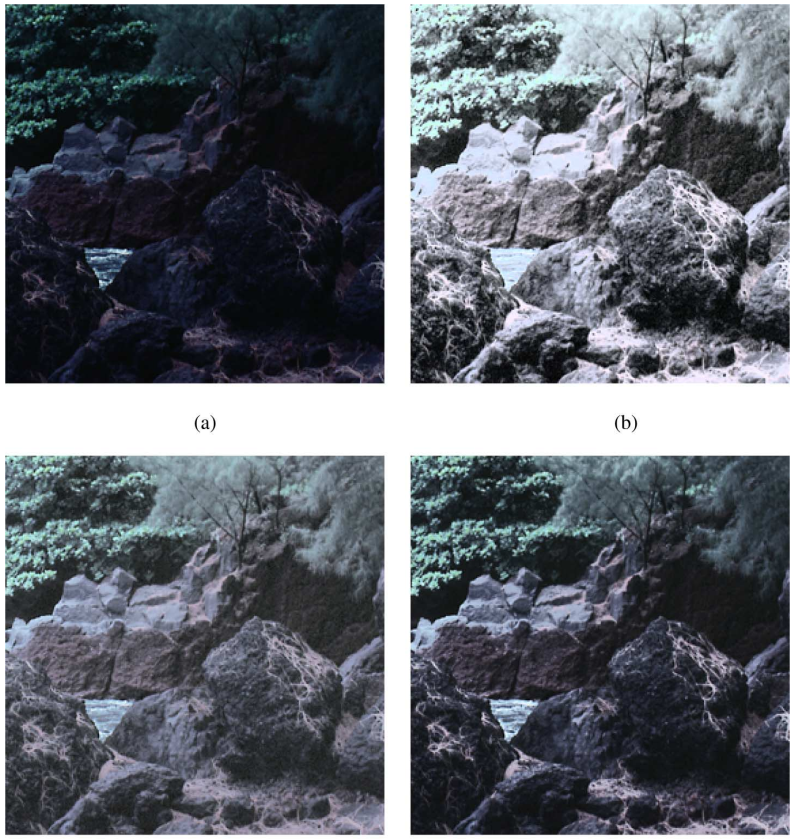
Fig. 5. Comparison of different methods on image Pollen. (a) Original image. (b) HE. (c) CLAHE. (d) OCTM.

trast gain means a neutral contrast level without any enhancement. The notion of neutral contrast can be generalized to the cases when $L \neq Ł$. We call $\tau = Ł/L$ the tone scale. In general, the transfer function

$$T(i) = \left\lfloor \frac{Ł - 1}{L - 1} i + 0.5 \right\rfloor, \ 0 \leq i < L \tag{5}$$

or equivalently $s_i = \tau$, $0 \leq i < L$, corresponds to the state of neutral contrast $G(\tau \mathbf{1}) = \tau$.

High contrast by itself does not equate high image quality. Another important aspect of image fidelity is the tone continuity. A single-minded approach of maximizing $G(\mathbf{s})$ would likely produce over-exaggerated, unnatural visual effects, as re-



(a) (b) (c) (d)

Fig. 6. Comparison of different methods on image Rocks. (a) Original image. (b) HE. (c) CLAHE. (d) OCTM.

vealed by Proposition 1. The resulting $T(i)$ degenerates a continuous-tone image to a binary image. This maximizes the contrast of a particular gray level but completely ignores accurate tone reproduction. We begin our discussions on the tradeoff between contrast and tone by stating the following simple and yet informative observation.

Proposition 2: The $\max \min s \quad \{s_0, s_1, \ldots, s_{L-1}\}$ is achieved if and only if $G(\tau \mathbf{1}) = \tau$, or $s_i = \tau$, $0 \leq i < L$.

As stated previously, the simple linear transfer function, i.e., doing nothing in the traditional sense of contrast enhancement, actually maximizes the minimum of context-free contrasts s_i of different levels $0 \leq i < L$, and the neutral contrast gain largest $G(\tau \mathbf{1}) = \tau$ is possible when satisfying this maxmin criterion.

In terms of visual effects, the reproduction of continuous tones demands the transfer function to meet the maxmin criterion of proposition 2. The collapse of distinct gray levels into one tends to create contours or banding artifacts. In this consideration, we define the tone distortion of a transfer function $T(i)$ by

$$D(\mathbf{s}) = \max_{1 \leq i,j \leq L} \{j - i \| T(i) = T(j); p_i > 0, p_j > 0\}. \quad (6)$$

In the definition we account for the fact that the transfer function $T(i)$ is not a one-to-one mapping in general. The smaller the tone distortion $D(\mathbf{s})$ the smoother the tone reproduced by $T(i)$. It is immediate from the definition that the smallest achievable tone distortion is

$$\tau = \min_{\mathbf{s}} D(\mathbf{s}).$$

However, since the dynamic range L of the output device is finite, the two visual quality criteria of high contrast and tone continuity are in mutual conflict. Therefore, the mitigation of such an inherent conflict is a critical issue in designing contrast enhancement algorithms, which is seemingly overlooked in the existing literature on the subject.

Following the previous discussions, the problem of contrast enhancement manifests itself as the following optimization problem

$$\max_{\mathbf{s}} \{G(\mathbf{s}) - \lambda D(\mathbf{s})\}. \quad (7)$$

The OCTM objective function (7) aims for sharpness of high frequency details and tone subtlety of smooth shades at the same time, using the Lagrangian multiplier λ to regulate the relative importance of the two mutually conflicting fidelity metrics.

Interestingly, the OCTM solution of (7) is $\mathbf{s} = \mathbf{1}$ if the input histogram of an image I is uniform. It is easy to verify that $G(\mathbf{s}) = 1$ for all $\mathbf{s}$ but $D(\mathbf{s}) = 0$ when $\mathbf{s} = \mathbf{1}$ for $p_0 = p_1 = \cdots = p_{L-1} = 1/L$. In other words, no other transfer functions can make any contrast gain over the identity transfer function $T(i) = i$ (or $\mathbf{s} = \mathbf{1}$), and at the same time the identity transfer function achieves the minimum tone distortion $D(\mathbf{1}) = \min_{\mathbf{s}} D(\mathbf{s}) = 0$. This concludes that an image of uniform histogram cannot be further enhanced in OCTM, lending a support for histogram equalization as a contrast enhancement technique. For a general input histogram, however, the transfer function of histogram equalization is not necessarily the OCTM solution, as we will appreciate in the following sections.

(a) (b) (c) (d)

Fig. 7. Comparison of different methods on image Tree. (a) Original image. (b) HE. (c) CLAHE. (d) OCTM.

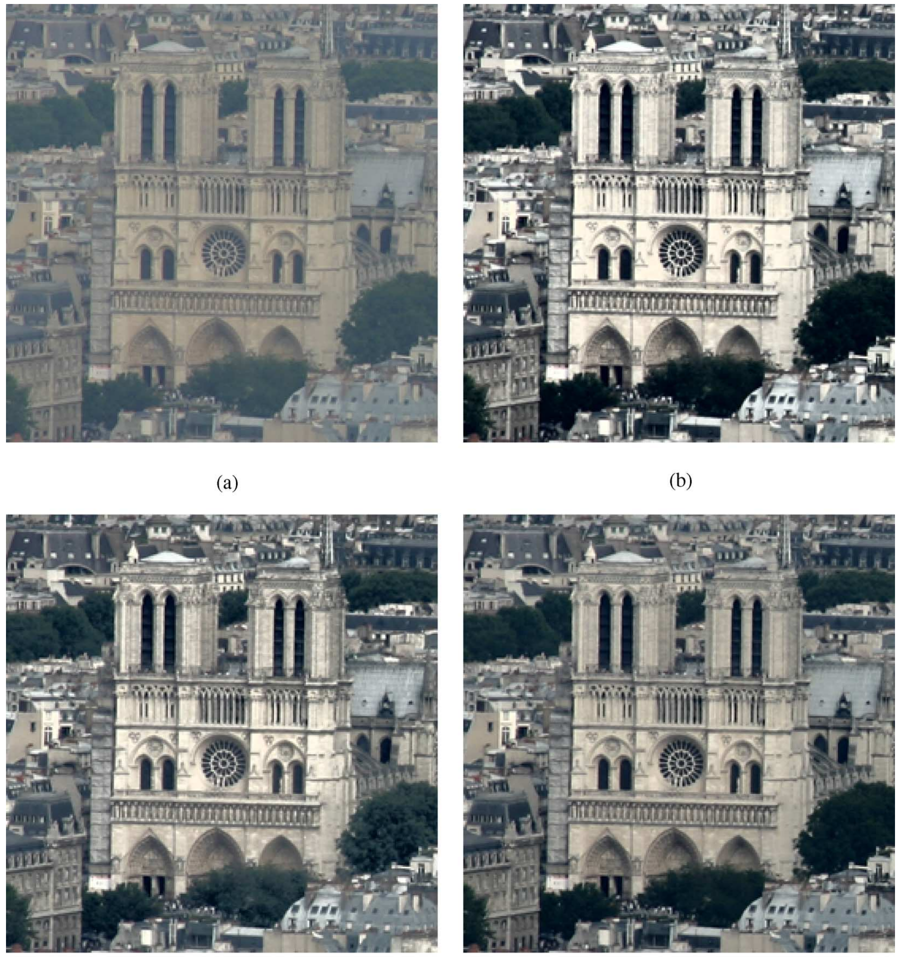
III. Contrast-Tone Optimization by Linear Programming

To motivate the development of an algorithm for solving (7), it is useful to view contrast enhancement as an optimal resource allocation problem with constraint. The resource is the output dynamic range and the constraint is tone distortion. The achievable contrast gain $G(\mathbf{s})$ and tone distortion $D(\mathbf{s})$ are physically confined by the output dynamic range Ł of the output device. In (4) the optimization variables $s_0, s_1, \ldots, s_{L-1}$ represent an allocation of Ł available output intensity levels, each competing for a larger piece of dynamic range. While contrast enhancement necessarily invokes a competition for dynamic range (an insufficient resource), a highly skewed allocation of Ł output levels to L input levels can deprive some input gray levels of necessary representations, incurring tone distortion. This causes unwanted side effects, such as flattened subtle shades, unnatural contour bands, shifted average intensity, and etc. Such artifacts were noticed by other researchers as drawbacks of the original histogram equalization algorithm, and they proposed a number of ad hoc. techniques to alleviate these artifacts by reshaping the original histogram prior to the equalization process. In OCTM, however, the control of undesired side effects of contrast enhancement is realized by the use of constraints when maximizing contrast gain $G(\mathbf{s})$.

Since the tone distortion function $D(\mathbf{s})$ is not linear in $\mathbf{s}$, directly solving (7) is difficult. Instead, we rewrite (7) as the following constrained optimization problem:

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0\le j<L} p_j s_j \\ &\text{subject to (a)} \sum_{0\le j<L} s_j < \text{Ł} \\ &\qquad\text{(b)}\ s_j \ge 0,\ 0 \le j < L \\ &\qquad\text{(c)} \sum_{j\le i<j+d} s_i \ge 1,\ 0 \le j < L-d. \end{aligned} \tag{8}$$

In (8), constraint (a) is to confine the output intensity level to the available dynamic range; constraints (b) ensure that the transfer function $T(i)$ be monotonically nondecreasing; constraints (c) specify the maximum tone distortion allowed, where d is an upper bound $D(\mathbf{s}) \le d$. The objective function and all the constraints are linear in $\mathbf{s}$. The choice of d depends upon user's re-



(a) (b) (c) (d)

Fig. 8. Comparison of different methods on image Notre Dame. (a) Original image. (b) HE. (c) CLAHE. (d) OCTM.

quirement on tone continuity. In our experiments, pleasing visual appearance is typically achieved by setting d to 2 or 3.

Computationally, the OCTM problem formulated in (8) is one of integer programming. This is because the transfer function $T(i)$ is an integer-to-integer mapping, i.e., all components of $\mathbf{s}$ are integers. But we relax the integer constraints on $\mathbf{s}$ and convert (8) to a linear programming problem. By the relaxation any solver of linear programming can be used to solve the real version of (8). The resulting real-valued solution $\mathbf{s} = (s_0, s_1, \ldots, s_{L-1})$ can be easily converted to an integer-valued transfer function

$$T(i) = \left\lfloor \sum_{0 \le j \le i} s_j + 0.5 \right\rfloor, \ 0 \le i < L. \tag{9}$$

For all practical considerations the proposed relaxation solution does not materially compromise the optimality. As a beneficial side effect, the linear programming relaxation simplifies constraint (c) in (8), and allows the contrast-tone optimization problem to be stated as

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0 \le j < L} p_j s_j \\ &\text{subject to} \sum_{0 \le j < L} s_j < \text{Ł}; \\ &\qquad s_j \ge 1/d, \ 0 \le j < L. \end{aligned} \tag{10}$$

IV. Fine Tuning of Visual Effects

The proposed OCTM technique is general and it can achieve desired visual effects by including additional constraints in (10). We demonstrate the generality and flexibility of the proposed linear programming framework for OCTM by some of many possible applications.

The first example is the integration of Gamma correction into contrast-tone optimization. The optimized transfer function $T(\mathbf{s})$ can be made close to the Gamma transfer function by adding to (10) the following constraint:

$$\sum_{0 \le i < L} \left| (L-1)^{-1} \sum_{0 \le j \le i} s_j - [i(L-1)^{-1}]^{\gamma} \right| \le \Delta \tag{11}$$

where γ is the Gamma parameter and Δ is the degree of closeness between the resulting $T(\mathbf{s})$ and the Gamma mapping $[i(L-1)^{-1}]^{\gamma}$.

In applications when the enhancement process cannot change the average intensity of the input image by certain amount Δ_μ, the user can impose this restriction easily in (10) by adding another linear constraint

$$\left| \frac{L}{\text{Ł}} \sum_{0 \le i < L} p_i \sum_{0 \le j \le i} s_j - \sum_{0 \le i < L} p_i i \right| \le \Delta_\mu. \tag{12}$$

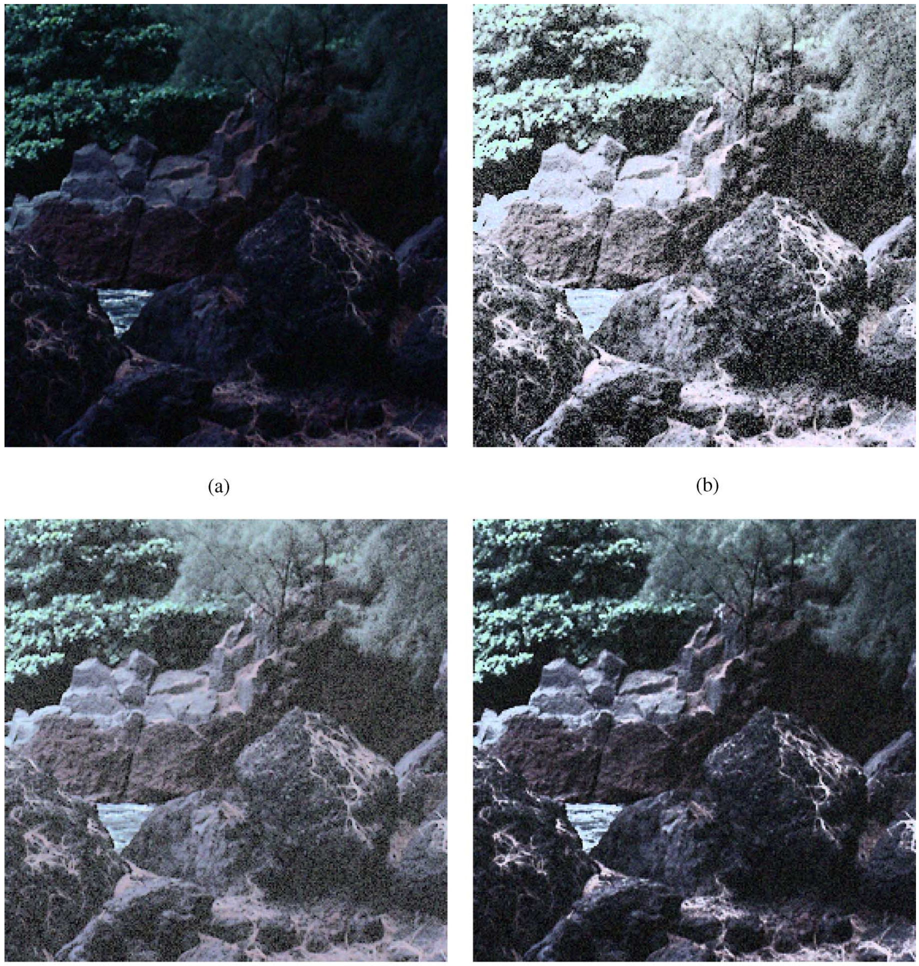


Fig. 9. Results of different methods on image Rocks of noise, to be compared with those in Fig. 6. (a) Noisy image. (b) HE. (c) CLAHE. (d) OCTM.

Besides the use of constraints in the linear programming framework, we can incorporate context-based or semantics-based fidelity criteria directly into the OCTM objective function. The contrast gain $G(\mathbf{s}) = \sum p_j s_j$ depends only upon the intensity distribution of the input image. We can augment $G(\mathbf{s})$ by weighing in the semantic or perceptual importance of increasing the contrast at different gray levels by w_j, $0 \le j < L$. In general, w_j can be set up to reflect specific requirements of different applications. In medical imaging, for example, the physician can read an image of L gray levels on an Ł-level monitor, $\text{Ł} < L$, with a certain range of gray levels $j \in [j_0, j_1] \subset [0, L)$ enhanced. Such a weighting function presents itself naturally if there is a preknowledge that the interested anatomy or lesion falls into the intensity range $[j_0, j_1]$ for given imaging modality. In combining image statistics and domain knowledge or/and user preference, we introduce a weighted contrast gain function

$$G_w(\mathbf{s}) = \sum_{0 \le j < L} p_j s_j + \lambda \sum_{0 \le j < L} w_j s_j \tag{13}$$

where λ is a Lagrangian multiplier to factor in user-prioritized contrast into the objective function.

In summarizing all of the previous discussions, we finally present the following general linear programming framework for visual quality enhancement

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0 \le j < L} (p_j + \lambda w_j) s_j \\ &\text{subject to} \sum_{0 \le j < L} s_j < \text{Ł} \\ &\qquad s_j \ge 1/d,\ 0 \le j < L; \\ &\qquad \sum_{0 \le i < L} \left| (\text{Ł} - 1)^{-1} \sum_{0 \le j \le i} s_j - [i(L-1)^{-1}]^{\gamma} \right| \le \Delta \\ &\qquad \left| \frac{L}{\text{Ł}} \sum_{0 \le i < L} p_i \sum_{0 \le j \le i} s_j - \sum_{0 \le i < L} p_i i \right| \le \Delta_\mu. \end{aligned} \tag{14}$$

In this paper, we focus on global contrast-tone optimization. The OCTM technique can be applied separately to different image regions and, hence, made adaptive to local image statistics. The idea is similar to that of local histogram equalization. However, in locally adaptive histogram equalization [8], [10], each region is processed independently of others. A linear

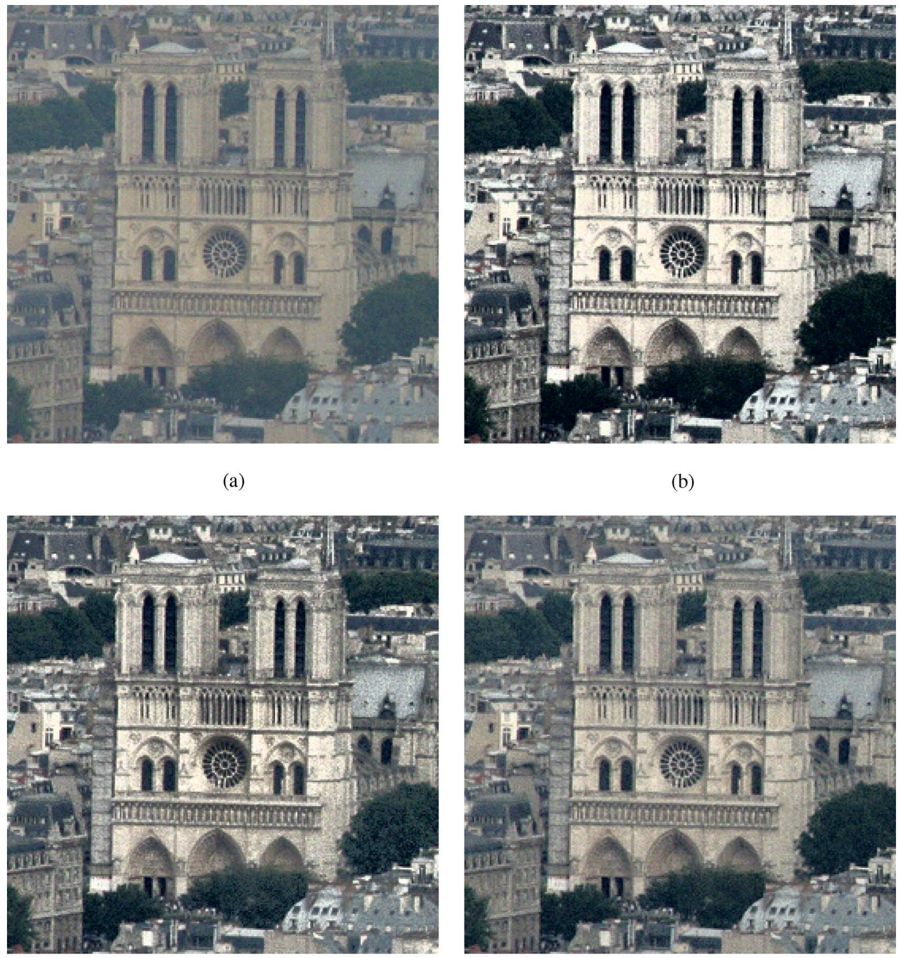


Fig. 10. Results of different methods on image Notre Dame of noise, to be compared with those in Fig. 8. (a) Noisy image. (b) HE. (c) CLAHE. (d) OCTM.

weighting scheme is typically used to fuse the results of neighboring blocks to prevent block effects. In contrast, the proposed linear programming approach can optimize the contrasts and tones of adjacent regions jointly while limiting the divergence of the transfer functions of these regions. The only drawback is the increase in complexity. Further investigations in locally adaptive OCTM are underway.

V. Experimental Results

Figs. 1–4 present some sample images that are enhanced by the OCTM technique in comparison with those produced by conventional histogram equalization (HE). The transfer functions of both enhancement techniques are also plotted in accompany with the corresponding input histograms to show different behaviors of the two techniques in different image statistics.

In image Beach (Fig. 1), the output of histogram equalization is too dark in overall appearance because the original histogram is skewed toward the bright range. But the OCTM method enhances the original image without introducing unacceptable distortion in average intensity. This is partially because of the constraint in linear programming that bounds the relative difference ($< 20\%$ in this instance) between the average intensities of the input and output images.

Fig. 2 compares the results of histogram equalization and the OCTM method when they are applied to a common portrait image. In this example histogram equalization overexposes the input image, causing an opposite side effect as in image Beach, whereas the OCTM method obtains high contrast, tone continuity and small distortion in average intensity at the same time.

Fig. 3 shows an example when the user assigns higher weights w_j in (14) to gray levels $j, j \in (a, b)$, where $(a, b) = (100, 150)$ is an intensity range of interest (brain matters in the head image). The improvement of OCTM over histogram equalization in this typical scenario of medical imaging is very significant.

In Fig. 4, the result of joint Gamma correction and contrast-tone optimization by the OCTM technique is shown, and compared with those in difference stages of the separate Gamma correction and histogram equalization process. The image quality of OCTM is clearly superior to that of the separation method.

The new OCTM approach is also compared with the well-known contrast-limited adaptive histogram equalization (CLAHE) [8] in visual quality. CLAHE is considered to be one of the best contrast enhancement techniques, and it alleviates many of the problems of histogram equalization, such as over- or under-exposures, tone discontinuities, and etc. Figs. 5–8 are side-by-side comparisons of OCTM, CLAHE, and HE. CLAHE is clearly superior to HE in perceptual quality, as well recognized in the existing literature and among practitioners, but it is somewhat inferior to OCTM in overall image quality, particularly in the balance of sharp details and subtle tones. In fact, the OCTM technique was assigned and implemented as a course project in one of the author's classes. There was a consensus on the superior subjective quality of OCTM over HE and its variants among more than one hundred students.

Finally, in Figs. 9 and 10, we empirically assess the sensitivity of OCTM to noises in comparison with histogram-based contrast enhancement methods. Since contrast enhancement tends to boost high frequency signal components, it is difficult but highly desirable for a contrast enhancement algorithm to resist noises. By inspecting the output images of different algorithms in Figs. 9 and 10, it should be apparent that OCTM is more resistant to noises than HE and CLAHE. This is because OCTM can better balance the increase in contrast and the smoothness in tone.

VI. Conclusion

A new, general image enhancement technique of optimal contrast-tone mapping is proposed. The resulting OCTM problem can be solved efficiently by linear programming. The OCTM solution can increase image contrast while preserving tone continuity, two conflicting quality criteria that were not handled and balanced as well in the past.

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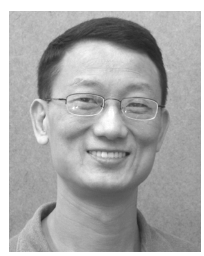

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