CoE4TN3
Medical Image Processing

Image Restoration

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Image Restoration

- Restoration: a process that attempts to reconstruct or recover a degraded image by using some a priori knowledge of the degradation phenomenon.
- Technique: model the degradation -> apply the inverse process to recover the original image.
- Enhancement technique are heuristic while restoration techniques are mathematical.

Degradation Model

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,\upsilon) = H(u,\upsilon)F(u,\upsilon) + N(u,\upsilon)$$

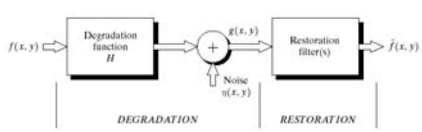
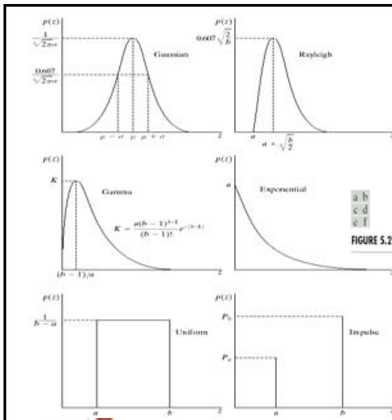


FIGURE 5.1 A model of the image degradation/restoration process.

Noise

- Image sensor might produce noise because of environmental conditions or quality of sensing elements.
- Interference in the image transmission channel.
- Assumptions: noise is independent of spatial coordinates (except for periodic noise) and independent of the image.
- Spatial description of noise: Gaussian noise, Rayleigh noise, Erlang (Gamma) noise, Exponential noise, Uniform noise, Impulse noise, etc.



a b
c d
e f

FIGURE 5.2 Some important probability density functions.

Noise Model

Different PDFs provide useful tools for modeling a broad range of noise corruption situations:

- Gaussian noise: due to factors such as electronic circuit noise, sensor noise (due to poor illumination or high temperature)
- Rayleigh noise: model noise in range imaging
- Exponential and Gamma: laser imaging
- Impulse noise: found in quick transients (e.g., faulty switches)

Noise Model

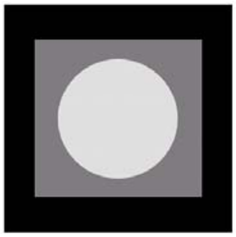


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

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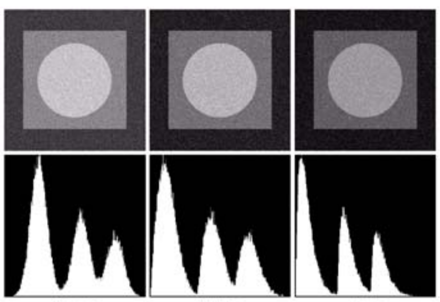
Noise Model



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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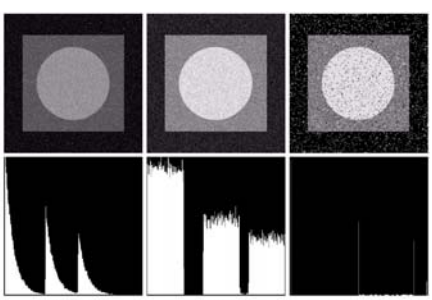
Noise Model



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

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Periodic Noise

- Periodic noise: from electrical or electromechanical interference during image acquisition.
- Frequency domain filtering can be used to remove this noise.
- Fourier transform of a pure sinusoid is a pair of conjugate impulses.
- In the Fourier transform of an image corrupted with periodic noise should have a pair of impulses for each sine wave.

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a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

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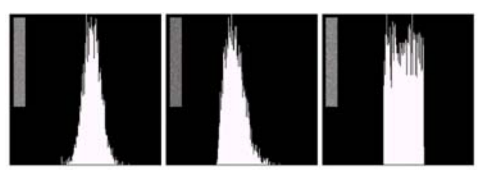
Estimation of Noise Parameters



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

$$\mu = \sum_{z_i \in S} z_i \, p(z_i) \qquad \sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 \, p(z_i)$$

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Restoration in the presence of noise

- When the only degradation is noise:
  g(x,y)=f(x,y)+n(x,y)
  G(u,v)=F(u,v)+N(u,v)
- Spatial filtering is the method of choice in this case: Mean filters, Order-statistics filters, Adaptive filters

Mean filters

- S_{xy} : subimage of size m*n
- Arithmetic mean filter:

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

- Geometric mean filter:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Mean filters

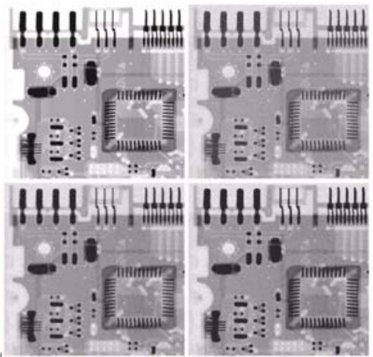
- Harmonic mean filter:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

- Contraharmonic mean filter

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

  - Negative Q: Suitable for salt noise
  - Positive Q: Suitable for pepper noise

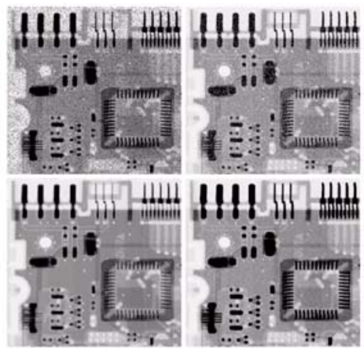


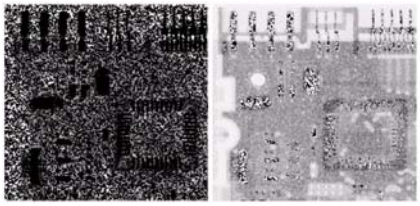
a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.





a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-statistics filters

- Median filters: $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\}$
  - Effective for salt and pepper noise
- Max and Min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}}\{g(s,t)\} \qquad \hat{f}(x,y) = \min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

  - Max filter: useful for finding brightest points in an image (remove pepper noise)
  - Min filter: useful for finding darkest points in an image (remove salt noise)

Order-statistics filters

- Midpoint filter:

$$\hat{f}(x,y) = \frac{1}{2}[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}]$$

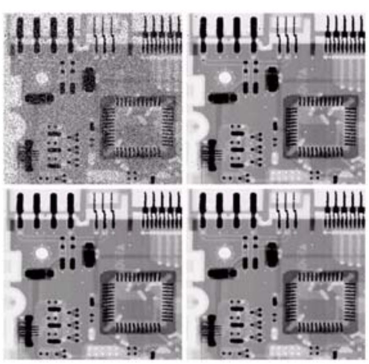
  - Works best for Gaussian and uniform noise
- Alpha-trimmed mean filter
  d/2 lowest and d/2 highest gray-levels are removed

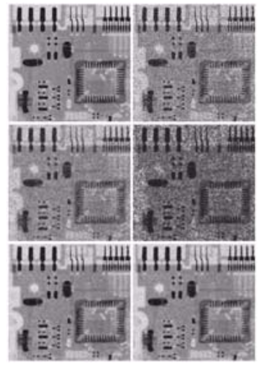
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

  - Useful for combination of salt-pepper and Gaussian noise

a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3 × 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.





a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5 × 5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Adaptive, local noise reduction filter

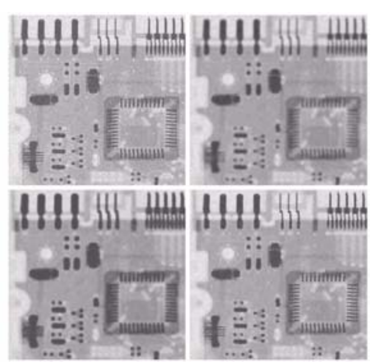
- Response of the filter is based on four quantities:

1. g(x,y)
2. σ_η^2 : variance of noise
3. m_L: mean of pixels in S_{xy}
4. σ^2_L: variance of pixels in S_{xy}

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.



Adaptive median filtering

- Adaptive median filter:
  - Handle dense impulse noise
  - Smoothes non-impulse noise
  - Preserves details
- z_{min}: minimum gray level in S_{xy}
- z_{max}: maximum gray level in S_{xy}
- z_{med}: median gray level of S_{xy}
- z_{xy}: gray level at coordinate (x,y)
- S_{max}: maximum allowed size of S_{xy}

Adaptive median filtering

- A
  - $A1=z_{med}-z_{min}$
  - $A2=z_{med}-z_{max}$
  - If A1>0 and A2<0 go to B else increase the window size
  - If window size<S_{max} repeat A
  - Else output z_{xy}
- B
  - $B1=z_{xy}-z_{min}$
  - $B2=z_{xy}-z_{max}$
  - If B1>0 and B2<0 output z_{xy}
  - Else output z_{med}

a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7 × 7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Periodic noise reduction

- Bandreject filters remove or attenuate a band of frequencies

$$H(u,v)=\begin{cases}1 & D(u,v)<D_0-\frac{W}{2}\\ 0 & D_0-\frac{W}{2}\le D(u,v)\le D_0+\frac{W}{2}\\ 1 & D(u,v)>D_0+\frac{W}{2}\end{cases}$$

Periodic noise reduction

- Butterworth Bandreject Filter

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

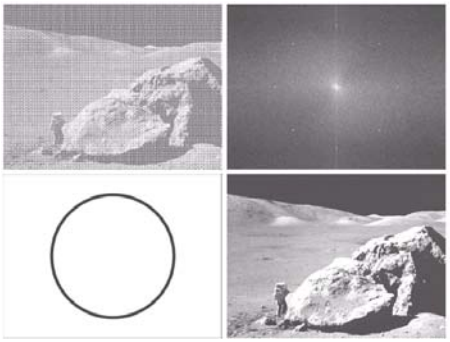
- Gaussian Bandreject Filter

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

Periodic noise reduction

a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

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Estimation of Degradation

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,\upsilon) = H(u,\upsilon)F(u,\upsilon) + N(u,\upsilon)$$

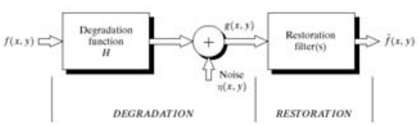

FIGURE 5.1 A model of the image degradation/restoration process.

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Estimation by image observation

- We look at a small section of the image containing simple structures (e.g., part of an object and the background)
- By using sample gray levels of the object and background, we can construct an unblurred "true" image of the subimage $\hat{f}_s(x,y)$

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

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Estimation by Experimentation

- If equipment similar to the equipment used to acquire the degraded image is available it is possible to obtain an accurate estimate of the degradation.
- The idea is to obtain the impulse response of the degradation by imaging an impulse (small dot of light) using the system

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Estimation by Experimentation

$$H(u,v) = \frac{G(u,v)}{A}$$

FT of an impulse is a constant

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

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Estimation by modeling

- Approach: derive a mathematical model starting from basic principles
- Example: Turbulence model

$$H(u,v) = e^{-k(u^2+\upsilon^2)^{5/6}}$$

36

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

Inverse Filtering

$$\hat{F}(u,\upsilon) = F(u,\upsilon) + \frac{N(u,\upsilon)}{H(u,\upsilon)}$$

Difficulty: if the degradation has zero or very small values, then the ratio N(u,v)/H(u,v) could easily dominate the estimation.

Cure: limit filter frequencies to values near the origin.

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Wiener Filtering

Goal: minimize the estimation error

$$e^2 = E\left\{(f - \hat{f})^2\right\}$$

Wiener filtering:

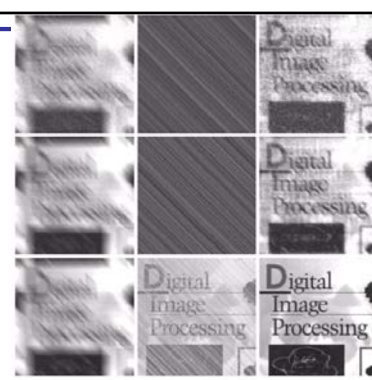
$$\hat{F}(u,\upsilon) = \left[\frac{1}{H(u,\upsilon)} \frac{|H(u,\upsilon)|^2}{|H(u,\upsilon)|^2 + K}\right] G(u,\upsilon)$$

K is a constant (1/SNR), adjusted in practice

Wiener Filtering

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Square Filtering

Vector-matrix representation of an image

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

Minimize criterion function

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 \hat{f}(x, y)]^2$$

subject to

$$\left\| \mathbf{g} - \mathbf{H}\hat{\mathbf{f}} \right\|^2 = \left\| \boldsymbol{\eta} \right\|^2$$

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Constrained Least Square Filtering

Solution:

$$\hat{F}(u, \upsilon) = \frac{H^*(u, \upsilon)}{|H(u, \upsilon)|^2 + \gamma |P(u, \upsilon)|^2} G(u, \upsilon)$$

where $P(u,v)$ is the FT of

$$p(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

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Constrained Least Square Filtering

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

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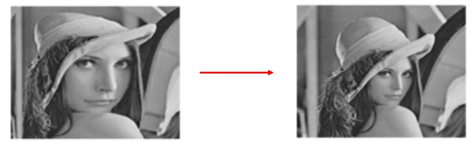
Geometric Transformation

- Geometrical transformations: modify the spatial relationships between pixels in an image



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Geometric Transformation

- Geometrical transformation consists of two basic operations:
  1. Spatial transformation: defines the rearrangement of pixels on the image plane
  2. Gray level interpolation: deals with the assignment of gray levels to pixels in the spatially transformed image

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Spatial Transformation

- Image f with pixels coordinates (x,y) has undergone geometric distortion to produce an image g with coordinates (x',y')
  - $x'=r(x,y)$, $y'=s(x,y)$
- Example: $x'=r(x,y)=x/2$, $y'=s(x,y)=y/2$
  - Distortion is a shrinking of the size of $f(x,y)$ by one-half in both directions.

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Spatial Transformation

- If $r(x,y)$ and $s(x,y)$ are known analytically: the inverse of r and s is applied to $g(x',y')$ to recover $f(x,y)$.
- In practice finding a single set of $r(x,y)$ and $s(x,y)$ is not possible
- Solution: spatial relocation is formulated by the use of tiepoints.
- Tiepoints: a set of pixels whose locations in distorted and corrected images are known

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Spatial Transformation

- Suppose the geometrical distortion process within the region is modeled by a pair of bilinear equations:
  - $x'=r(x,y)=c_1x+c_2y+c_3yx+c_4$
  - $y'=s(x,y)=c_5x+c_6y+c_7yx+c_8$
  - 8 known tiepoints, 8 unknown c_i
  - The model is used for all the points inside the region

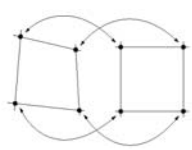

FIGURE 5.32 Corresponding tiepoints in two image segments.

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Spatial Transformation

```
for x=1 to horizontal size {
 for y=1 to vertical size {
    x'=r(x,y)=c1x+c2y+c3yx+c4
    y'=s(x,y)=c5x+c6y+c7yx+c8
    f^(x,y)=g(x',y')
    }
    }
```

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Gray-level Interpolation

- Depending on the values of c_i, x' and/or y' can be noninteger for integer values of (x,y)
  - $x'=r(x,y)=c_1x+c_2y+c_3yx+c_4$
  - $y'=s(x,y)=c_5x+c_6y+c_7yx+c_8$
- g is a digital image and its pixel values are defined only at integer values of (x,y).
- We need inferring gray-level values at noninteger locations (gray-level interpolation)

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Gray-level Interpolation

- Simplest scheme: nearest neighbor approach (zero-order interpolation)
  1. Mapping (x,y) to (x',y')
  2. Selection of closest integer coordinate neighbor to (x',y')
  3. Assign the gray-level of this nearest neighbor to the pixel at (x,y)

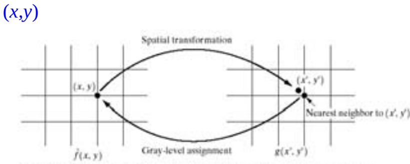

FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

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Gray-level Interpolation

- Nearest neighbor interpolation: simple to implement, has the drawback of producing undesirable artifacts
- Example: distortion of straight edges in an image
- More sophisticated techniques: better results, costly in terms of computations
- A reasonable compromise: bilinear interpolation approach

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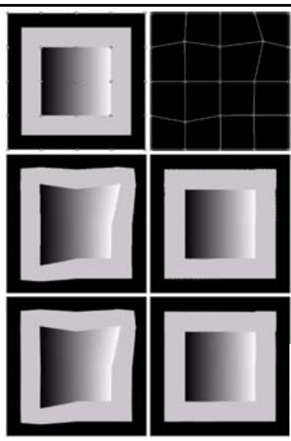
Gray-level Interpolation

- (x', y'): a noninteger coordinate
- $v(x', y')$: the gray-level at (x', y')
- $v(x', y') = ax' + by' + cx'y' + d$
- The four unknowns (a, b, c, d) are determined using the gray-levels of four neighbors of (x', y')
- When the coefficients (a, b, c, d) have been determined, $v(x', y')$ is computed and this values is assigned to the location (x, y).



a b
c d
e f

FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

End of Lecture