

CoE4TN3 Image Processing

Chapter 9 Morphological Image Processing



Preliminaries

- Reflection of a set:

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

- Translation:

$$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$$



FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.



Image Morphology

- Morphology: a branch of biology that deals with the form and structure of animals and plants
- A tool for extracting image components that are useful in representation and description
- Language of morphology: set theory
- Objects in an image are represented by a sets
- For binary images each element of the set is a 2-D vector with the (x,y) coordinates of a black (or white depending on the object) pixel



Dilation & Erosion

- Dilation:

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

- Obtain the reflection of B and shift it by z.
- Dilation of A by B is the set of all displacements z such that \hat{B}^z and A overlap at least one element
- B is called the structuring element
- One of the applications of dilation is for bridging gaps



Preliminaries

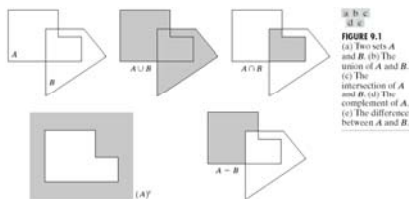


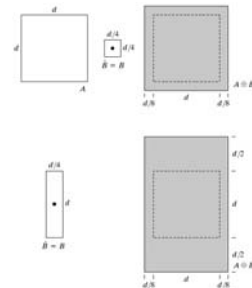
FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.



Dilation

FIGURE 9.4

- (a) Set A.
- (b) Square structuring element (dot is the center).
- (c) Dilation of A by B, shown shaded.
- (d) Elongated structuring element.
- (e) Dilation of A using this element.



Dilation

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

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FIGURE 9.5
 (a) Sample text of poor resolution with broken characters (magnified view).
 (b) Structuring element.
 (c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

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Dilation & Erosion

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1x, 15 pixels on the side. (c) Dilation of (b) with the same structuring element.

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Dilation & Erosion

- Erosion:

$$A(-)B = \{z \mid (B)_z \subseteq A\}$$
- Erosion of A by B is the set of all displacements z such that B translated by z is contained in A
- B is called the structuring element
- One of the applications of erosion is elimination of irrelevant details

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Opening and closing

- Opening: smoothes the contour of an object, breaks narrow strips and eliminates thin protrusions (bulges)
- Opening of set A by structuring element B is defined as:

$$A \circ B = (A(-)B) \oplus B$$
- Closing of A by structuring element B is defined as:

$$A \bullet B = (A \oplus B)(-)B$$

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Erosion

FIGURE 9.8 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

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Opening and closing

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

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Hit-or-Miss transform

- Hit-or-miss transform is a basic tool for shape detection
- Set A consists of three shapes: X, Y, Z
- The objective is to find the location of X
- Let X be enclosed by a small window W
- Local background of X with respect to W is W-X
- Set of locations for which X exactly fits inside A is the intersection of the erosion of A by X and the erosion of A^c by (W-X)

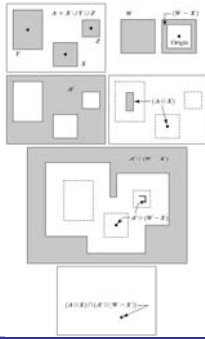
$$A(*)B = (A(-)X) \cap [A^c(-)(W - X)]$$

Boundary



FIGURE 9.14 (a) A simple binary image, with 1's represented as white. (b) Result of using Fig. 9.13(b) with the structuring element on Fig. 9.13(a).

Hit-or-Miss transform



Region filling

- Let's A denote the boundary points of a region and p is a point inside the boundary
- Objective is to fill the entire region with 1s.

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

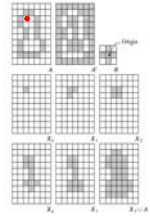
$$X_0 = p$$

B is the structuring element

Algorithm terminates if $X_k = X_{k-1}$

Union of X_k and A is the region filled

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) First point inside the boundary. (e) A. (f) Union of A and B.



Boundary Extraction

- Boundary of a set A, denoted by $\beta(A)$ can be obtained by first eroding A by B and then performing the set difference between A and its erosion:

$$\beta(A) = A - (A(-)B)$$



Extraction of connected components

- Extraction of connected components in a binary image is central to many automated image analysis applications
- Let Y represent a connected component contained in a set A and assume that a point p of Y is known. Then the following iterative expression yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A$$

$$X_0 = p$$

B is the structuring element

Algorithm terminates if $X_k = X_{k-1}$

Extraction of connected components

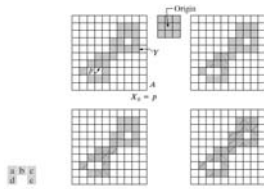


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.