Chapter 1 (Answers) (Prepared by Payam Abolghasem : payam@grads.ece.mcmaster.ca)

1.1-1:

a) $E_f = \pi$

b) $E_f = \pi$

c) $E_f = \int_{2\pi}^{4\pi} (\sin t)^2 dt = \int_{2\pi}^{4\pi} \sin^2 t\, dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\, dt = \pi + 0 = \pi$

d) $E_f = 4\pi$

1.1-2:

$E_f = \frac{1}{3}$

$E_{f_1} = \frac{1}{3}$

$E_{f_2} = \frac{1}{3}$

$E_{f_3} = \int_1^2 (t-1)^2 dt = \int_0^1 x^2 dx = \frac{1}{3}.$

$E_{f_4} = \int_0^1 (2t)^2 dt = \frac{4}{3} t^3 \Big|_{-1}^{0} = 4/3.$

1.1-3:

a) $E_x = \int_0^2 (1)^2 dt = 2$

$E_y = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt = 2$

$E_{x+y} = \int_0^1 (2)^2 dt = 4$

$E_{x-y} = \int_1^2 (2)^2 dt = 4$

$\Rightarrow E_{x\pm y} = E_x + E_y.$

b) $E_x = 2\pi$, $E_y = 2\pi$; $E_{x+y} = 4\pi$, $E_{x-y} = 4\pi$ $\Rightarrow E_{x\pm y} = E_x + E_y.$

c) $E_x = \pi$; $E_y = \pi$; $E_{x+y} = \pi$; $E_{x-y} = 3\pi \Rightarrow$ In general $E_{x\pm y} \neq E_x + E_y$.

1.1-4:

$P_f = \frac{1}{4}\int_{-2}^{2} (t^3)^2 dt = 64/7$ (a) $P_{-f} = 64/7$ (b) $P_{2f} = \frac{1}{4}\int_{-2}^{2} (2t^3)^2 dt = 256/7$ (c) $P_{cf} = \frac{64}{7}c^2$

Comment: Sign change of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor c^2.

1.1-5 $P_f = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t) f^*(t)\, dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n} \sum_{r=m}^{n} D_k D_r^* e^{j(\omega_k - \omega_r)t} dt$

The integrals of the cross product terms (when $k \neq r$) are finit because the integrands are periodic signals (made up of sinusoids). These terms when divided by

$T\to\infty$, yield zero. The remaining terms $(k=r)$ yeild:

$$P_f = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=m}^{n} |D_k|^2 dt = \sum_{k=m}^{n} |D_k|^2$$

1.1-6

a) In general, the power of a sinusoid of amplitude C is $C^2/2$, regardless of its frequency $(\omega\neq 0)$ and phase. In this case $P = (10)^2/2 = 50$

b) Power of a sum of sinusoids is equal to the sum of the powers of the sinusoids [Eq (1.5b)]. In this case $P = \frac{(10)^2}{2} + \frac{(16)^2}{2} = 178$.

c) $(10 + 2\sin 3t)\cos(10t) = 10\cos(10t) + \sin(13t) - \sin(7t) \Rightarrow P = \frac{(10)^2}{2} + \frac{1}{2} + \frac{1}{2} = 51$

d) $P = \frac{(5)^2}{2} + \frac{(5)^2}{2} = 25$

e) $P = \frac{(5)^2}{2} + \frac{(-5)^2}{2} = 25$

f) $P_f = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f(t)f^*(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} \underbrace{|e^{j\alpha t}|^2}_{1} \cos^2\omega_0 t\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\cos^2\omega_0 t\,dt = 1/2$

1.3-1:

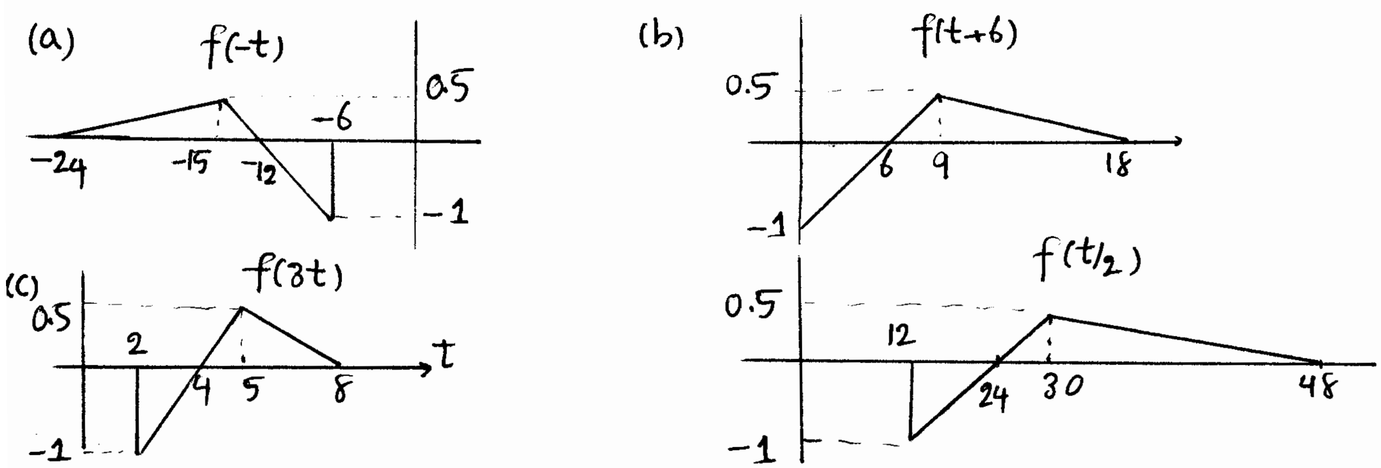
$f_2(t) = f(t-1) + f_1(t-1)$

$f_3(t) = f(t-1) + f_1(t+1)$

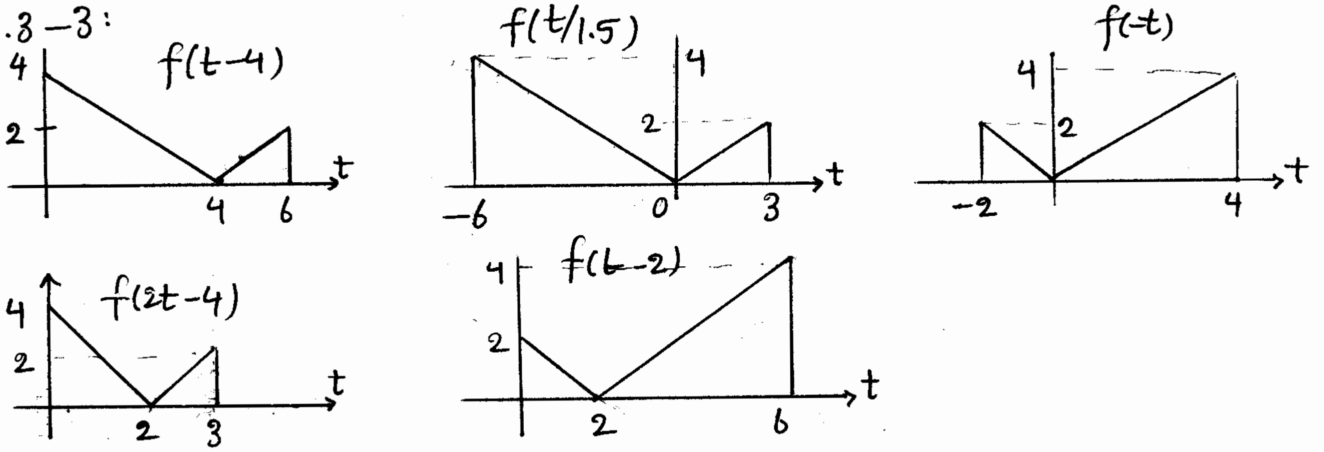
$f_4(t) = f(t-0.5) + f_1(t+0.5)$

$f_5(t) = 1.5 f(t/2 - 1)$

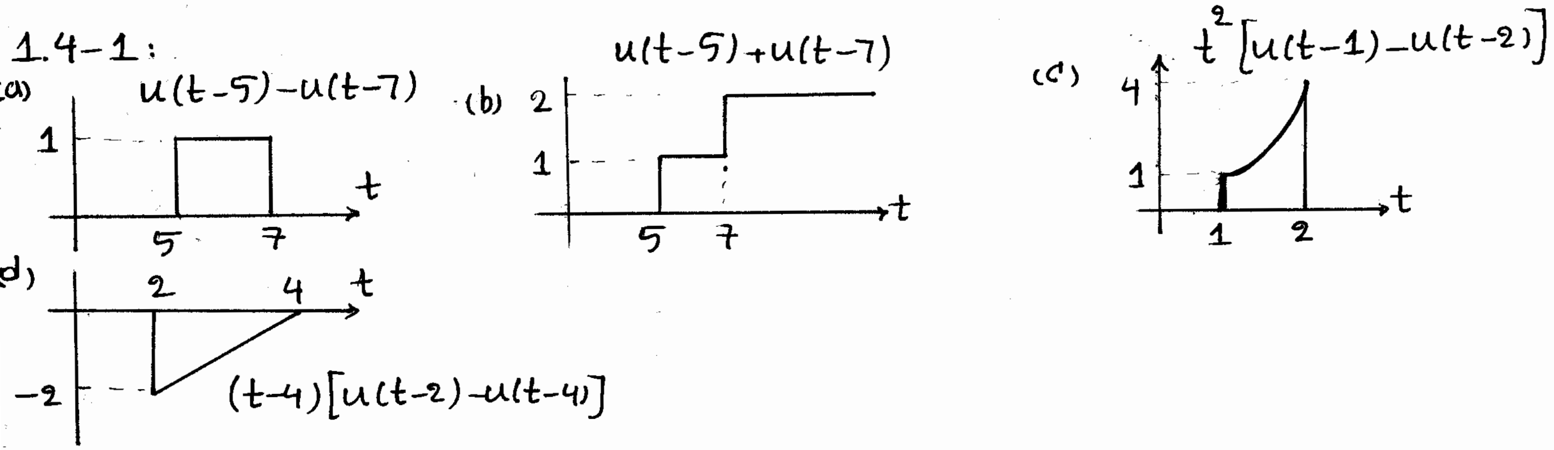
1.3-2:



1.3-3:



1.4-1:



1.4-2:

(a) $f_1(t) = (4t+4)[u(t+1)-u(t)] + (-2t+4)[u(t)-u(t-2)]$

$= 4(t+1)u(t+1) - 6tu(t) + 3u(t) + (2t-4)u(t-2)$

(b) $f_2(t) = t^2[u(t)-u(t-2)] + (2t-8)[u(t-2)-u(t-4)] = t^2u(t) - (t^2-2t+8)u(t-2) - (2t-8)u(t-4)$

1.4-3

$E_{-f} = \int_{-\infty}^{+\infty}[-f(t)]^2dt = \int_{-\infty}^{+\infty} f^2(t)dt = E_f$; $E_{f(-t)} = \int_{-\infty}^{+\infty}[f(-t)]^2dt \xrightarrow{-t\to x} \int_{-\infty}^{+\infty} f^2(x)\,dx = E_f$.

$E_{f(t-T)} = \int_{-\infty}^{+\infty}[f(t-T)]^2dt \xrightarrow{t-T\to x} \int_{-\infty}^{+\infty} f^2(x)\,dx = E_f$.

The rest of equalities can be proven the same way:

$E_{f(at)} = E_f/a$; $E_{f(at-b)} = E_f/a$; $E_{f(t/a)} = aE_f$; $E_{af(t)} = a^2E_f$.

1.4-4: Using the fact that: $f(x)\delta(x) = f(0)\delta(x)$ we have:

(a) 0 (b) $\frac{2}{9}\delta(\omega)$ (c) $\frac{1}{2}\delta(t)$ (d) $-\frac{1}{5}\delta(t-1)$ (e) $\frac{1}{2-j3}\delta(\omega+3)$

(f) $k\delta(\omega)$ (using Hopital's rule)

1.4-5:

a) $\int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau)\,d\tau \xrightarrow[d\tau=-dt']{t-\tau=t'} \int_{+\infty}^{-\infty} f(t-t')\delta(t')(-dt') = \int_{-\infty}^{+\infty} f(t-t')\delta(t')\,dt'$

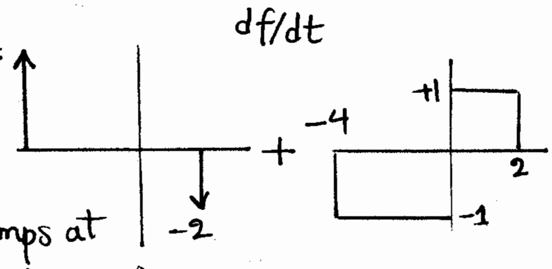
$= \int_{-\infty}^{+\infty} f(t)\,\delta(t')\,dt' = f(t)\int_{-\infty}^{+\infty}\delta(t')\,dt' = f(t)$

b) Using the same arguments as (a) we get $\int_{-\infty}^{+\infty}\delta(\tau)f(t-\tau)\,d\tau = f(t)$

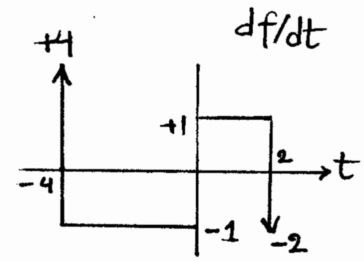
(c) 1 (d) 0 (e) e^{3} (f) 5 (g) $f(-1)$ (h) e^{-2}

1.4-6:

(a)

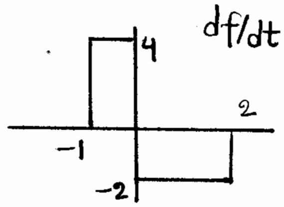


The two graphs can be shown in one graph →



For jumps at $t=-4$ and $+2$ ↗

(b)



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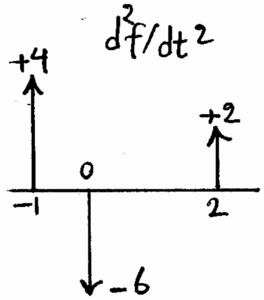
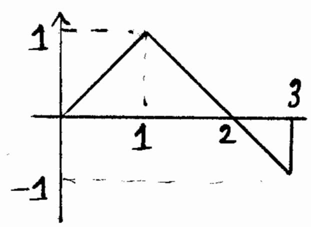


*Comment:
Recall that the derivative of a function at the jump discontinuity is equal to an impulse of strength equal to the amount of discontinuity.

1.4-7

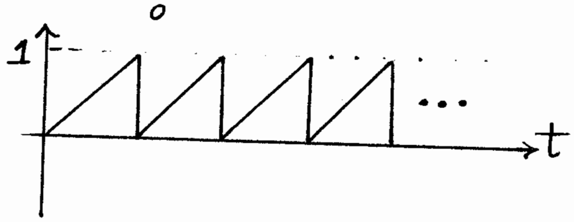
a) $f(t) = \begin{cases} 0 & t<0 \\ 1 & 0\le t<1 \\ -1+\delta(t-3) & 1\le t\le 3 \\ 0 & t>3 \end{cases} \longrightarrow \int_{-\infty}^{t} f(t)\,dt = \begin{cases} 0 & t<0 \\ t & 0\le t<1 \\ -t+2 & 1\le t\le 3 \\ 0 & t>3 \end{cases}$



b) $f(t) = 1-\delta(t-1)-\delta(t-2)-\delta(t-3)----$

$\Rightarrow \int_{-\infty}^{t} f(t)\,dt = \int_{0}^{t}[1-\delta(t-1)-\delta(t-2)-\delta(t-3)----]\,dt = \int_0^t dt - \int_0^t \delta(t-1)\,dt - \int_0^t \delta(t-2)\,dt ---$

$= tu(t) - u(t-1) - u(t-2) - u(t-3) ---$



1.4-8 $\int_{-\infty}^{+\infty}\phi(t)\delta(t)\,dt = \phi(0)$

$\int_{-\infty}^{+\infty}\phi(t)\delta(-t)\,dt \xrightarrow{t'=-t} \int_{+\infty}^{-\infty}\phi(-t')\delta(t')(-dt') = \int_{-\infty}^{+\infty}\phi(-t')\delta(t')\,dt' = \phi(0)$

$\Rightarrow \delta(t) = \delta(-t)$ Therefore $\delta(t)$ is an even function.

1.4-9

Two cases should be considered:

$a>0$: $\int_{-\infty}^{+\infty}\phi(t)\,\delta(at)\,dt \xrightarrow{t'=at} \int_{-\infty}^{+\infty}\phi(\frac{t'}{a})\delta(t')\frac{dt'}{a} = \frac{\phi(0)}{a}$

$a<0$ $\int_{-\infty}^{+\infty}\phi(t)\,\delta(at)\,dt \xrightarrow{t'=at} \int_{+\infty}^{-\infty}\phi(\frac{t'}{a})\delta(t')(\frac{-dt'}{a}) = \int_{-\infty}^{+\infty}\phi(\frac{-t'}{a})\delta(t')\frac{dt'}{a} = \frac{\phi(0)}{a}$

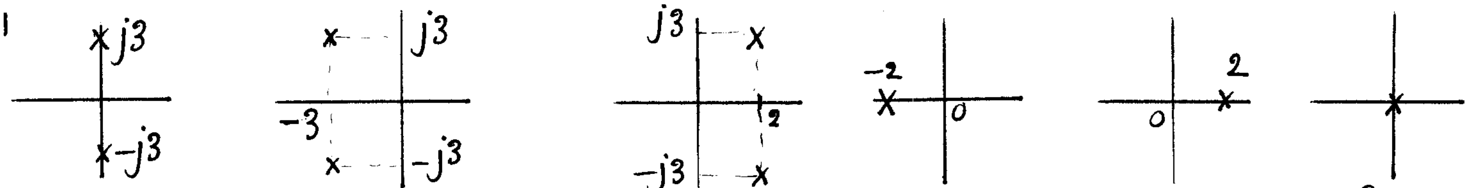
$\rightarrow \delta(at) = \frac{1}{|a|}\delta(t)$

1.4-10 $\int_{-\infty}^{+\infty}\dot{\delta}(t)\phi(t)dt = \int_{-\infty}^{+\infty}\frac{d}{dt}[\delta(t)]\phi(t)dt = \int_{-\infty}^{+\infty}\phi(t)\,d[\delta(t)]$.

using integration by part: $\int_{-\infty}^{+\infty}\phi(t)d[\delta(t)] = \phi(t)\delta(t)\Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty}\delta(t)dt = -\int_{-\infty}^{+\infty}\delta(t)d\phi(t)$

$= -\int_{-\infty}^{+\infty}\delta(t)\dot{\phi}(t)\,dt = -\dot{\phi}(0)$

1.4-11



(a): $s_{1,2} = \pm j3$ (b) $s_{1,2} = -3\pm j3$ (c) $s_{1,2} = 2\pm j3$ (d) $s=-2$ (e) $s=2$ (f) $s=0$

1.5-1

(a) $f_e(t) = 0.5[u(t)+u(-t)] = 0.5$

$f_o(t) = 0.5[u(t)-u(-t)]$

(b) $f_e(t) = 0.5[tu(t)-tu(-t)] = 0.5|t|$

$f_o(t) = 0.5[tu(t)+tu(-t)] = 0.5t$

(c) $f_e(t) = 0.5[\sin(\omega_0 t)u(t) - \sin(\omega_0 t)u(-t)]$

$f_o(t) = 0.5[\sin(\omega_0 t)u(t) + \sin(\omega_0 t)u(t)] = 0.5\sin(\omega_0 t)$

(d) $f_e(t) = 0.5\cos\omega_0 t$

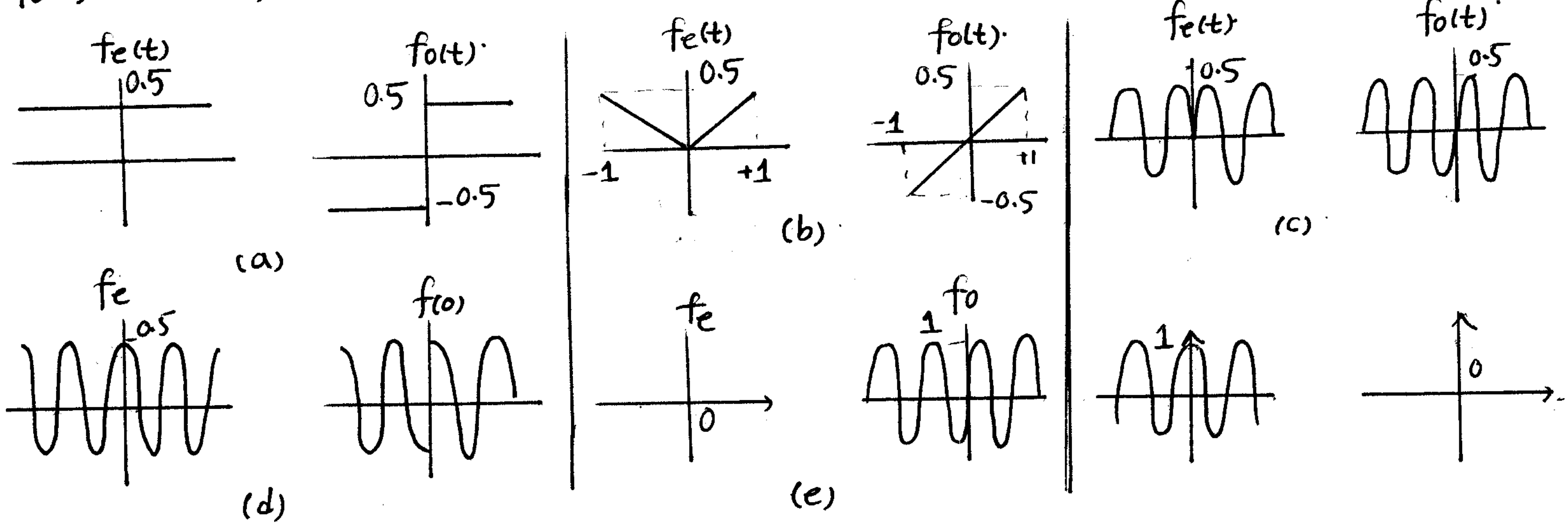
$f_o(t) = 0.5[\cos(\omega_0 t)u(t) - \cos(\omega_0 t)u(-t)]$

e) $f_e(t) = 0$

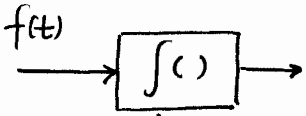
$f_o(t) = \sin(\omega_0 t)$

(f) $f_e(t) = \cos(\omega_0 t)$

$f_o(t) = 0$



1.6-1.



$y(t)=\int_{-\infty}^{t} f(\tau)\,d\tau$

$$y(t)=\int_{-\infty}^{t} f(\tau)\,d\tau=\int_{-\infty}^{0} f(\tau)\,d\tau+\int_{0}^{t} f(\tau)\,d\tau=\underbrace{y(0)}_{\text{Zero-input}}+\underbrace{\int_{0}^{t} f(\tau)\,d\tau}_{\text{Zero-State}}.$$

1.7-1

a) $\frac{dy}{dt}+2y(t)=f^2(t)$

$$\begin{cases}\frac{dy_1}{dt}+2y_1(t)=f_1^2(t) & (I)\\ \frac{dy_2}{dt}+2y_2(t)=f_2^2(t) & (II)\end{cases}$$

multiplying equation (I) by k_1 and equation (II) by K_2 and adding two equations; we have:

$$\frac{d}{dt}\left[k_1y_1(t)+k_2y_2(t)\right]+2\left[k_1y_1(t)+k_2y_2(t)\right]=\left(k_1f_1^2(t)+k_2f_2^2(t)\right).$$

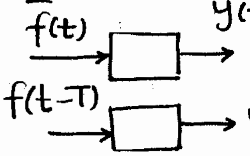
The above equation is not the response of this system when the input is $k_1f_1(t)+k_2f_2(t)$; therefore the system is not linear (also study example 1.9 of text book).

(b) linear (c) non-linear (d) non-linear (e) non-linear (f) non-linear

(g) linear (h) linear.

1.7-2

(a)

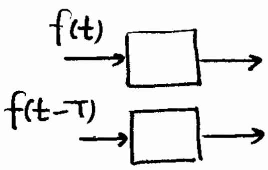


$y(t)=f(t-2)$

$y_1(t)=f(t-T-2)=y(t-T)$

$\Rightarrow y_1(t)=y(t-T)\Rightarrow$ System is time-invariant.

b)



$y(t)=f(-t)$

$y_1(t)=f(-t-T)=y(t+T)$

$\Rightarrow y_1(t)\neq y(t-T)\Rightarrow$ System is time-varying.

c) $f(t)\longrightarrow y(t)=f(at)$

$f(t-T)\longrightarrow y_1(t)=f(at-T)=f[a(t-\frac{T}{a})]=y(t-\frac{T}{a})$

$\Rightarrow y_1(t)\neq y(t-T)\Rightarrow$ System is time-varying.

(d) time-varying (e) time-varying (f) time-invariant.

1.7-3

The system is linear, and because of linearity property, the given data can be multiplied by a non-zero constant or can be added/Subtracted to give new data. Using such operations four new rows are added to the initial rows:

Row	$f(t)$	$x_1(0)$	$x_2(0)$	$y(t)$
r_1	0	1	-1	$e^{-t}u(t)$
r_2	0	2	1	$e^{-t}(3t+2)u(t)$
r_3	$u(t)$	-1	-1	$2u(t)$
$r_4 = 1/3(r_1+r_2)$	0	1	0	$(t+1)e^{-t}u(t)$
$r_5 = \frac{1}{2}(r_1+r_3)$	$1/2\,u(t)$	0	-1	$(\frac{1}{2}e^{-t}+1)u(t)$
$r_6 = (r_4+r_5)$	$1/2\,u(t)$	1	-1	$(1.5e^{-t}+te^{-t}+1)u(t)$
$r_7 = 2(r_1+r_6)$	$u(t)$	0	0	$(e^{-t}+2te^{-t}+2)u(t)$

$f(t)=u(t+5)-u(t-5)$: From r_7 and superposition and time invariance property:

$$y(t) = r_7(t+5) - r_7(t-5) = \left[e^{-(t+5)} + 2(t+5)e^{-(t+5)} + 2\right]u(t+5) - \left[e^{-(t-5)} + 2(t-5)e^{-(t-5)} + 2\right]u(t-5)$$

1.7-4

homogeneity property: $f(t) \longrightarrow y(t)$, $cf(t) \longrightarrow cy(t)$

In this problem: $f(t) \longrightarrow y(t) = f^2(t)/(df/dt)$

$$cf(t) \longrightarrow y_1(t) = [cf(t)]^2 \Big/ \frac{d}{dt}(cf(t)) = \frac{c^2 f^2(t)}{c\, df/dt} = c\, f^2(t)\Big/\frac{df}{dt} = cy(t)$$

$\Rightarrow$ System satisfies homogeneity property.

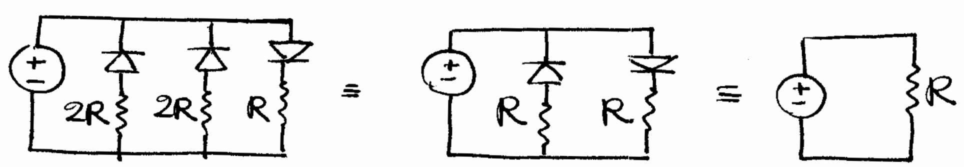
$$f_1(t) \longrightarrow y_1(t) = f_1^2 \Big/ \frac{df_1}{dt}$$

$$f_2(t) \longrightarrow y_2(t) = f_2^2 \Big/ \frac{df_2}{dt}$$

$$f_1+f_2 = f_3(t) \longrightarrow y_3(t) = (f_1+f_2)^2 \Big/ \frac{d}{dt}(f_1+f_2) \neq y_1 + y_2$$

1.7-5

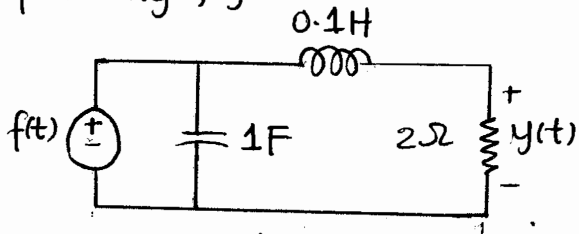
From the hint it is clear that when $V_C(0)=0$ the capacitor may be removed and the circuit behaves as shown below:



It is clearly zero-state linear. To show that it is zero-input non-linear consider the circuit with $f(t)=0$ (zero-input). The current $y(t)$ has the same direction regardless of the polarity of V_C (because the input branch is a short). Thus the system is zero-input non-linear.

1.7-6

The input is a current source. As far as the output y(t) concerned, the circuit behaves as shown in the following figure:

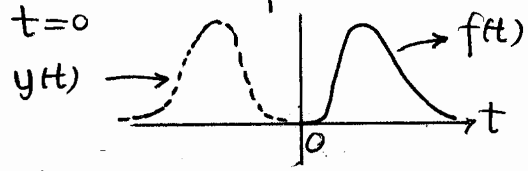


The non-linear elements are irrelevant in computing the output y(t). Hence the output y(t) satisfies the linearity condition, yet the circuit is not linear because it contains non-linear elements, and the output associated with non-linear elements L and C, will not satisfy linearity conditions.

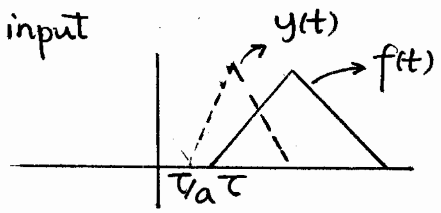
1.7-7

(a) Causal

(b) non-causal, since if the input starts at t=0 the output starts before t=0



(c) Non-causal as the output can start befor input



(d) Non-causal, like part (c) the output may start before input.

1.7-8

(a) Invertable (inverse system is a differentiator).

(b) Invertable

(c) If n is even the system is not invertable as the information about the sign can be lost. ($f^n(t) = (-f(t))^n$), but for n-odd the system is invertable.

(d) Cosine is a multiple valued function which means that $\cos^{-1}[f(t)]$ is not unique, hence system is not invertable.

1.8-1 $Df(t) = (3+D)y_2(t)$ $\quad f(t) = (3+D)y_1(t)$

1.8-2 $Df(t) = (D^2+2D+2)y_2(t)$ $\quad (D^2+2D+2)y_1(t) = D^2 f(t)$

1.8-3 $(D+a)h(t) = \frac{1}{A} q_i(t)$ where $a = R/A$.