Chapter 4 (Answers)

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4.1-4 (a) $F(\omega) = \dfrac{1 - e^{-(j\omega + a)T}}{j\omega + a}$ (b) $F(\omega) = \dfrac{1 - e^{-(j\omega - a)T}}{j\omega - a}$

4.1-5 (a) $F(\omega) = \dfrac{4 - 2e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$ (b) $F(\omega) = \dfrac{2}{\tau\omega^2}\left(\cos\omega\tau + \omega\tau \sin\omega\tau - 1\right)$

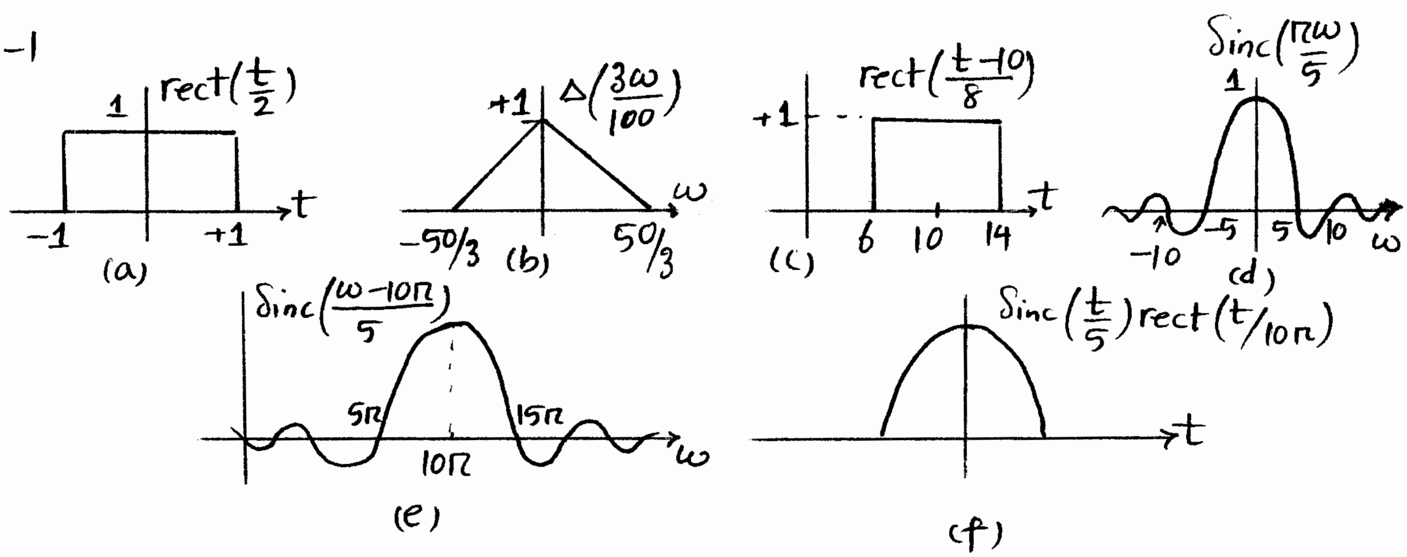
4.1-6 (a) $f(t) = \dfrac{(\omega_0^2 t^2 - 2)\sin\omega_0 t + 2\omega_0 t \cos\omega_0 t}{\pi t^3}$

(b) $f(t) = \dfrac{\sin 2t + \sin t}{\pi t}$

4.1-7 (a) $f(t) = \dfrac{1}{\pi(1-t^2)} \cos\left(\dfrac{\pi t}{2}\right)$

(b) $f(t) = \dfrac{1}{\pi\omega_0 t^2}\left(\cos\omega_0 t + \omega_0 t \sin\omega_0 t - 1\right)$

4.2-1



4.2-2 $F(\omega) = \text{Sinc}\left(\frac{\omega}{2}\right) e^{-j5\omega}$

4.2-3 $f(t) = \frac{e^{j10t}}{j2\pi\omega}\left(2j\,\text{Sin}\,\pi t\right) = \text{Sinc}(\pi t)\, e^{j10t}$

4.2-4 (a) $f(t) = \frac{\omega_0}{\pi}\,\text{Sinc}(\omega_0(t-t_0))$

(b) $f(t) = \frac{1-\text{Cos}\,\omega_0 t}{\pi t}$

4.3-1

(b) $\underbrace{\text{Cos}(\omega_0 t)}_{f(t).} \longleftrightarrow \underbrace{\pi\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]}_{F(\omega)}$

Using duality property:

$$\pi\left[\delta(t+\omega_0) + \delta(t-\omega_0)\right] \longleftrightarrow \underbrace{2\pi\,\text{Cos}(-\omega_0\omega)}_{2\pi f(-\omega)} = 2\pi\,\text{Cos}(\omega_0\omega)$$

Setting $\omega_0 = T$ yeilds:

$$\delta(t+T) + \delta(t-T) \longleftrightarrow 2\,\text{Cos}\,T\omega.$$

4.3-2

(b) $f_1(t) = f(-t)$; $F_1(\omega) = \frac{1}{\omega^2}\left(e^{-j\omega} + j\omega e^{-j\omega} - 1\right) = F(-\omega)$

(c) $f_2(t) = f(t-1) + f_1(t-1)$; $F_2(\omega) = \frac{2e^{-j\omega}}{\omega^2}\left(\text{Cos}\,\omega + \omega\,\text{Sin}\,\omega - 1\right)$

(d) $f_3(t) = f(t-1) + f_1(t-1)$; $F_3(\omega) = \text{Sinc}^2\left(\frac{\omega}{2}\right)$

(e) $f_4(t) = f(t-\frac{1}{2}) + f_1(t+\frac{1}{2})$; $F_4(\omega) = \text{Sinc}\left(\frac{\omega}{2}\right).$

(f) $f_5(t) = 1.5 f\left(\frac{t-2}{2}\right)$; $F_5(\omega) = \frac{3}{4\omega^2}\left(1 - j2\omega - e^{-j2\omega}\right)$

4.3-3 (a) $F(\omega) = \frac{j4}{\omega} \sin^2\left(\frac{\omega T}{2}\right)$

(b) $F(\omega) = \frac{1}{1-\omega^2}\left(1 + e^{-j\pi\omega}\right)$

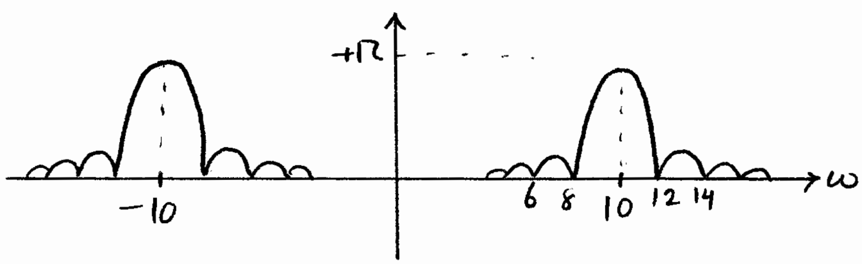
(c) $F(\omega) = \frac{1}{1-\omega^2}\left(j\omega + e^{-j\pi\omega/2}\right)$

(d) $F(\omega) = \frac{1}{j\omega + a}\left(1 - e^{-(a+j\omega)T}\right)$

4.3-4. (a) $4\,\text{Sinc}(\omega)\cos(3\omega)$ (b) $2\,\text{Sinc}^2\left(\frac{\omega}{2}\right)\cos(3\omega)$

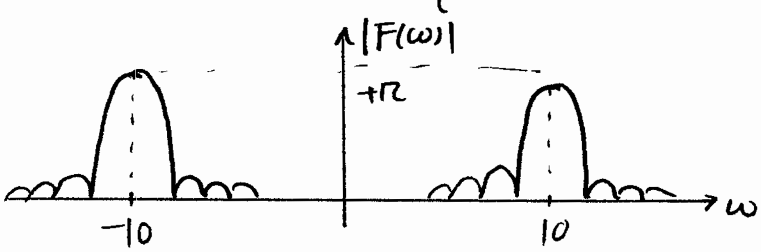
4.3-5: $4j\,\text{Sinc}(\omega)\sin(3\omega)$

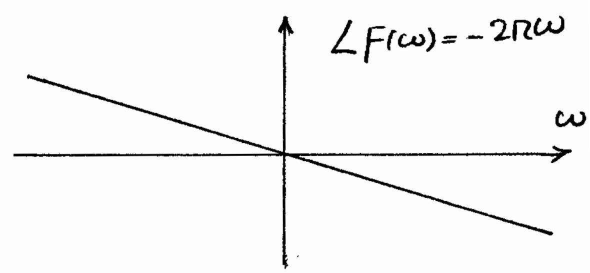
4.3-6 (a) $F(\omega) = \frac{\pi}{2}\left\{\text{Sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{Sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$



(a)

(b) $F(\omega) = \frac{\pi}{2}\left\{\text{Sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \text{Sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\} e^{-j2\pi\omega}$





(b)

(c) $F(\omega) = \pi\left\{\text{Sinc}[\pi(\omega+10)] + \text{Sinc}[\pi(\omega-10)]\right\} e^{-j2\pi\omega}$

4.3-7 $f(t) = \frac{2}{\pi} \operatorname{Sinc}^2(t) \cos 4t$

4.3-10 $F(\omega) = \frac{j4}{\omega} \sin^2\left(\frac{\omega T}{2}\right)$

4.3-11 (b) $te^{-at}u(t) \longleftrightarrow \frac{1}{(j\omega + a)^2}$

4.4-1 (a) $y(t) = (e^{-t} - e^{-2t})u(t)$

(b) $y(t) = te^{-at}u(t)$

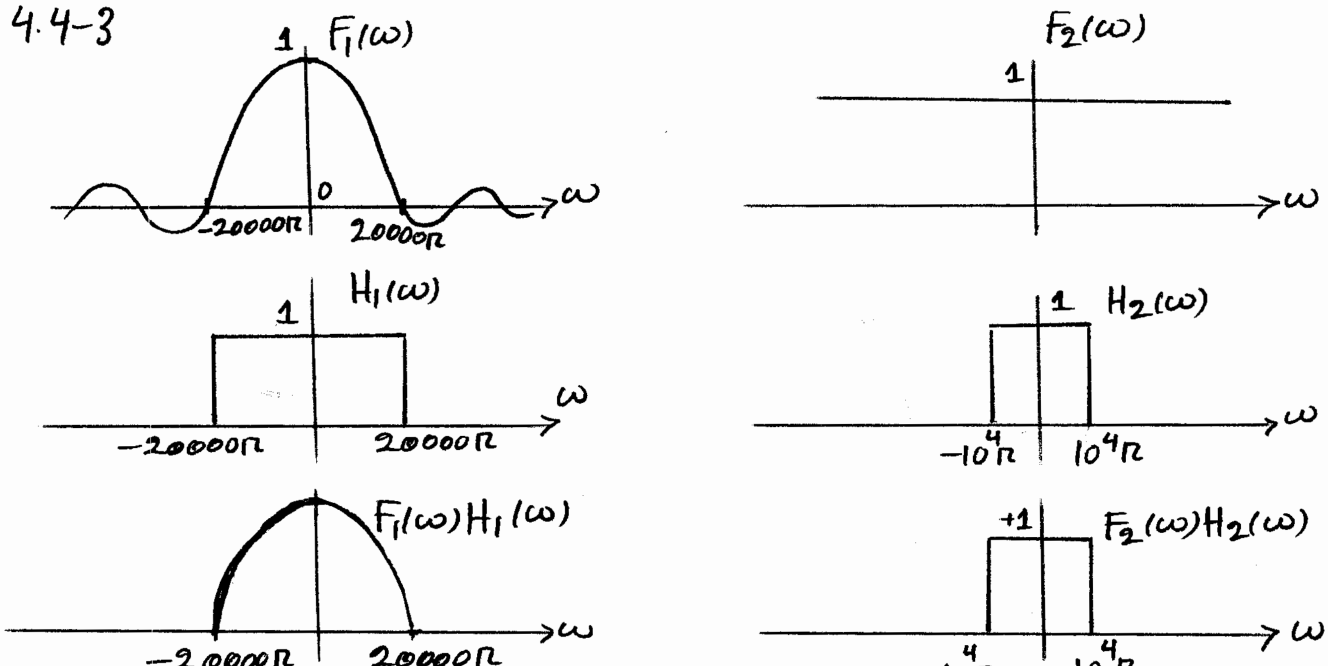
(c) $y(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)$

(d) $y(t) = (1 - e^{-t})u(t)$

4.4-2 (a) $y(t) = \frac{1}{3}\left[e^{-t}u(t) + e^{2t}u(-t)\right]$

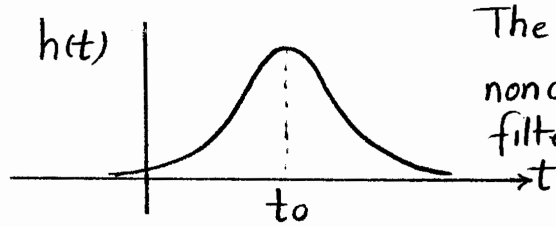
(b) $y(t) = (e^{t} - e^{2t})u(-t)$

4.4-3



The bandwidth of $Y(\omega)$ is 15 KHz.

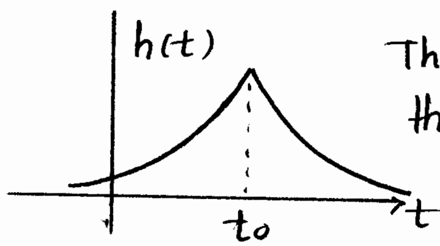
4.5-1 $h(t) = \frac{1}{\sqrt{4\pi k}} e^{-(t-t_0)^2/4k}$



The impulse response is noncausal. Hence the filter is unrealizable.

h(t) is a Gaussian function delayed by t_0 as shown in the above figure. Choosing $t_0 = 3\sqrt{2k}$, $h(0) = e^{-4.5} = 0.011$ or 1.1% of its peak value. Hence $t_0 = 3\sqrt{2k}$ is a reasonable choice to make the filter approximately realizable.

4.5-2 $h(t) = e^{-10^5|t-t_0|}$



The impulse response is noncausal, and the filter is unrealizable.

The exponential delays to 1.8% at 4 times constants. Hence $t_0 = 4/a = 40\mu s$ is a reasonable choice to make this filter approximately realizable.

4.5-3 (a) $h(t) = 0.5\,\text{rect}\left(\frac{t}{2\times10^{-6}}\right)$.

Dalying h(t) by 1 μs makes it realizable.

(b) $h(t) = \text{Sinc}^2(10{,}000\pi t)$

(c) $h(t) = 1$

4.6-5 Using duality property:

$$\frac{2a}{t^2+a^2} \longleftrightarrow 2\pi e^{-a|\omega|}$$

The signal energy is given by:

$$E_f = \frac{1}{\pi}\int_0^{\infty} \left|2\pi e^{-a\omega}\right|^2 d\omega = 4\pi \int_0^{\infty} e^{-2a\omega} d\omega = 2\pi/a$$

The energy contained within the band (0 to w) is:

$$E_W = 4\pi \int_0^{W} e^{-2aW} dW = \frac{2\pi}{a}\left(1 - e^{-2aW}\right)$$

If $E_W = 0.99\, E_f$ then:

$$e^{-2aW} = 0.01 \longrightarrow W = \frac{2.3025}{a} \text{ rad/s} = \frac{0.366}{a} \text{ Hz}.$$

4.7-1

i) $\phi_{DSB-SC} = \frac{1}{2}\left[\underbrace{\cos(9000t)}_{LSB} + \underbrace{\cos(11000t)}_{USB}\right]$

ii) $\phi_{DSB-SC} = \underbrace{\left[\cos(9000t) + \frac{1}{2}\cos(8000t)\right]}_{LSB} + \underbrace{\left[\cos(11000t) + \frac{1}{2}\cos(12000t)\right]}_{USB}.$

iii) $\phi_{DSB-SC} = \underbrace{\frac{1}{2}\left[\cos(8000t) + \cos(6000t)\right]}_{LSB} + \underbrace{\frac{1}{2}\left[\cos(12000t) + \cos(14000t)\right]}_{USB}$