Chapter 6 (detailed Solutions of selected problems)

Prepared by: Payam Abolghasem (payam@grads.ece.mcmaster.ca).

6.1-1.

(a) $f(t)=u(t)-u(t-1) \longrightarrow F(s)=\int_0^1 e^{-st}dt = -\frac{e^{-st}}{s}\Big|_0^1 = -\frac{1}{s}(e^{-s}-1)=\frac{1}{s}(1-e^{-s})$.

The region of convergence is the entire s-plane. The abscissa of convergence is $\sigma_0=-\infty$

(c) $f(t)=t\cos\omega_0 t\, u(t) \longrightarrow F(s)=\int_0^\infty t\cos(\omega_0 t)e^{-st}dt$

$\rightarrow F(s)=\frac{1}{2}\left\{\int_0^\infty\left[te^{(j\omega_0-s)t}+te^{-(j\omega_0+s)t}\right]dt\right\}=\frac{1}{2}\left[\frac{1}{(s-j\omega_0)^2}+\frac{1}{(s+j\omega_0)^2}\right]$ $Re(s)>0$

$=\frac{s^2-\omega_0^2}{(s^2+\omega_0^2)^2}$

(e) $f(t)=\cos\omega_1 t\cos\omega_2 t\, u(t)=\left\{\frac{1}{2}\cos[(\omega_1+\omega_2)t]+\frac{1}{2}\cos[(\omega_1-\omega_2)t]\right\}u(t)$

$\rightarrow F(s)=\frac{1}{2}\int_0^\infty \cos[(\omega_1+\omega_2)t]e^{-st}dt+\frac{1}{2}\int_0^\infty\cos[(\omega_1-\omega_2)t]e^{-st}dt$

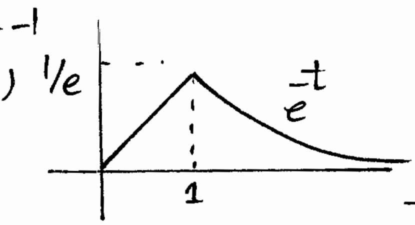
$=\frac{1}{2}\left[\frac{s}{s^2+(\omega_1+\omega_2)^2}+\frac{s}{s^2+(\omega_1-\omega_2)^2}\right]$ provided that $R(s)>0$

(g) $f(t)=\sinh(at)u(t)=\left(\frac{1}{2}e^{at}-\frac{1}{2}e^{-at}\right)u(t)$

$\rightarrow F(s)=\frac{1}{2}\left[\int_0^\infty e^{-(s-a)t}dt-\int_0^\infty e^{-(s+a)t}dt\right]=\frac{a}{s^2-a^2}$ $Re(s)>|a|$

6.2-1

(c)



$f(t)=\begin{cases}\frac{t}{e} & 0\le t<1\\ e^{-t} & t>1\end{cases}$

$\rightarrow F(s)=\int_0^1\frac{t}{e}e^{-st}dt+\int_1^\infty e^{-t}e^{-st}dt$

$=\frac{1}{e}\int_0^1 te^{-st}dt+\int_1^\infty e^{-(s+1)t}dt$

$\rightarrow F(s)=\frac{e^{-st}}{es}(-st-1)\Big|_0^1-\frac{1}{(s+1)}e^{-(s+1)}\Big|_1^\infty=\frac{1}{es^2}(1-e^{-s}-se^{-s})+\frac{1}{s+1}e^{-(s+1)}$

6.3-1

(c) $F(s)=\dfrac{(s+1)^2}{s^2-s-6}=1+\dfrac{3s+7}{s^2-s-6}=1+\dfrac{3s+7}{(s+2)(s-3)}$

Define $G(s)=\dfrac{3s+7}{(s+2)(s-3)}=\dfrac{A}{s+2}+\dfrac{B}{s-3}$: $A=(s+2)G(s)\Big|_{s=-2}=\dfrac{3s+7}{s-3}\Big|_{s=-2}=-0.2$

$B=(s-3)G(s)\Big|_{s=3}=\dfrac{3s+7}{s+2}\Big|_{s=+3}=3.2$

$F(s)=1+G(s)=1-\dfrac{0.2}{s+2}+\dfrac{3.2}{s-3}$

$\rightarrow f(t)=\delta(t)+\left(3.2e^{3t}-0.2e^{-0.2t}\right)u(t)$

(d) $F(s)=\dfrac{5}{s^2(s+2)}=\dfrac{A}{s}+\dfrac{B}{s^2}+\dfrac{C}{s+2}$

$A=\dfrac{d}{ds}\left[s^2F(s)\right]_{s=0}=\dfrac{d}{ds}\left(\dfrac{5}{s+2}\right)_{s=0}=\left(\dfrac{-5}{(s+2)^2}\right)_{s=0}=-1.25$

$B=s^2F(s)\Big|_{s=0}=\dfrac{5}{(s+2)}\Big|_{s=0}=2.5$

$C=(s+2)F(s)\Big|_{s=-2}=\dfrac{5}{s^2}\Big|_{s=-2}=1.25$

$\rightarrow F(s)=-\dfrac{1.25}{s}+\dfrac{2.5}{s^2}+\dfrac{1.25}{s+2}\rightarrow f(t)=1.25\left(-1+2t+e^{-2t}\right)u(t)$

(e) $F(s)=\dfrac{2s+1}{(s+1)(s^2+2s+2)}=\dfrac{A}{s+1}+\dfrac{Bs+C}{s^2+2s+2}$ (I)

$A=(s+1)F(s)\Big|_{s=-1}=\dfrac{2s+1}{s^2+2s+2}\Big|_{s=-1}=-1$

multiplying both sides of (I) by s and let $s\rightarrow\infty$ yeilds:

$0=-1+B\rightarrow B=+1$

Setting both sides of (I) with $s=0$ yeilds:

$\dfrac{1}{2}=-1+\dfrac{C}{2}\rightarrow C=3$

$\rightarrow F(s)=\dfrac{-1}{s+1}+\dfrac{s+3}{s^2+2s+2}$ Pair (10.c) of Table 6.1 can be used to determine the inverse Laplace transform of second fraction:

with $a=1$, $b=1$, $c=2$:

$$r=\sqrt{\frac{2+9-6}{2-1}}=\sqrt{5}\quad,\quad \theta=\tan^{-1}\left(-\frac{2}{1}\right)=-63.4^\circ$$

$$\rightarrow f(t)=\left[-e^{-t}+\sqrt{5}\,e^{-t}\cos(t-63.4^\circ)\right]u(t)$$

(i) $$F(S)=\frac{S^3}{(S+1)^2(S^2+2S+5)}=\frac{A}{S+1}+\frac{B}{(S+1)^2}+\frac{CS+D}{S^2+2S+5}\qquad (I)$$

$$A=\frac{d}{dS}\left[(S+1)^2F(S)\right]_{S=-1}=\frac{d}{dS}\left[\frac{S^3}{S^2+2S+5}\right]_{S=-1}=3/4$$

$$B=(S+1)^2F(S)\Big|_{S=-1}=\frac{S^3}{(S^2+2S+5)}\Big|_{S=-1}=-1/4$$

Multiplying both sides of (I) by s and let $s\to\infty$

$$1=A+C\rightarrow C=1-3/4=1/4$$

Set $S=0$ in both sides:

$$0=A+B+\frac{D}{5}\rightarrow D=-5(A+B)=-5/2$$

$$\rightarrow F(S)=\frac{3/4}{S+1}-\frac{1/4}{(S+1)^2}+\frac{1}{4}\left(\frac{S-10}{S^2+2S+5}\right)$$

Use pair (10.c) of Table 6.1 for last term with $a=1$, $b=2$ and $c=5$;

$$r=\sqrt{\frac{5+100+20}{5-1}}=5.59\quad,\quad \theta=\tan^{-1}(11/4)=70^\circ$$

Therefore:

$$f(t)=\left(\frac{3}{4}e^{-t}-\frac{1}{4}te^{-t}+\frac{5.59}{4}\cos(2t+70^\circ)\,e^{-t}\right)u(t)\,.$$

$$\rightarrow f(t)=\left[\frac{1}{4}(3-t)+1.3975\cos(2t+70^\circ)\right]e^{-t}u(t)$$

▨

6.2-1 (d) $f(t)=e^{-t}u(t-\tau)=e^{-\tau}\cdot e^{+\tau}e^{-t}u(t-\tau)=e^{-\tau}\cdot e^{-(t-\tau)}u(t-\tau)$

Observe that $e^{-(t-\tau)}u(t-\tau)$ is $e^{-t}u(t)$ delayed by τ. Therefore:

$F(s)=e^{-\tau}\left(\frac{1}{s+1}\right)e^{-s\tau}=\left(\frac{1}{s+1}\right)e^{-(s+1)\tau}$.

(e) $f(t)=te^{-t}u(t-\tau)=(t-\tau+\tau)e^{-\tau}e^{\tau}e^{-t}u(t-\tau)$

$=[(t-\tau)+\tau]e^{-\tau}e^{-(t-\tau)}u(t-\tau)$

$=(t-\tau)e^{-\tau}\cdot e^{-(t-\tau)}u(t-\tau)+\tau e^{-\tau}e^{-(t-\tau)}u(t-\tau)$.

$\rightarrow F(s)=e^{-\tau}\frac{1}{(s+1)^2}e^{-s\tau}+\frac{\tau}{s+1}e^{-s\tau}$

$\rightarrow F(s)=\frac{e^{-(s+1)\tau}[1+\tau(s+1)]}{(s+1)^2}$

(h) $f(t)=\sin(\omega_0 t)u(t-\tau)=\sin[\omega_0(t-\tau+\tau)]u(t-\tau)$

$=\cos(\omega_0\tau)\sin[\omega_0(t-\tau)]u(t-\tau)+\sin(\omega_0\tau)\cos[\omega_0(t-\tau)]u(t-\tau)$

$\rightarrow F(s)=\left[\cos(\omega_0\tau)\left(\frac{\omega_0}{s^2+\omega_0^2}\right)+\sin(\omega_0\tau)\left(\frac{s}{s^2+\omega_0^2}\right)\right]e^{-s\tau}$

6.2-2

(a) $f(t)=t[u(t)-u(t-1)]=tu(t)-tu(t-1)$

$=tu(t)-(t-1+1)u(t-1)$

$=tu(t)-(t-1)u(t-1)-u(t-1)$

$\rightarrow F(s)=\frac{1}{s^2}-\frac{1}{s^2}e^{-s}-\frac{1}{s}e^{-s}$

(b) $f(t)=\sin t\, u(t)+\sin(t-\pi)u(t-\pi)\longrightarrow F(s)=\frac{1}{s^2+1}(1+e^{-\pi s})$

(c) $f(t)=t[u(t)-u(t-1)]+e^{-t}u(t-1)$

$=tu(t)-(t-1)u(t-1)-u(t-1)+e^{-1}e^{-(t-1)}u(t-1)$

Therefore: $F(s) = \frac{1}{s^2}(1 - e^{-s} - s e^{-s}) + \frac{e^{-s}}{e(s+1)}$

▨

6.2-3

(a) $F(s) = \frac{(2s+5)e^{-2s}}{s^2+5s+6} = \hat{F}(s) e^{-2s}$

Using time-shift property it is clear that $f(t) = \hat{f}(t-2)$

$\hat{F}(s) = \frac{2s+5}{s^2+5s+6} = \frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$\hat{f}(t) = (e^{-2t} + e^{-3t})u(t) \to f(t) = \hat{f}(t-2) = [e^{-2(t-2)} + e^{-3(t-2)}]u(t-2)$

(d) $F(s) = \frac{e^{-s}+e^{-2s}+1}{s^2+3s+2} = (e^{-s}+e^{-2s}+1)\left(\frac{1}{s^2+3s+2}\right) = (e^{-s}+e^{-2s}+1)\left(\frac{1}{s+1} - \frac{1}{s+2}\right)$

Define $F(s) = (e^{-s}+e^{-2s}+1)\hat{F}(s)$ where $\hat{F}(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$\to \hat{f}(t) = (e^{-t} - e^{-2t})u(t)$.

Moreover

$f(t) = \hat{f}(t-1) + \hat{f}(t-2) + \hat{f}(t)$

$\to f(t) = [e^{-(t-1)} - e^{-2(t-1)}]u(t-1) + [e^{-(t-2)} - e^{-2(t-2)}]u(t-2) + (e^{-t} - e^{-2t})u(t).$

▨

6.2-4

(a) $y(t) = f(t) + f(t-T_0) + f(t-2T_0) + \cdots$

and $G(s) = F(s) + F(s)e^{-sT_0} + F(s)e^{-2sT_0} + \cdots$

$= F(s)[1 + e^{-sT_0} + e^{-2sT_0} + e^{-3sT_0} + \cdots] = \frac{F(s)}{1-e^{-sT_0}}$ $|e^{-sT_0}| < 1$ or $Re(s) > 0$

(b) $f(t) = u(t) - u(t-2)$ and $F(s) = \frac{1}{s}(1-e^{-2s})$

$G(s) = \frac{F(s)}{1-e^{-8s}} = \frac{1}{s}\left(\frac{1-e^{-2s}}{1-e^{-8s}}\right).$

▨

6.3-1

(c) $(D^2+6D+25)y(t)=(D+2)f(t)$.

$$\rightarrow \left[s^2Y(s)-sy(0^-)-\dot{y}(0^-)+6\left(sY(s)-y(0^-)\right)+25Y(s)\right]=\left(sF(s)-f(0^-)+2F(s)\right)$$

$$\rightarrow \left[(s^2Y(s)-s-1)+6(sY(s)-1)+25Y(s)\right]=(s+2)\left(\frac{25}{s}\right)=25+\frac{50}{s}$$

$$\rightarrow (s^2+6s+25)Y(s)=s+32+\frac{50}{s}=\frac{s^2+32s+50}{s}$$

$$\rightarrow Y(s)=\frac{s^2+32+50}{s(s^2+6s+25)}=\frac{2}{s}+\frac{-s+20}{s^2+6s+25}$$

$$\rightarrow y(t)=\left[2+5.836e^{-3t}\cos(4t-99.86^\circ)\right]u(t).$$

6.3-2

(a) All initial condition are zero. The zero input response is zero. The entire response found in problem 6.3-2a is zero-state response. Hence:

$$y_{zs}(t)=(e^{-t}-e^{-2t})u(t)$$
$$y_{zi}(t)=0$$

(b) The Laplace transform of the differential equation is:

$$\left(s^2Y(s)-2s-1\right)+4\left(sY(s)-2\right)+4Y(s)=(s+1)\frac{1}{s+1}$$

or
$$(s^2+4s+4)Y(s)-\underbrace{(2s+9)}_{\text{Initial Cond. term}}=\underbrace{1}_{\text{Input}}$$

or
$$Y(s)=\underbrace{\frac{2s+9}{s^2+4s+4}}_{\text{Zero-Input}}+\underbrace{\frac{1}{s^2+4s+4}}_{\text{Zero-state}}=\underbrace{\frac{2}{s+2}+\frac{5}{(s+2)^2}}_{\text{zero-input}}+\underbrace{\frac{1}{(s+2)^2}}_{\text{zero-state}}$$

$$\rightarrow y(t)=\underbrace{(2+5t)e^{-2t}}_{\text{zero-input}}+\underbrace{te^{-2t}}_{\text{zero-state}}$$

(c) The Laplace transform of the equation is:

$$\left(s^2Y(s)-s-1\right)+6\left(sY(s)-1\right)+25Y(s)=25+\frac{50}{s}$$

or $$(s^2+6s+25)Y(s)=\underbrace{(s+7)}_{\text{Initial Cond. term}}+\underbrace{\left(25+\frac{50}{s}\right)}_{\text{Input}}$$

$$\rightarrow Y(s)=\underbrace{\frac{s+7}{s^2+6s+25}}_{\text{zero-input}}+\underbrace{\frac{25+50}{s(s^2+6s+25)}}_{\text{zero-state}}=\underbrace{\left(\frac{s+7}{s^2+6s+25}\right)}_{\text{zero-input}}+\underbrace{\left(\frac{2}{s}+\frac{-2s+13}{s^2+6s+25}\right)}_{\text{zero-state}}.$$

$$\rightarrow y(t)=\underbrace{\left[\sqrt{2}e^{-3t}\cos\left(4t-\frac{\pi}{4}\right)\right]}_{\text{zero-input}}+\underbrace{\left[2+5.154e^{-3t}\cos(4t-112.83^\circ)\right]}_{\text{zero-state.}}$$

6.3-3

(a) Laplace transform of the two equations yeilds:

$$\begin{cases}(s+3)Y_1(s)-2Y_2(s)=\frac{1}{s}\\ -2Y_1(s)+(2s+4)Y_2(s)=0\end{cases}$$

In matrix form:

$$\begin{bmatrix}(s+3) & -2\\ -2 & (2s+4)\end{bmatrix}\begin{bmatrix}Y_1(s)\\ Y_2(s)\end{bmatrix}=\begin{bmatrix}\frac{1}{s}\\ 0\end{bmatrix}.$$

let Define $A=\begin{bmatrix}s+3 & -2\\ -2 & (2s+4)\end{bmatrix}\longrightarrow$ $\det(A)=2(s^2+5s+4)$

$$A^{-1}=\begin{bmatrix}\frac{s+2}{s^2+5s+4} & \frac{1}{s^2+5s+4}\\ \frac{1}{s^2+5s+4} & \frac{s+3}{2(s^2+5s+4)}\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}Y_1(s)\\ Y_2(s)\end{bmatrix}=A^{-1}\begin{bmatrix}\frac{1}{s}\\ 0\end{bmatrix}\longrightarrow\begin{cases}Y_1(s)=\dfrac{s+2}{s(s^2+5s+4)}=\dfrac{1/2}{s}-\dfrac{1/3}{s+1}-\dfrac{1/6}{s+4}\\[2ex] Y_2(s)=\dfrac{1}{s(s^2+5s+4)}=\dfrac{1/4}{s}-\dfrac{1/3}{s+1}+\dfrac{1/12}{s+4}\end{cases}$$

Therefore:

$$y_1(t) = \left(\frac{1}{2} - \frac{1}{3}e^{-t} - \frac{1}{6}e^{-4t}\right)u(t)$$

$$y_2(t) = \left(\frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t}\right)u(t).$$

If $H_1(s)$ and $H_2(s)$ are the transfer functions relating $y_1(t)$ and $y_2(t)$, respectively to the input $f(t)$, thus:

$$H_1(s) = \frac{s+2}{s^2+5s+4} \quad \text{and} \quad H_2(s) = \frac{1}{s^2+5s+4}$$

6.3-4

At $t=0^-$ the inductor current $y_1(0)=4$ and the capacitor voltage is 16 volts. After $t=0$, the loop equations are:

$$\frac{2dy_1}{dt} - 2\frac{dy_2}{dt} + 5y_1(t) - 4y_2(t) = 40$$

$$-2\frac{dy_1}{dt} - 4y_1(t) + 2\frac{dy_2}{dt} + 4y_2(t) + \int_{-\infty}^{t} y_2(\tau)d\tau = 0$$

If $y_1(t) \longleftrightarrow Y_1(s) \Rightarrow \frac{dy_1(t)}{dt} = sY_1(s) - 4$

$y_2(t) \longleftrightarrow Y_2(s) \Rightarrow \frac{dy_2(t)}{dt} = sY_2(s)$

$$\int_{-\infty}^{t} y_2(\tau)d\tau = \frac{1}{s}Y_2(s) + \frac{16}{s}$$

Laplace transform of the loop equations are:

$$2(sY_1(s)-4) - 2sY_2(s) + 5Y_1(s) - 4Y_2(s) = \frac{40}{s}$$

$$-2(sY_1(s)-4) - 4Y_1(s) + 2sY_2(s) + 4Y_2(s) + \frac{1}{s}Y_2(s) + \frac{16}{s} = 0$$

or

$$(2s+5)Y_1(s) - (2s+4)Y_2(s) = 8 + \frac{40}{s}$$

$$-(2s+4)Y_1(s) + \left(2s+4+\frac{1}{s}\right)Y_2(s) = -8 - \frac{16}{s}$$

The above set of equations can be written in matrix form as:

$$AY(s) = B \quad \text{where} \quad A = \begin{bmatrix} 2s+5 & -(2s+4) \\ -(2s+4) & (2s+4+\frac{1}{s}) \end{bmatrix};\ Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix},\ B = \begin{bmatrix} 8+\frac{40}{s} \\ -8-\frac{16}{s} \end{bmatrix}$$

$$\det(A) = \frac{2s^2+6s+5}{s}, \quad A^{-1} = \begin{bmatrix} \frac{2s^2+4s+1}{2s^2+6s+5} & \frac{s(2s+4)}{2s^2+6s+5} \\ \frac{s(2s+4)}{2s^2+6s+5} & \frac{s(2s+5)}{2s^2+6s+5} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = A^{-1} \begin{bmatrix} \frac{8s+40}{s} \\ -\left(\frac{8s+16}{s}\right) \end{bmatrix}.$$

replacing A^{-1} in the above linear equation and after some algebraic simplification, we get:

$$Y_1(s) = \frac{4(6s^2+13s+5)}{s(s^2+3s+2.5)} = \frac{8}{s} + \frac{16s+28}{s^2+3s+2.5}$$

$$\Rightarrow y_1(t) = \left[8 + 17.89\, e^{-1.5t} \cos\left(\frac{t}{2} - 26.56^\circ\right)\right] u(t)$$

$$Y_2(s) = \frac{20(s+2)}{s^2+3s+2.5} \rightarrow y_2(t) = 20\sqrt{2}\, e^{-1.5t} \cos\left(\frac{t}{2} - \frac{\pi}{4}\right) u(t).$$

6.3-7

(a)

(ii) $f(t) = e^{-4t} u(t) \longleftrightarrow F(s) = \frac{1}{s+4}$

$$H(s) = \frac{Y(s)}{F(s)} \rightarrow Y(s) = \left(\frac{1}{s+4}\right)\left(\frac{s+5}{s^2+5s+6}\right) = \frac{(s+5)}{(s+4)(s+2)(s+3)}$$

$$= \frac{3/2}{s+2} - \frac{2}{s+3} + \frac{1/2}{(s+4)}$$

$$\rightarrow y(t) = \left(\frac{3}{2} e^{-2t} - 2e^{-3t} + \frac{1}{2} e^{-4t}\right) u(t).$$

(iv) The input here is equal to the input in (ii) multiplied by e^{20} because $e^{-4(t-5)} = e^{20} e^{-4t}$. Therefore the output is equal to the output in (ii) multiplied by e^{20}:

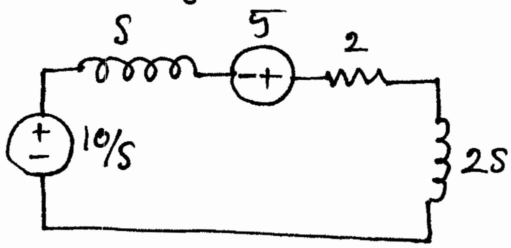
$$y(t) = e^{20}\left(\frac{3}{2} e^{-2t} - 2e^{-3t} + \frac{1}{2} e^{-4t}\right) u(t).$$

b) $H(S) = \dfrac{Y(S)}{F(S)} = \dfrac{S+5}{S^2+5S+6} \longrightarrow (S^2+5S+6)\,Y(S) = (S+5)\,F(S).$

$\longrightarrow (D^2+5D+6)\,y(t) = (D+5)\,f(t)$

6.4-2

Before the switch is opened, the inductor current is 5A, that is $y(0)=5$. The following figure shows the transformed cirut for $t \geq 0$ with initial condition generator. The current $Y(S)$ is given by:



$$Y(S) = \frac{(10/S)+5}{3S+2} = \frac{5S+10}{S(3S+2)} = \frac{5}{3}\left[\frac{3}{S} - \frac{2}{S+(2/3)}\right]$$

$$\rightarrow y(t) = \left(5 - \frac{10}{3}e^{-2t/3}\right)u(t)$$

6.4-4

At $t=0$ the steady-state values of currents y_1 and y_2 is $y_1(0)=2$, $y_2(0)=1$. The following figure shows the transformed circuit for $t \geq 0$ with initial condition generators. The loop equations are:

$$(S+2)Y_1(S) - Y_2(S) = 2 + \frac{6}{S}$$

$$-Y_1(S) + (S+2)Y_2(S) = 1$$

Using the same approach in problems 6.3-3 and 6.3-4, we can calculate $Y_1(S)$ and $Y_2(S)$:

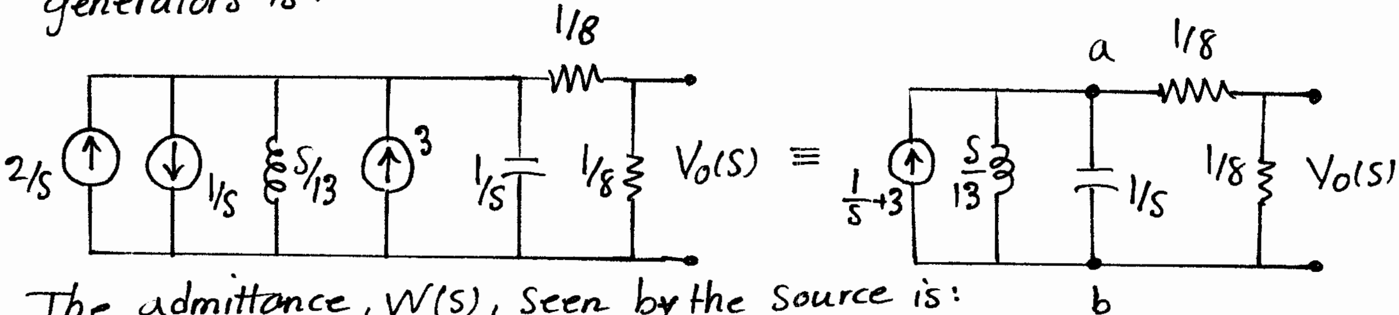
$$Y_1(S) = \frac{2S^2+11S+12}{S(S+1)(S+3)} = \frac{4}{S} \quad \frac{3/2}{S+1} - \frac{1/2}{S+3}$$

$$Y_2(S) = \frac{S^2+4S+6}{S(S+1)(S+3)} = \frac{2}{S} - \frac{3/2}{S+1} + \frac{1/2}{S+3}$$

$$\rightarrow \begin{cases} y_1(t) = \left(4 - \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t}\right)u(t) \\ y_2(t) = \left(2 - \frac{3}{2}e^{-t} + \frac{1}{2}e^{-3t}\right)u(t) \end{cases}$$

6.4-7

The transformed circuit with parallel form of initial condition generators is:



The admittance, $W(S)$, seen by the source is:

$$W(S) = \frac{13}{S} + S + 4 = \frac{S^2+4S+13}{S}$$

The voltage accross terminal a-b is:

$$V_{ab} = \frac{I(S)}{W(S)} = \frac{\frac{1}{S}+3}{\frac{S^2+4S+13}{S}} = \frac{3S+1}{S^2+4S+13}$$

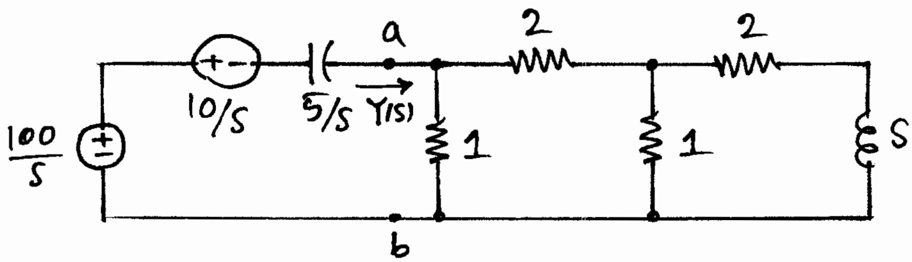
Also

$$V_o(S) = \frac{1}{2}V_{ab} = \frac{3S+1}{2(S^2+4S+13)} \longrightarrow v_o(t) = 1.716\, e^{-2t}\cos(3t+29^\circ)u(t)$$

▨

6.4-8

The capacitor voltage at $t=0$ is 10 volts. The inductor current is zero. The transformed circuit with initial condition generators is shown below for $t>0$:



To determine the current $Y(S)$, we determine $Z_{ab}(S)$:

$$Z_{ab}(S) = \frac{1}{1+\left(\frac{1}{2+\frac{S+2}{S+3}}\right)} = \frac{3S+8}{4S+11}$$

Also

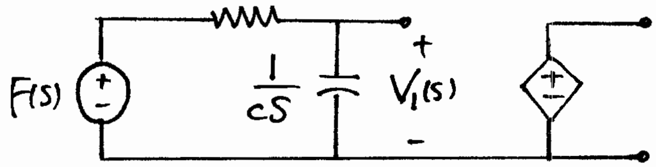
$$Y(S) = \frac{\frac{90}{S}}{\frac{5}{S}+\left(\frac{3S+8}{4S+11}\right)} = \frac{90(4S+11)}{3S^2+28S+55} = \frac{30(4S+11)}{S^2+\frac{28}{3}S+\frac{55}{3}} = \frac{30(4S+11)}{(S+2.8)(S+6.53)}$$

$$\rightarrow Y(S) = -\frac{1.61}{S+2.8} + \frac{121.61}{S+6.53} \longrightarrow y(t) = (121.6\,e^{-6.53t} - 1.61\,e^{-2.8t})\,u(t).$$

▨

6.4-9

The transfered circuit with non-inverting op-amp replaced by its equivalent is shown below:



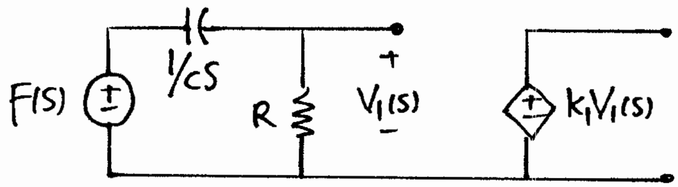
(a)

$$V_o(S) = K V_1(S) = K \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} F(S) = \frac{Ka}{S+a} \qquad a = \frac{1}{RC}$$

Therefore

$$H(S) = \frac{Ka}{S+a} \qquad a = \frac{1}{RC}, \quad K = 1 + \frac{R_b}{R_a}$$

Similarly for the circuit shown in Fig. P6.4-9:



From Figure we can see that: $H(S) = \frac{Ks}{S+a}$

▨

6.4-11 (a) $Y(S) = \frac{6S^2+3S+10}{S(2S^2+6S+5)}$

$$y(0^+) = \lim_{S\to\infty} sY(S) = 3$$

$$y(\infty) = \lim_{S\to 0} sY(S) = 2$$

(b) $Y(S) = \frac{6S^2+3S+10}{(S+1)(2S^2+6S+5)}$

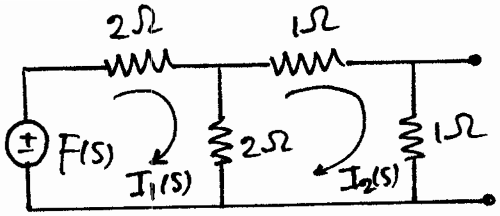
$$y(0^+) = \lim_{S\to 0} sY(S) = 3$$

$$y(\infty) = \lim_{S\to 0} sY(S) = 0$$

▨

6.5.1

(a)



The loop equations are:

For left loop: $4I_1(S) - 2I_2(S) = F(S)$.

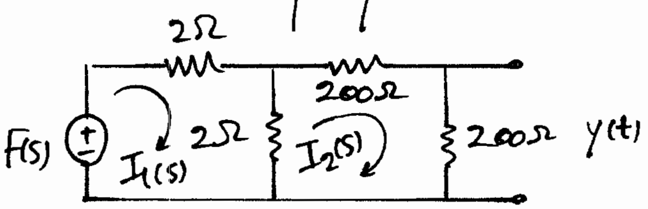
For right " : $-2I_1(S) + 4I_2(S) = 0$

Solving for $I_1(S)$ and $I_2(S)$ we get $\begin{cases} I_1(S) = F(S)/5 \\ I_2(S) = F(S)/10 \end{cases}$

$$Y(S) = I_2(S)(1\Omega) = F(S)/10 \rightarrow H(S) = \frac{Y(S)}{F(S)} = 1/10$$

Therefore $H(S) = 1/10$ not $1/4$.

(b) The loop equations when $R_3 = R_4 = 200\Omega$ are:



$$\begin{cases} 4I_1(S) - 2I_2(S) = F(S). \\ -2I_1(S) + 402\,I_2(S) = 0 \end{cases}$$

Solving for $I_1(S)$ and $I_2(S)$ yeilds:

$I_1(S) = 0.2506\,F(S)$

$I_2(S) = 0.0012\,F(S)$.

$Y(S) = 200\,I_2(S) = 0.24\,F(S)$.

$H(S) = \dfrac{Y(S)}{F(S)} = 0.24$

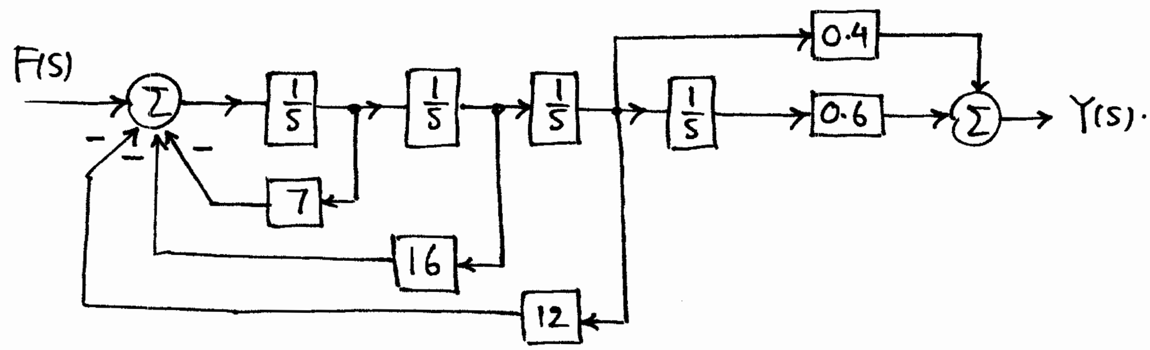
In this case $H(S)$ is very close to $1/4 = 0.25$. This is because the second ladder section causes a negligible load on the first. The cascade rule implies only when the successive subsystems do not load the preceding subsystems.

6.6-3

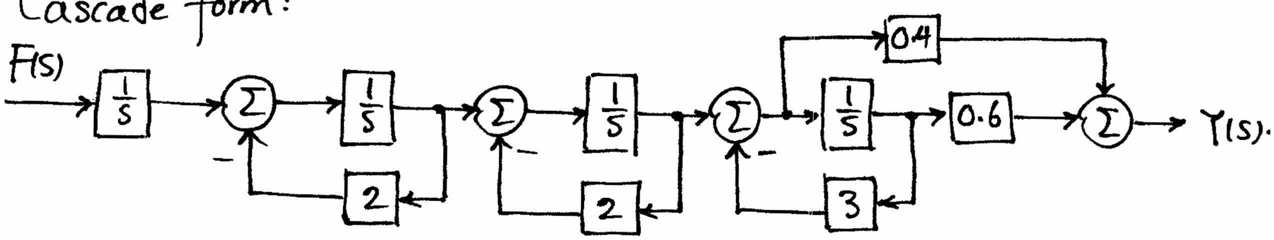
$$H(S) = \frac{2S+3}{5(S^4+7S^3+16S^2+12S)} = \frac{0.4S+0.6}{S^4+7S^3+16S^2+12S}$$

$$= \left(\frac{1}{S}\right)\left(\frac{1}{S+2}\right)\left(\frac{1}{S+2}\right)\left(\frac{0.4S+0.6}{S+3}\right) = \frac{\frac{1}{20}}{S} - \frac{\frac{1}{4}}{S+2} + \frac{\frac{1}{10}}{(S+2)^2} + \frac{\frac{1}{5}}{S+3}$$

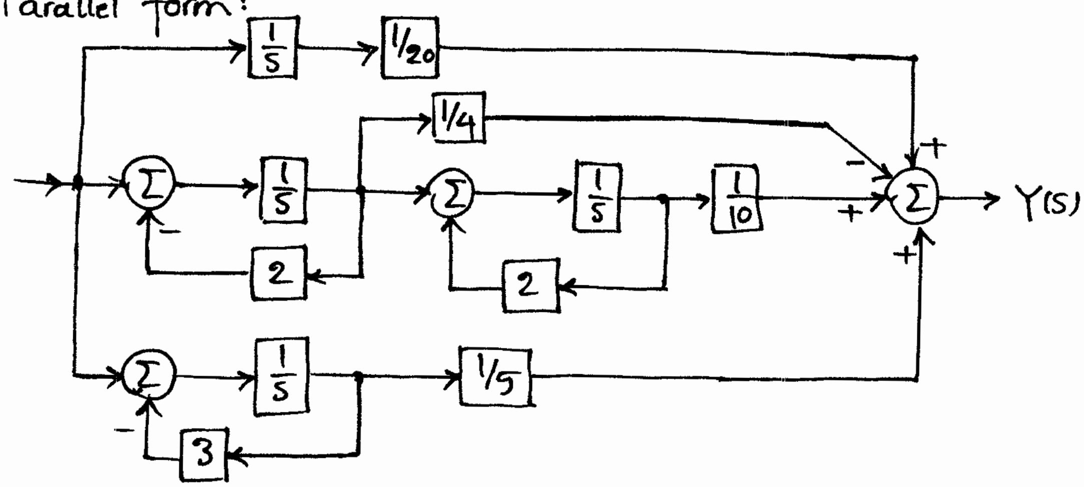
Canonical form:



Cascade form:



Parallel form:

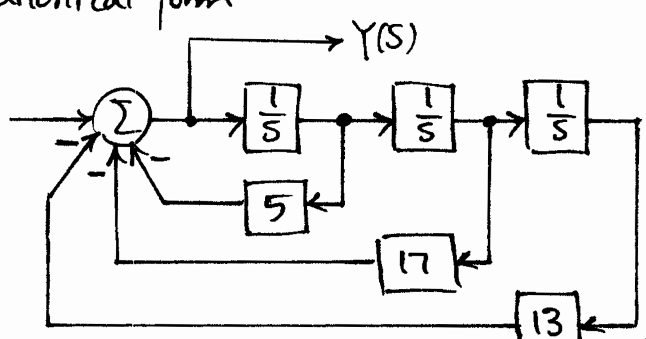


6.6-6

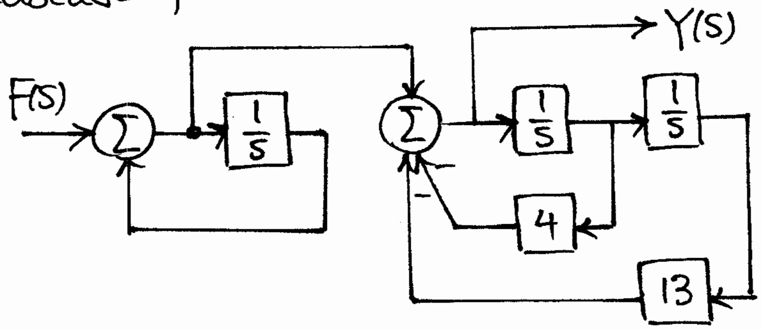
$$H(s) = \frac{s^3}{(s+1)(s^2+4s+13)} = \frac{s^3}{s^3+5s^2+17s+13}$$

$$= \left(\frac{s}{s+1}\right)\left(\frac{s^2}{s^2+4s+13}\right) = \frac{-0.1}{s+1} + \frac{s^2-0.9s+1.3}{s^2+4s+13} = 1 - \frac{0.1}{s+1} - \frac{4.9s+11.7}{s^2+4s+13}$$

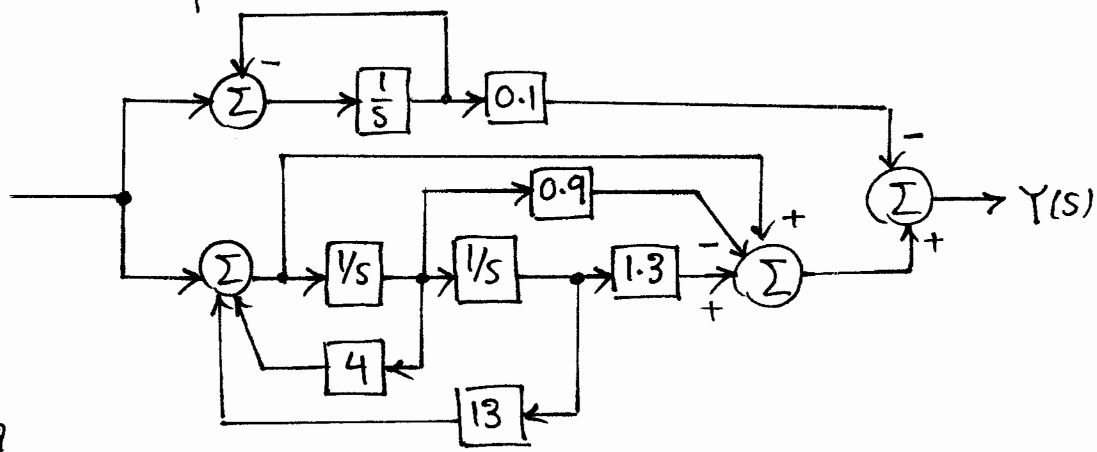
Canonical form



Cascade form:



Parallel form:



6.7-1.

(a) $T(S) = \dfrac{9}{s^2+3s+9} \longrightarrow \omega_n = 3\,,\; 2\zeta\omega_n = 3 \longrightarrow \zeta = 0.5$

From Fig. 6.39 (page 436), $PO \approx 17\%$ and $\omega_n t_r \approx 1.63$ which yeilds $t_r = 0.526$. Also

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.5} = 2.67 \qquad e_s = \lim_{s\to 0}[1-T(s)] = 0 \qquad e_r = \lim_{s\to 0}[1-T(s)]/s = 1/3$$

$$e_p = \lim_{s\to 0}[1-T(s)]/s^2 = \infty$$

(b) $T(S) = \dfrac{4}{s^2+3s+4} \longrightarrow \omega_n = 2\;,\; 2\zeta\omega_n = 3 \longrightarrow \zeta = 0.75$

From Fig. 6.39, $PO \approx 3\%$ and $\omega_n t_r \approx 2.3$ which yeilds $t_r = 1.15$, also

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.5} = 2.67 \qquad e_s = \lim_{s\to 0}[1-T(s)] = 0 \qquad e_r = \lim_{s\to 0}[1-T(s)]/s = 0.75$$

$$e_p = \lim_{s\to 0}[1-T(s)]/s^2 = \infty$$

6.7-2

$$T(S) = K_1\left[\frac{\frac{K_2}{S(S+a)}}{1+\frac{K_2}{S(S+a)}}\right] = \frac{K_1K_2}{S^2+aS+K_2}$$

$$PO = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.09 \rightarrow \zeta = 0.608.$$ Moreover

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \pi/4 \longrightarrow \omega_n\sqrt{1-\zeta^2} \rightarrow \omega_n = 5.04 \text{ for } \zeta = 0.608$$

Thus

$$s^2+2\zeta\omega_n s+\omega_n^2 = s^2+as+K_2 \longrightarrow a = 6.128 \text{ and } K_2 = 25.4$$

The steady_state value of the output is given to be 2. But the steady_state value of the output is:

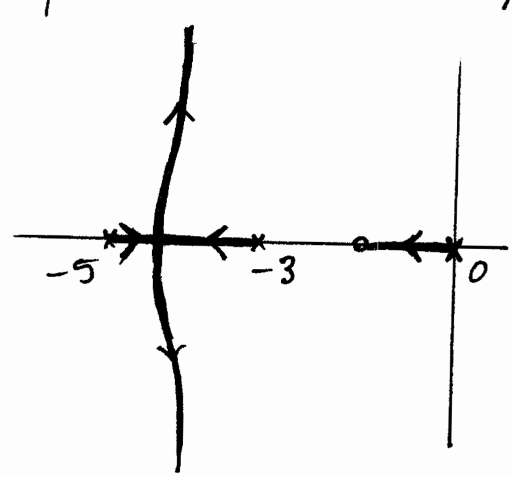
$$y_{ss} = \lim_{S\to 0} \frac{K_aK_2}{S^2+aS+K_2} = K_1 = 2$$

Thus the parameters are $K_1 = 2$, $K_2 = 25.4$ and $a = 6.128$.

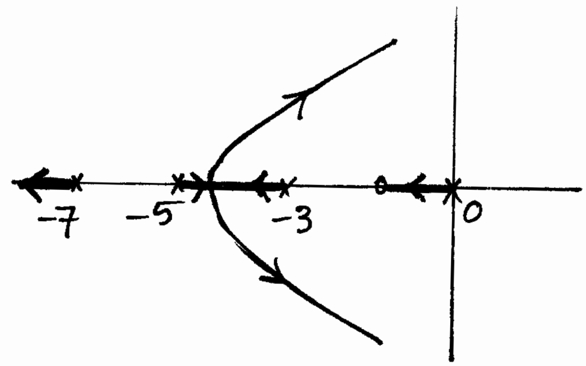
▨

6.7-4

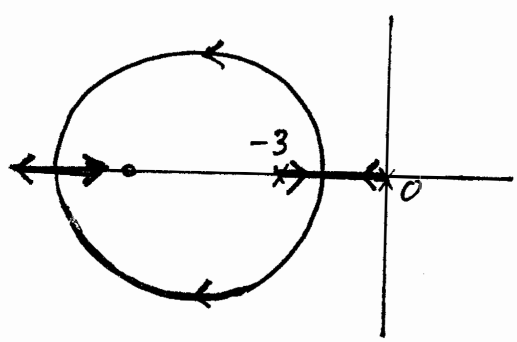
(a) The open loop poles are at $0, -3$ and -5. Hence there are three root loci starting at $0, -3$ and 5. (when $K=0$). More over, the segments 0 to -1 and -3 to -5 of the real axis are a part of the root locus. There is only one open loop zero at -1. Hence one locus will terminate at -1 (when $K=\infty$). The other two branches terminate at ∞ (when $K=\infty$) along asymptotes at angles $k\pi/n-m = \pi/2$ and $3\pi/2$. The centroid of the asymptotes is $\sigma = (0-3-5+1)/2 = -3.5$.



(b) The open loop poles are at $0, -3, -5$ and -7. Hence there are four root loci starting at $0, -3, -5$ and -7 (when $K = 0$). Moreover the entire real axis in the LHP, except two segments 0 to -3 and -5 to -7 are a part of the root locus. There is only one loop zero at -1. Hence one locus will terminate at -1 (when $K = \infty$). The other three branches terminate at ∞ (when $K = \infty$) along asymptotes at angles $k\pi/n-m = \pi/3$ and π and $5\pi/3$. The centroid of the asymptotes is $\sigma = (0-3-5-7+1)/3 = -4.67$.

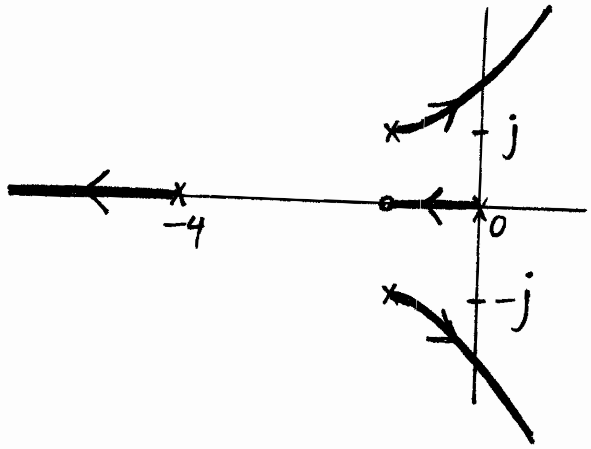


(c) The open loop poles are at $0, -3$. Hence there are two root loci Starting at 0 and -3 (when $K = 0$). Moreover the entire real axis in the LHP except the segment from -3 to -5 is a part of the root locus. There is only one open loop zero at -5. Hence one locus will terminate at -5 (when $K = \infty$). The other branch terminate at ∞ (when $K = \infty$) along asymptote at angles $k\pi/(n-m) = \pi$.



(d) The open loop poles are at $0, -4$ and $-1 \pm j$. Hence there are four root loci starting at $0, -4$ and $-1 \pm j$ (when $K = 0$). Moreover, the entire real axis, except the segment from -1 to -4 is a part of the root locus.

There is only one open loop zero at -1. Hence one locus will terminate at -1 (when $K=\infty$). The other three branches terminate at ∞ (when $K=\infty$) along asymptotes at angles $k\pi/(n-m) = \pi/3, \pi$ and $5\pi/3$. The centroid of the asymptotes is $\sigma = (0-4-1-1+1)/3 = -1.67$.



6.8-2

(b) $f(t) = e^{-|t|}\cos t = e^{-t}\cos t\, u(t) + e^{t}\cos t\, u(-t) = f_1(t) + f_2(t)$

Hence $F_1(s) = \dfrac{s+1}{(s+1)^2+1}$ and $F_2(-s) = \dfrac{s+1}{(s+1)^2+1}$ $\quad \sigma < 1$

$$F(s) = F_1(s) + F_2(s) = \frac{s+1}{(s+1)^2+1} - \frac{s-1}{(s-1)^2+1} = \frac{4-2s^2}{s^4+4} \qquad -1 < \sigma < +1$$

(d) $f(t) = e^{-tu(t)} = \begin{cases} e^{-t} & \text{for } t > 0 \\ 1 & \text{for } t < 0 \end{cases}$

$f_1(t) = e^{-t}u(t)$, $f_2(t) = u(-t)$ Hence $F_1(s) = \dfrac{1}{s+1}$ $\quad \sigma > -1$

and $F_2(-s) = \dfrac{1}{s} \rightarrow F_2(s) = -\dfrac{1}{s}$ $\quad \sigma < 0$

Hence: $F(s) = \dfrac{1}{s+1} - \dfrac{1}{s} = \dfrac{-1}{s(s+1)}$ $\quad -1 < \sigma < 0$

(f) $f(t) = \cos(\omega_0 t)u(t) + e^{t}u(-t) = f_1(t) + f_2(t)$

$F_1(s) = \dfrac{s}{s^2+\omega_0^2}$ $\quad \sigma > 0$

and $F_2(-s) = \dfrac{1}{s+1} \longrightarrow F_2(s) = \dfrac{1}{1-s}$ $\quad \sigma < 1$

$$F(s) = F_1(s) + F_2(s) = \frac{-(s+\omega_0)^2}{(s-1)(s^2+\omega_0^2)} \qquad 0<\sigma<1$$

▨

6.8-3

(c) $$F(s) = \frac{2s+3}{(s+1)(s+2)} \qquad \sigma>-1$$

$$= \frac{1}{(s+1)} + \frac{1}{(s+2)} \qquad \sigma>-1$$

both poles lie to the left of the region of convergence, and:

$$f(t) = (e^{-t} + e^{-2t})u(t).$$

(e) $$F(s) = \frac{3s^2-2s-17}{(s+1)(s+3)(s-5)} \qquad -1<\sigma<5$$

$$= \frac{1}{s+1} + \frac{1}{s+3} + \frac{1}{s-5}$$

The poles -1 and -3 lie to the left of the region of convergence, whereas the pole 5 lies to the right:

$$f(t) = (e^{-t} + e^{-3t})u(t) - e^{5t}u(-t).$$

▨

6.8-5

(a) $f(t) = e^{-|t|/2}$ $\qquad H(s) = \frac{1}{s+1} \quad \sigma>-1$

$$F(s) = \frac{1}{s+0.5} - \frac{1}{s-0.5} \qquad -\tfrac{1}{2}<\sigma<\tfrac{1}{2}$$

Hence $Y(s) = H(s)F(s) = \frac{1}{s+1}\left[\frac{1}{s+0.5} - \frac{1}{s-0.5}\right] \quad -\tfrac{1}{2}<\sigma<+\tfrac{1}{2}$

$$Y(s) = \frac{-2}{s+1} + \frac{2}{s+0.5} + \frac{2/3}{s+1} - \frac{2/3}{s-0.5}$$

$$= \frac{-4/3}{s+1} + \frac{2}{s+0.5} - \frac{2/3}{s-0.5} \qquad -1/2<\sigma<+1/2$$

The poles -1 and -0.5 which are to the left of the strip of convergence yeild the causal signal, and the pole, 0.5, which is to the right of the strip of convergence, yields the anticausal signal. Hence:

$$y(t) = \left(-\frac{4}{3}e^{-t} + 2e^{-t/2}\right)u(t) + \frac{2}{3}e^{t/2}u(-t)$$

(c) $f(t) = e^{-t/2}u(t) + e^{-t/4}u(-t)$

$$F(s) = \frac{1}{s+0.5} - \frac{1}{s+0.25} = \frac{-1/4}{(s+0.5)(s+0.25)} \qquad -\frac{1}{2} < \sigma < +\frac{1}{2}$$

Also $H(s) = \frac{1}{s+1}$ $\sigma > -1$

Hence: $$Y(s) = H(s)F(s) = \frac{-1/4}{(s+1)(s+0.5)(s+0.25)} \qquad -\frac{1}{2} < \sigma < \frac{1}{4}$$

$$= \frac{-2/3}{s+1} + \frac{2}{s+0.5} - \frac{4/3}{s+0.25} \qquad -\frac{1}{2} < \sigma < \frac{1}{4}$$

and $y(t) = \left(-\frac{2}{3}e^{-t} + 2e^{-t/2}\right)u(t) + \frac{4}{3}e^{-t/4}u(-t)$

(d) $f(t) = e^{2t}u(t) + e^{t}u(-t) = f_1(t) + f_2(t)$

$F_1(s) = \frac{1}{s-2}$ $\sigma > 2$; $F_2(s) = -\frac{1}{s-1}$ $\sigma < 1$ and $H(s) = \frac{1}{s+1}$ $\sigma > -1$

In this case there is no region of convergence that is common to $F_1(s)$ and $F_2(s)$. However each of $F_1(s)$ and $F_2(s)$ have a region of convergence that is common to $H(s)$. Hence the output can be computed by finding the system response to $f_1(t)$ and $f_2(t)$ separately and then adding these two components.

$$Y(s) = Y_1(s) + Y_2(s) \text{ where } Y_1(s) = F_1(s)H(s) = \frac{1}{(s+1)(s-2)} \qquad \sigma > 2$$

Observe that both poles $(-1, 2)$ are two the left of the region of convergence, hence both term are causal and: $y_1(t) = \left(-\frac{1}{3}e^{-t} + \frac{1}{3}e^{2t}\right)u(t)$

$$= \frac{-1/3}{s+1} + \frac{1/3}{s-2} \qquad \sigma > 2$$

$$Y_2(s) = F_2(s)H(s) = \frac{-1}{(s+1)(s-1)} = \frac{1/2}{s+1} - \frac{1/2}{s-1} \qquad -1 < \sigma < 1$$

The poles -1 and 1 are to the left and right, respectively, of the strip of convergence hence the first term yields causal signal and the second yeild anticausal signal. Hence: $y_2(t) = -\frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)$.

Therefore: $y(t) = y_1(t) + y_2(t) = \left(\frac{1}{6}e^{-t} + \frac{1}{3}e^{2t}\right)u(t) + \frac{1}{2}e^{t}u(-t)$