Chapter 7 (Answers and detailed Solutions).

Prepared by: Payam Abolghasem (payam@ece.mcmaster.ca).

7.1.1 $H(j\omega)=\dfrac{j\omega+2}{(j\omega)^2+5j\omega+4}=\dfrac{j\omega+2}{(4-\omega^2)+j5\omega}$

$$|H(j\omega)|=\sqrt{\frac{\omega^2+4}{(4-\omega^2)^2+(5\omega)^2}}=\sqrt{\frac{\omega^2+4}{\omega^4+17\omega^2+16}}$$

$$\angle H(j\omega)=\tan^{-1}\left(\frac{\omega}{2}\right)-\tan^{-1}\left(\frac{5\omega}{4-\omega^2}\right)$$

(a) $f(t)=5\cos(2t+30^\circ)$. Here $\omega=2$ and:

$$|H(j\omega)|=\sqrt{2/25}=\sqrt{2}/5 \qquad \angle H(j\omega)=\tan^{-1}(1)-\tan^{-1}(\infty)=45^\circ-90^\circ=-45^\circ$$

$$y(t)=5\frac{\sqrt{2}}{5}\cos(2t+30^\circ-45^\circ)=\sqrt{2}\cos(2t-15^\circ)$$

(b) $f(t)=10\sin(2t+45^\circ)$

$$\rightarrow y(t)=10\left(\frac{\sqrt{2}}{5}\right)\sin(2t+45^\circ-45^\circ)=2\sqrt{2}\sin 2t$$

(c) $f(t)=10\cos(3t+40^\circ)$. Here $\omega=3$:

$$|H(j\omega)|=\sqrt{\frac{13}{250}}=0.228 \text{ and } \angle H(j3)=56.31^\circ-108.43^\circ=-52.12^\circ$$

Therefore:

$$y(t)=10(0.228)\cos(3t+40^\circ-52.12^\circ)=2.28\cos(3t-12.12^\circ)$$

7.1-2

$$H(j\omega)=\frac{j\omega+3}{(j\omega+2)^2}\rightarrow |H(j\omega)|=\frac{\sqrt{\omega^2+9}}{\omega^2+4} \text{ and } \angle H(j\omega)=\tan^{-1}\left(\frac{\omega}{3}\right)-\tan^{-1}\left(\frac{\omega}{2}\right)$$

(a) $f(t)=10u(t)=10e^{j0t}u(t)$. Here $\omega=0$ and $H(j0)=1$. Therefore

$$y(t) = 1 \times 10\, e^{j0t} u(t) = 10\, u(t)$$

(b) $f(t) = \cos(2t + 60^\circ) u(t)$. Here $\omega = 2$:

$$|H(j2)| = \frac{\sqrt{13}}{8} \quad \text{and} \quad \angle H(j2) = 33.69^\circ - 90^\circ = -56.31^\circ$$

Therefore:

$$y(t) = \frac{\sqrt{13}}{8} \cos(2t + 60^\circ - 56.31^\circ) u(t) = \frac{\sqrt{13}}{8} \cos(2t + 3.69^\circ) u(t)$$

(c) $f(t) = \sin(3t - 45^\circ) u(t)$. Here $\omega = 3$ and:

$$|H(j\omega)| = \frac{\sqrt{18}}{13} \quad \text{and} \quad \angle H(j\omega) = 45^\circ - 112.62^\circ = -67.62^\circ$$

Therefore:

$$y(t) = \frac{\sqrt{18}}{13} \sin(3t - 45^\circ - 67.62^\circ) u(t) = \frac{\sqrt{18}}{13} \sin(3t - 112.62^\circ) u(t)$$

(d) $f(t) = e^{j3t} u(t)$

$$y(t) = H(j3) e^{j3t} = |H(j3)|\, e^{j[3t + \angle H(j3)]} u(t) = \frac{\sqrt{18}}{13} e^{j[3t - 67.62^\circ]} u(t)$$

7.1-3

$$H(j\omega) = \frac{-(j\omega - 10)}{j\omega + 10} = \frac{10 - j\omega}{10 + j\omega} \rightarrow \begin{cases} |H(j\omega)| = \sqrt{\dfrac{\omega^2 + 100}{\omega^2 + 100}} = 1 \\ \angle H(j\omega) = \tan^{-1}\left(-\dfrac{\omega}{10}\right) - \tan^{-1}\left(\dfrac{\omega}{10}\right) = -2\tan^{-1}\left(\dfrac{\omega}{10}\right) \end{cases}$$

(a) $f(t) = e^{j\omega t}$

$$y(t) = H(j\omega) e^{j\omega t} = |H(j\omega)|\, e^{j[\omega t + \angle H(j\omega)]} = e^{j[\omega t - 2\tan^{-1}(\omega/10)]}$$

(b) $f(t) = \cos(\omega t + \theta)$

$$y(t) = \cos\left[\omega t + \theta - 2\tan^{-1}\left(\frac{\omega}{10}\right)\right]$$

(c) $f(t) = \cos t$. Here $\omega = 1$

$|H(j1)| = 1 \quad \angle H(j1) = -2\tan^{-1}(1/10) = -11.42^\circ$

$\rightarrow y(t) = \cos(t - 11.42^\circ)$

(d) $f(t) = \sin(2t)$. Here $\omega = 2$:

$|H(j2)| = 1 \quad \angle H(j2) = -2\tan^{-1}(2/10) = -22.62^\circ$

$\rightarrow y(t) = \sin(2t - 22.62^\circ)$

(e) $f(t) = \cos(10t)$. Here $\omega = 10$

$|H(j10)| = 1. \quad \angle H(j10) = -2\tan^{-1}(10/10) = -90^\circ$

$\rightarrow y(t) = \cos(10t - 90^\circ) = 10\sin(10t)$.

(f) $f(t) = \cos(100t)$. Here $\omega = 100$

$|H(j100)| = 1 \quad \angle H(j100) = -2\tan^{-1}(100/10) = -168.58^\circ$

$\rightarrow y(t) = \cos(100t - 168.58^\circ)$

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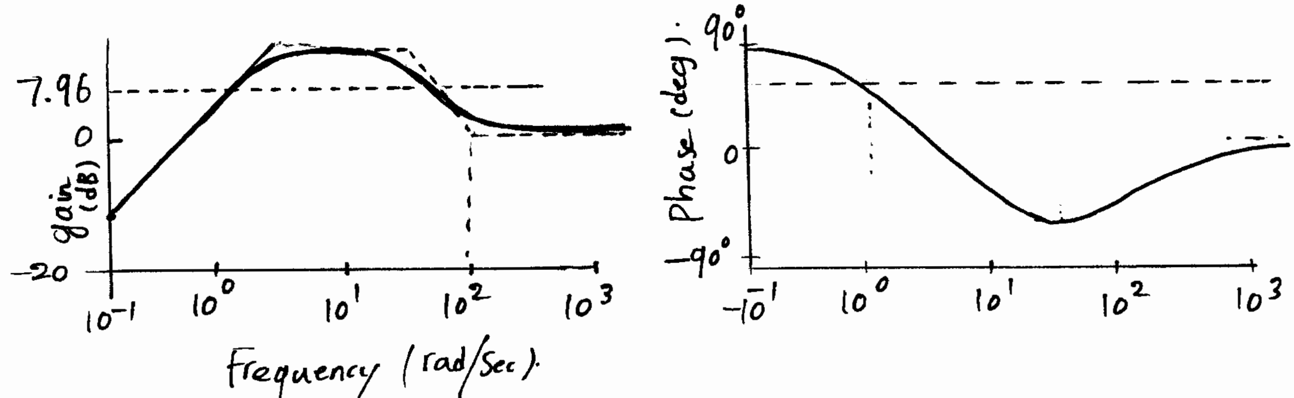
7.2-1.

(a) The transfer function can be expressed as:

$$H(s) = \frac{100}{2\times 20}\,\frac{s(\frac{s}{100}+1)}{(\frac{s}{2}+1)(\frac{s}{20}+1)} = 2.5\,\frac{s(\frac{s}{100}+1)}{(\frac{s}{2}+1)(\frac{s}{20}+1)}$$

The amplitude response:

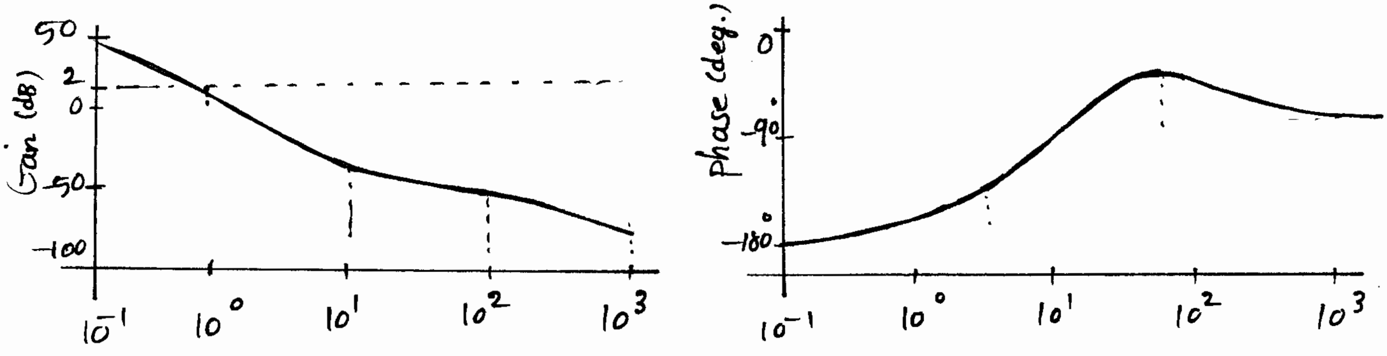
The horizontal axis where the asymptotes begin is 2.5 which 7.6 dB. We draw the asymptotes at $\omega = 1$ (20 dB/dec), 2 (-20 dB/dec) and 100 (20 dB/dec). As shown in the following figure. The corrections are applied at various points as discussed in examples 7.3 and 7.4.



(b) The transfer function can be expressed as:

$$H(S) = \frac{10 \times 20}{100} \frac{\left(\frac{S}{10}+1\right)\left(\frac{S}{20}+1\right)}{S^2\left(\frac{S}{100}+1\right)} = 2\, \frac{\left(\frac{S}{10}+1\right)\left(\frac{S}{20}+1\right)}{S^2\left(\frac{S}{100}+1\right)}$$

The horizontal axis where the asymptotes begin is 2 which is 6dB. Asymptotes start at $\omega = 1$ (−40 dB/dec), 10 (20 dB/dec), 20 (20 dB/dec) and 100 (−20dB/dec). The corrections are applied at various points as discussed in examples 7.3 and 7.4 to obtain the Bode plot.



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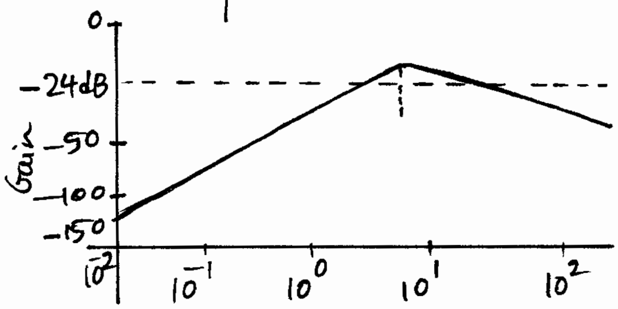
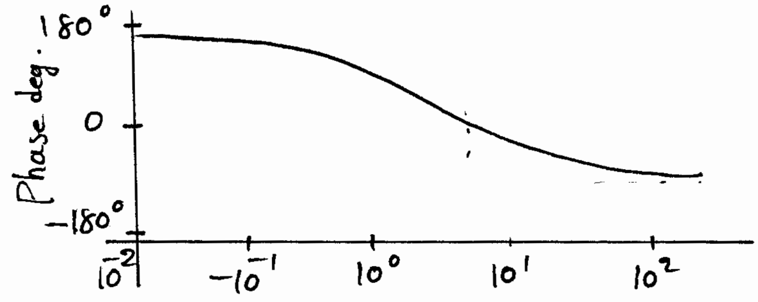
7.2-2

(a) The transfer function can be expressed as:

$$H(S) = \frac{1}{16} \frac{S^2}{\left(\frac{S}{1}+1\right)\left(\frac{S^2}{16}+\frac{S}{4}+1\right)}$$

The amplitude response: The horizontal axis where the asymptotes begin is 1/16 which is −24 dB. Asymptotes start at $\omega = 1$ (40 dB/dec), 1 (20 dB/dec), 4 (−40 dB/dec). The corrections are applied at various points as discussed in examples 7.3 and 7.4 to obtain the Bode plot.



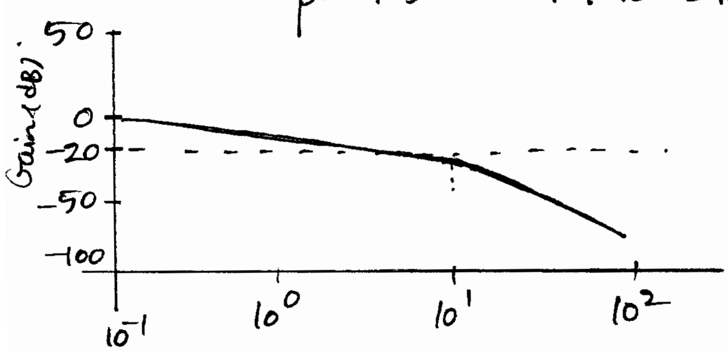


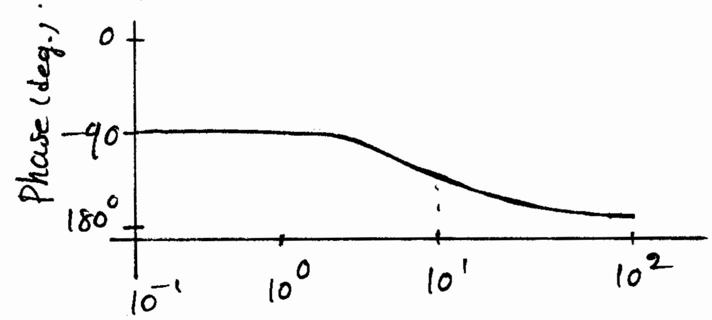
(c)

The transfer function can be expressed as:

$$H(S) = \frac{10}{100} \frac{\frac{S}{10}+1}{S\left(\frac{S^2}{100}+0.1414S+1\right)}$$

The amplitude response: The horizontal axis where the asymptotes begin is $1/10$, which is -20 dB. Asymptotes start at $\omega=1$ (-20dB/dec), 10 (20 dB/dec), 10 (-40 dB/dec). The corrections are applied at various points as discussed in example 7.3 and 7.4 to obtain the Bode plot:





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7.3-1

(a) In this case:

$$H(j\omega) = \frac{\omega_c}{j\omega+\omega_c} \longrightarrow |H(j\omega)| = \frac{\omega_c}{\sqrt{\omega^2+\omega_c^2}}$$

The dc gain is $H(0)=1$ and the gain at $\omega=\omega_c$ is $1/\sqrt{2}$ which is -3dB below the dc gain. Hence 3dB bandwidth is ω_c. Also the dc gain is unity, hence the gain-bandwidth product is ω_c

We could derive this result indirectly as follows. The system is a lowpass filter with a single pole at $\omega = \omega_c$. The dc gain is $H(0) = 1$ (0dB). Because there is a single pole at ω_c (and no zero) there is only one asymptote starting at $\omega = \omega_c$ (at a rate -20dB/dec). The breakpoint is ω_c where there is a correction of -3dB. Hence the amplitude response at ω_c is 3dB below 0 dB (the dc gain). Thus the 3dB bandwidth of this filter is ω_c.

(b). The transfer function of this system is:

$$H(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{\omega_c}{s+\omega_c}}{1 + \frac{9\omega_c}{s+\omega_c}} = \frac{\omega_c}{s + 10\omega_c}$$

We use the same argument as part (a) to duduce that the dc gain is 0.1 and the 3dB bandwidth is $10\omega_c$. Hence the gain bandwidth product is ω_c.

(c). The transfer function of the system is:

$$H(s) = \frac{G(s)}{1 - G(s)H(s)} = \frac{\frac{\omega_c}{s+\omega_c}}{1 - \frac{0.9\omega_c}{s+\omega_c}} = \frac{\omega_c}{s + 0.1\omega_c}$$

We use the same argument as in part (a) to deduce that the dc gain is 10 and the 3dB bandwidth is $0.1\omega_c$. Hence, the gain bandwidth product is ω_c.

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