Chapter 8 (Solution of Selected problems).

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8.2-3

(a) $\cos(0.5\pi k + 0.2)$.

$\Omega = 0.5\pi$. From Equ (8.9-b) we have $N_0 = m\left(\frac{2\pi}{\Omega}\right) = m(4)$ which is an integer. for the smallest $m=1$. Hence the signal is periodic with period $N_0 = 4$.

(b). $\cos(\sqrt{2}\pi k + 1.2)$. $\Omega = \sqrt{2}\pi$.

$\frac{\Omega}{2\pi} = \frac{\sqrt{2}\pi}{2\pi} = \frac{\sqrt{2}}{2}$ which is not rational. Hence the signal is aperiodic.

8.2-4

(c) $\cos(0.6\pi k + 0.3) + 0.3\sin(0.5\pi k + 0.4) + 8\cos(0.8\pi k - \frac{\pi}{3})$.

For $\cos(0.6\pi k + 0.3)$; $\Omega_1 = 0.6\pi \rightarrow N_0 = m\left(\frac{2\pi}{\pi}\right) = m\left(\frac{20}{6}\right) \xrightarrow{m=3} N_0 = 10$

" $\sin(0.5\pi k + 0.4)$: $\Omega_2 = 0.5\pi \rightarrow N_0 = m\left(\frac{2\pi}{\Omega}\right) = m(4) \xrightarrow{m=1} N_0 = 4$

" $\cos(0.8\pi k - \pi/3)$; $\Omega_3 = 0.8\pi \rightarrow N_0 = m\left(\frac{2\pi}{\Omega}\right) = m\left(\frac{20}{8}\right) \xrightarrow{m=2} N_0 = 5$

The smallest number with contains an integer number of cycles of each of the above three signals is 20, since $20 = 2(10) = 4(5) = 5(4)$. Hence the signal is periodic with period $N_0 = 20$.

8.2-9.

(a) $E_f = \sum_{-\infty}^{+\infty} |f[k]|^2 = \sum_{k=-2}^{2} |f[k]|^2 = 2(1)^2 + 2(2)^2 + (3)^2 = 19$.

8.2-10

(d) $f[k] = (-1)^k u[k]$.

$$P_f = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |(-1)^k u[k]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (-1)^{2k} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{0}^{N} (1)^k$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (N+1) = 0.5 \longrightarrow P_f = 0.5$$

(e) $f[k] = \cos\left(\frac{\pi}{3}k + \frac{\pi}{6}\right)$.

$$N_0 = 6; \quad P_f = \frac{1}{6} \sum_{0}^{5} \left(\cos\left[\frac{\pi}{3}k + \frac{\pi}{6}\right]\right)^2$$

$$= \frac{1}{6}\left[\left(\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(-\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 + 0 + \left(\frac{\sqrt{3}}{2}\right)^2\right] = 0.5 \rightarrow P_f = 0.5$$

8.3-3.

$f(t) = 10\cos(2000\pi t) + \sqrt{2}\sin(3000\pi t) + 2\cos(5000\pi t + \pi/4)$.

Here $T = 1/4000$ (s). The sampled signal is obtained by replacing t with $kT = k(1/4000)$. which is:

$$f[k] = 10\cos\left(\frac{\pi}{2}k\right) + \sqrt{2}\sin\left(\frac{3\pi}{4}k\right) + 2\cos\left(\frac{5\pi}{4}k + \frac{\pi}{4}\right).$$

$$= 10\cos\left(\frac{\pi}{2}k\right) + \sqrt{2}\sin\left(\frac{3\pi}{4}k\right) + 2\cos\left(\frac{3\pi}{4}k - \frac{\pi}{4}\right) \xrightarrow{\text{expanding the last term}}$$

$$= 10\cos\left(\frac{\pi}{2}k\right) + \sqrt{2}\sin\left(\frac{3\pi}{4}k\right) + \sqrt{2}\cos\left(\frac{3\pi}{4}k\right) + \sqrt{2}\sin\left(\frac{3\pi}{4}k\right).$$

$$= 10\cos\left(\frac{\pi}{2}k\right) + 2\sqrt{2}\sin\left(\frac{3\pi}{4}k\right) + \sqrt{2}\cos\left(\frac{3\pi}{4}k\right) = 10\cos\left(\frac{\pi}{2}k\right) + \sqrt{10}\cos\left(\frac{3\pi}{4}k - 1.107\right).$$

We can see that the frequency $5\pi/4$ has been reduced to $3\pi/4$. This indicates aliasing.

In the continous signal, $f(t)$, the highest frequency is $\omega = 5000\pi$ or 2500 Hz

The maximum value of T that can be used without aliasing is:

$$T = \frac{1}{2(2500)} = 1/5000$$