EE 3TP4: Signals and Systems
Lab 4: Sampling

Objective

To examine, both theoretically and audibly, the effects of aliasing in the sampling of continuous-time signals.

Report

A report for this lab is due one week after you do the experiment. In the report you should answer all questions, derive the requested mathematical expressions, interpret them, and demonstrate how they describe the experiments. You should also include your Matlab code and all required graphs. You may submit one report between two people, if you so wish.

Background: Sampling in the telephone system

During a telephone conversation, the digital telephone network transports streams of sampled voice signals between the phone exchanges to which the two connected handsets are attached. Voice signal transmission between the phone exchange and telephone handsets is normally done in analog form. At the phone exchanges, the required analog-to-digital conversion (sampling and quantization) and digital-to-analog conversion (reconstruction) are performed. The design is based on the fact that a large fraction of human voice energy is typically below about 3.5 kHz. Assuming 4 kHz just to be safe, the Nyquist sampling rate is 8000 samples-per-second (i.e., $f_s = 2\times4$ kHz), which is the standard sampling rate used in the telephone system. At each sampling instant an 8-bit sample is taken, which results in a total bit rate of 64 kbps (i.e., 8 bits×8 kHz) for each one-way voice stream. (In this lab we will not look at quantization effects and we will consider that the number of bits-per-sample is large enough for the resulting errors to have negligible effects.)

In each telephone exchange, the incoming analog voice signal is first low-pass filtered to reduce the frequency components above 3.5 kHz, so that significant aliasing does not occur. Following this, the signal is sampled as discussed above, and transported to its destination telephone. In the first part of the lab we will consider what would happen if this pre-filtering operation were not performed and therefore aliasing may occur.

1 Aliasing

In this section we will consider the effects of aliasing on a sinusoid which is sampled and transmitted through the phone system. (You can assume that the person on one end of the phone conversation hums various sinusoids into the handset.) We are interested in what emerges at the other telephone.

1. Start by writing a Matlab function file that produces samples of the sinusoidal signal, $x(t) = A\sin(2\pi f_o t + \varphi)$:

The samples are to be taken at periodic intervals of the sampling period, $T = 1/f_s$, where f_s is the sampling rate, and φ can be any phase. The sampled signal is given by

$x[k] = A\sin(2\pi\ f_{\text{o}} kT + \varphi) = A\sin(2\pi k(f_{\text{o}} / f_s) + \varphi)$

Your function file should have inputs f_{o}, A, f_s, φ, and k_1 and k_2, where k_1T is the starting time of the time interval of interest, and k_2T is the end time. The output of your function should be a vector of time samples, say tvec, and a vector, say xkT, whose ith element is $x((k_1 + i - 1)T)$. Your function gensin.m from Lab. 1 almost meets these specifications. Make the small modifications required.

2. Set f_s = 8kHz and f_{o} = 100 Hz, and calculate $x(kT)$ over the interval 0 to 10 ms. Plot $x(kT)$ using the Matlab stem command. Make a choice for A and φ and justify why these are reasonable choices in your report. Describe what you see. Save an electronic copy of the graph so that you can print it later and include it with your report. You can save an Encapsulated Postscript file using the command print -deps mygraph.eps.

3. When this sampled signal is transported to its destination, it is converted back into analog form and then transmitted out to the destination telephone. For simplicity we will assume that this digital-to-analog conversion is done by connecting the voice sample values with straight lines. This can be seen in Matlab simply by using the plot command rather than stem. In practice a more smooth reconstruction is done by performing analog filtering at the output.

Over the interval from 0 to 10 ms, plot $x(kT)$ using the plot command when f_{o} = 100, 200, 400, 800 Hz. Rather than plotting your graphs separately, plot all four output frequency plots on a single page using the subplot command. Label each sub-plot with its value of frequency, f_{o}. Save this graph so you can print it later for use in your report.

4. Play each individual sinusoid using the command soundsc(xkT, f_s), where f_s is your sampling frequency. To make these sinusoids easier to hear, make xkT one second long, rather than 10 milliseconds. Describe what you hear. Choose a different value for φ and repeat this experiment. Can you hear the difference? Explain why or why not.

5. Now concatenate four one-second tone segments at each of the above four frequencies into a single vector. Play this signal using the soundsc function. Describe what you hear, and explain why this is what you would expect.

6. Now repeat item 5, but this time use sinusoids of frequencies f_{o} = 7200; 7600; 7800; 7900 Hz. Describe what you hear, and again explain why this is exactly what you would expect.

7. Based on what you have observed, discuss why the performance of the telephone system would degrade significantly if anti-aliasing pre-filtering were not used. Explain how this filtering prevents these negative effects. What would happen in the above experiments if this filtering were in place?

2 Aliasing of a Chirp Signal

In this section we will illustrate the aliasing of a chirp signal. A chirp is a signal whose frequency is a linear function of time. We will use the following chirp signal,

$$c(t) = A\cos(\pi\mu t^2 + 2\pi f_1 t + \varphi): \qquad (1)$$

What is the frequency of this signal? Recall that $\cos(2\pi f_o t)$ has a frequency of $2\pi f_o$ radians-per- second, or f_o cycles-per-second (Hz). Also note that $\theta = 2\pi f_o t$ is the phase of $\cos(2\pi f_o t)$, and that $2\pi f_o = d\theta/dt$. By taking the derivative of the phase of the signal in (1), it can be seen that the instantaneous frequency of the chirp signal is given by $f(t) = \mu t + f_1$: That is, the frequency is increasing linearly with time, starting at f_1 Hz.

1. Write a Matlab function file which produces samples of the above signal. The format of your function should be similar to that of the function from Section 1 of this lab. The inputs to your function should be f_1, μ, A, φ, f_s, k_1 and k_2. Your outputs should be a vector of sampling times, and a vector, say ckT, of samples of $c(t)$. Choose f_1 = 100 Hz and set μ= 2000. Start by using a sampling frequency of f_s = 32kHz. Sample $c(t)$ for an 8 second period of time. Plot the first 2000 samples to see what the sampled signal looks like. Then using Matlab, play the sampled signal out through the speakers, using soundsc(ckT,f_s). Describe and explain what you see and hear in both cases.

2. Repeat the above experiment but this time use a sampling frequency of 16 kHz. Explain what you hear now. Using the theory that you know, explain in detail what you have heard. Try the same thing for 8 kHz, which indicates what would happen if this signal were sent through the digital telephone network without anti-aliasing pre-filtering. What would you hear over a telephone connection that includes the anti-aliasing filtering? Experiment with other f_1, f_s and μ values. In all cases explain what you hear using the theory you know about sampling.