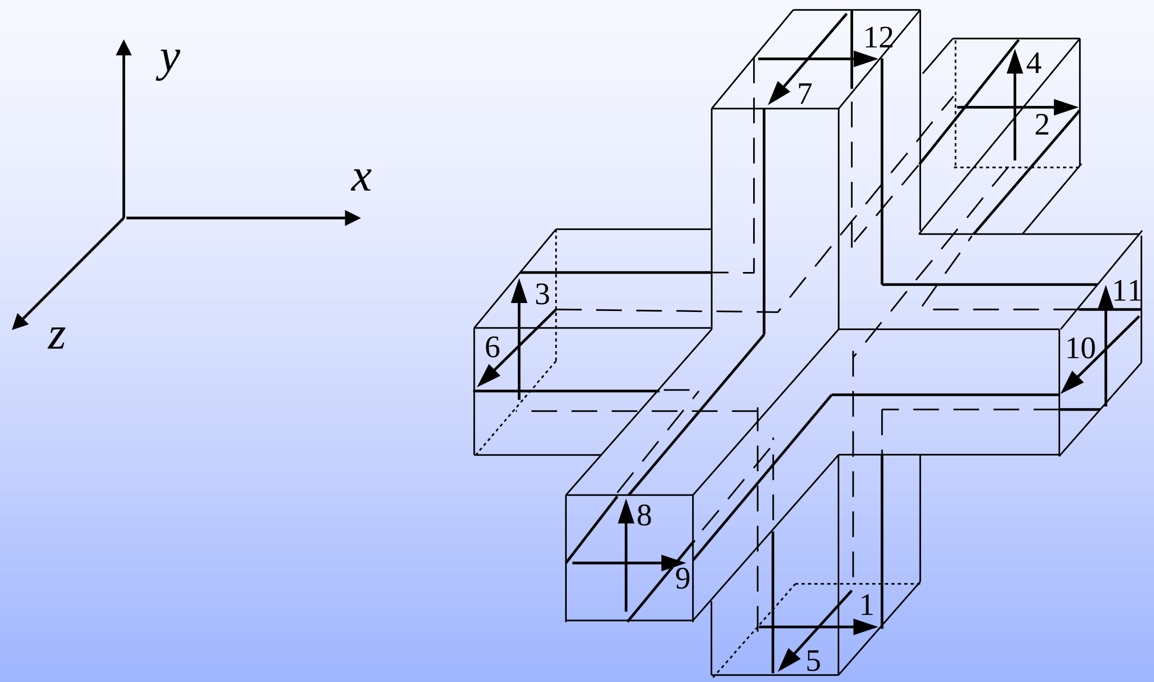
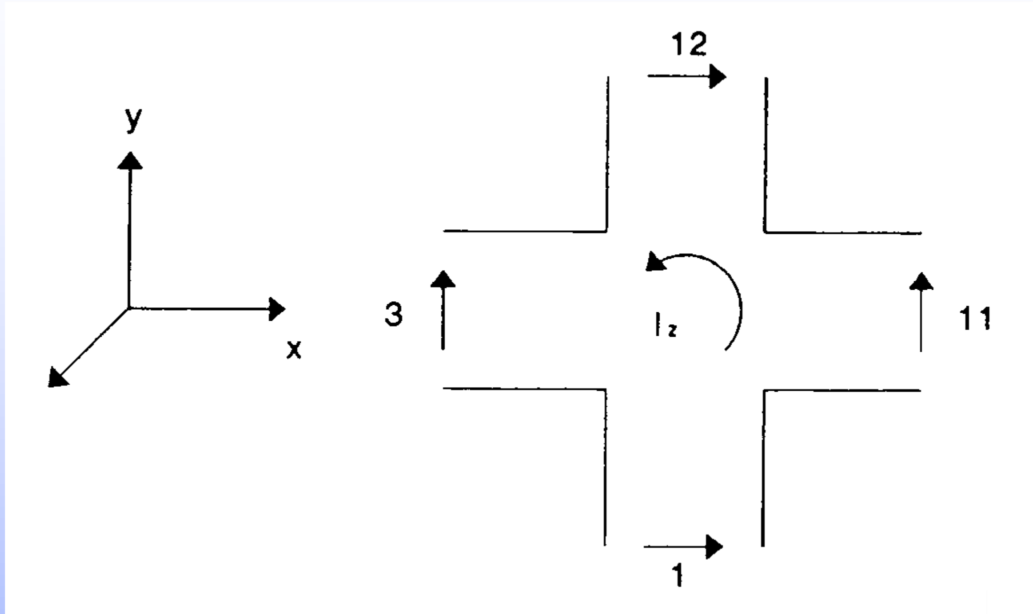
EE757
Numerical Techniques in Electromagnetics
Lecture 6

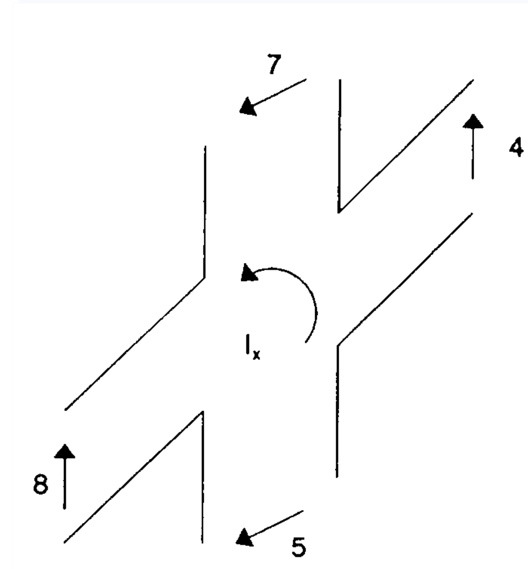
Modeling of a Homogenuous Medium



- The Scattering matrix has 12 rows and 12 columns
- Only few of these components are non zeros

Decomposition of the SCN

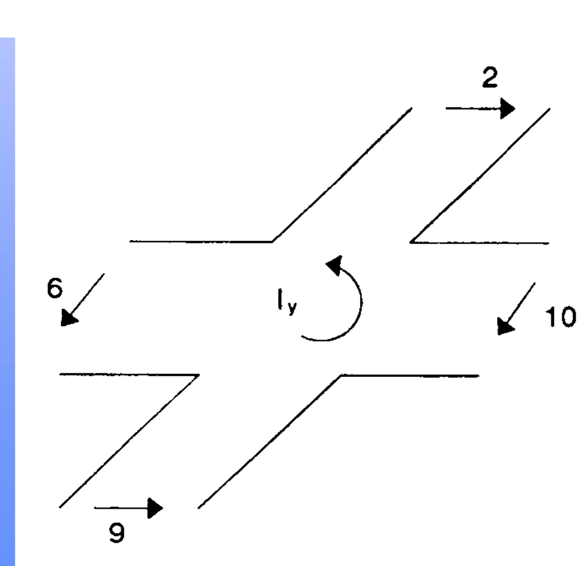




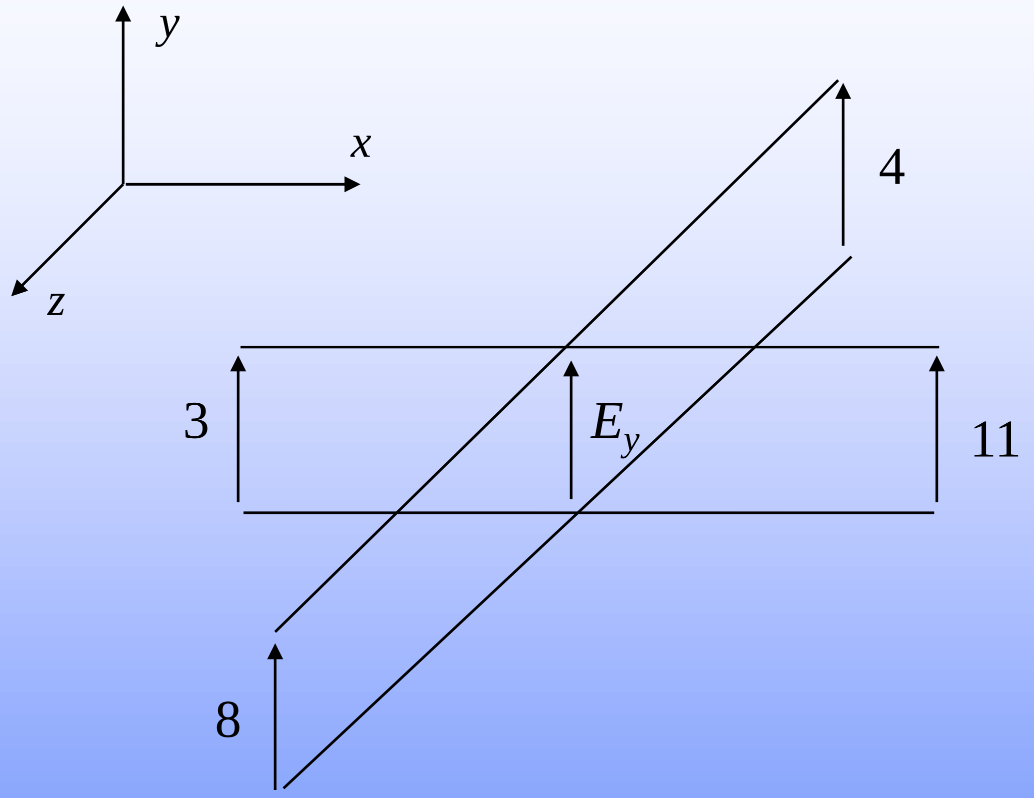
$$H_x = I_x / \Delta x\,,$$

$$H_y = I_y / \Delta y\,,$$

$$H_z = I_z / \Delta z$$



Decomposition of the SCN (Cont'd)

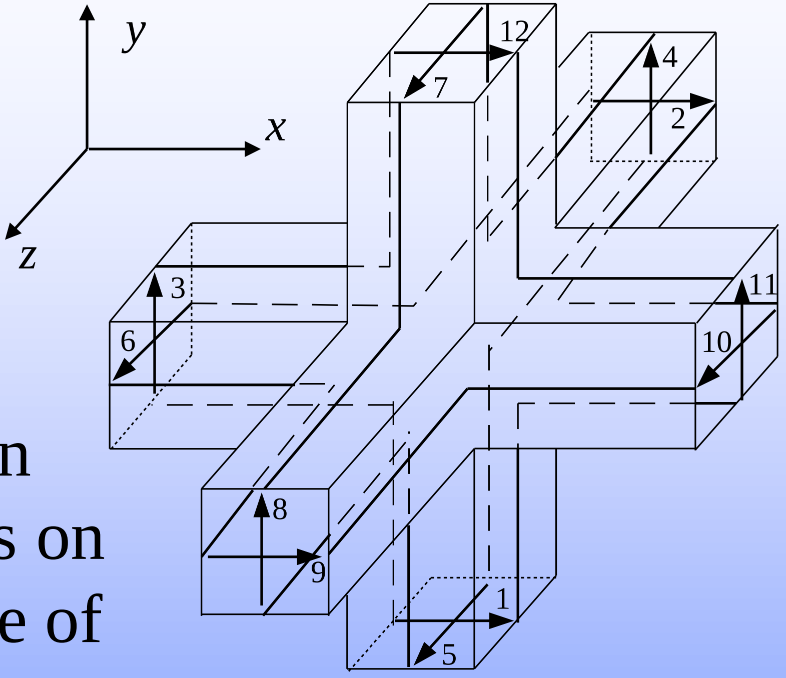


$$E_y = -V_y / \Delta y, \quad E_x = -V_x / \Delta x, \quad E_z = -V_z / \Delta z$$

The Scattering Matrix

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \varepsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t}$$



- Incident Impulses at port 1 can give rise to reflected impulses on ports 1, 2, 9, 12, 3, 11 because of Maxwell's equations

- Similar approach can be applied to all other links

The Scattering Matrix (Cont'd)

$$
S = \begin{bmatrix}
a & b & d & 0 & 0 & 0 & 0 & 0 & b & 0 & -d & c \\
b & a & 0 & 0 & 0 & d & 0 & 0 & c & -d & 0 & b \\
d & 0 & a & b & 0 & 0 & 0 & b & 0 & 0 & c & -d \\
0 & 0 & b & a & d & 0 & -d & c & 0 & 0 & b & 0 \\
0 & 0 & 0 & d & a & b & c & -d & 0 & b & 0 & 0 \\
0 & d & 0 & 0 & b & a & b & 0 & -d & c & 0 & 0 \\
0 & 0 & 0 & -d & c & b & a & d & 0 & b & 0 & 0 \\
0 & 0 & b & c & -d & 0 & d & a & 0 & 0 & b & 0 \\
b & c & 0 & 0 & 0 & -d & 0 & 0 & a & d & 0 & b \\
0 & -d & 0 & 0 & b & c & b & 0 & d & a & 0 & 0 \\
-d & 0 & c & b & 0 & 0 & 0 & b & 0 & 0 & a & d \\
c & b & -d & 0 & 0 & 0 & 0 & 0 & b & 0 & d & a
\end{bmatrix}
$$

- Applying the unitary conditions $S^T S = I$, and Maxwell's equations we obtain the components

The Scattering Matrix (Cont'd)

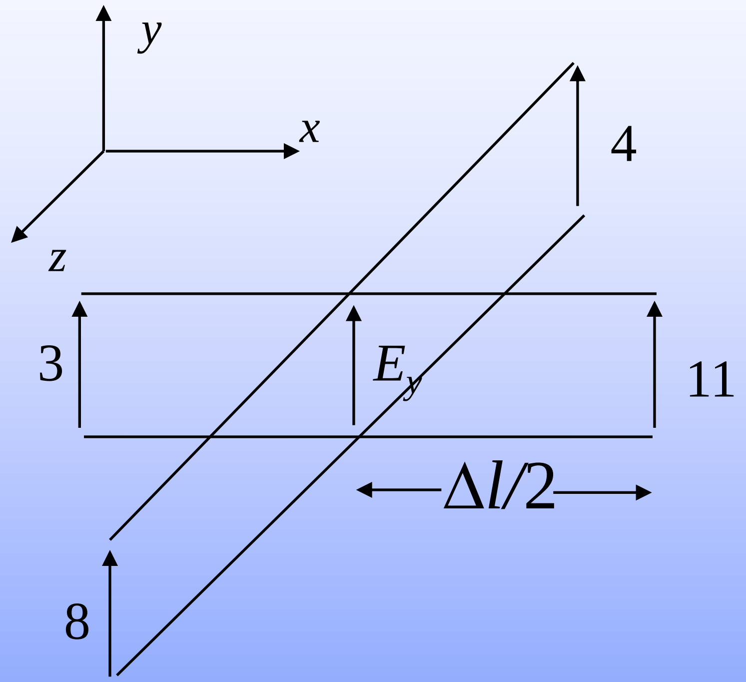
$$\mathbf{S} = 0.5\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Links Properties

- The capacitance of a space block in the *y* direction is $\varepsilon\Delta x\Delta z/\Delta y$

$$\varepsilon\frac{\Delta l\Delta l}{\Delta l}=4(C_d\,\Delta l/2)$$

$$C_d=\varepsilon/2$$



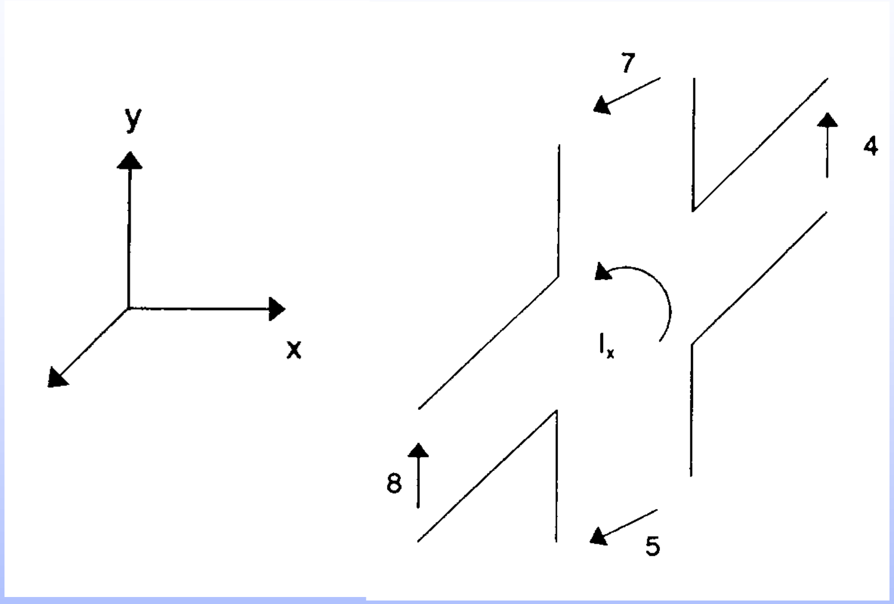
Link Properties (Cont'd)

- The inductance of a space block in the *y*-*z* plane is $\mu\,\Delta y \Delta z/\Delta x$

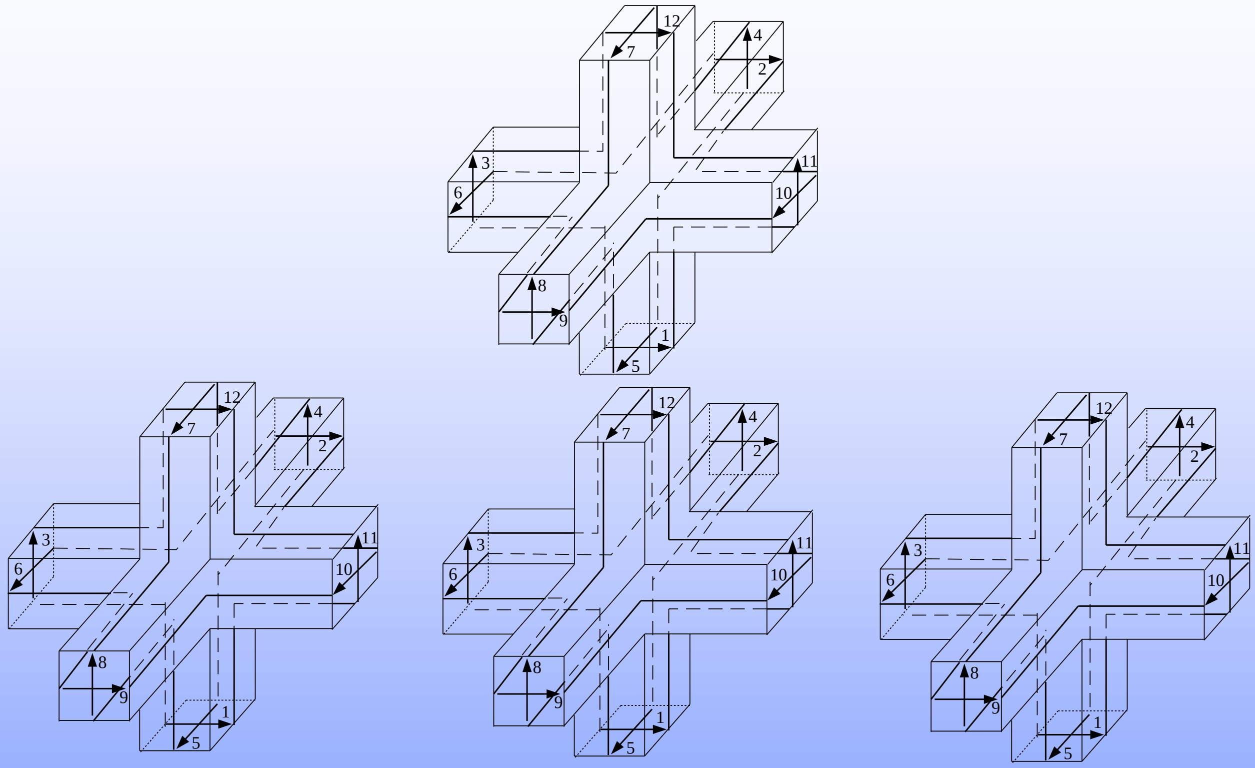
$$\mu \frac{\Delta l \Delta l}{\Delta l} = 4(L_d\,\Delta l / 2)$$

$$L_d = \mu / 2$$

$$\frac{1}{\sqrt{L_d C_d}} = 2 v_o, \qquad \sqrt{\frac{L_d}{C_d}} = Z_o$$

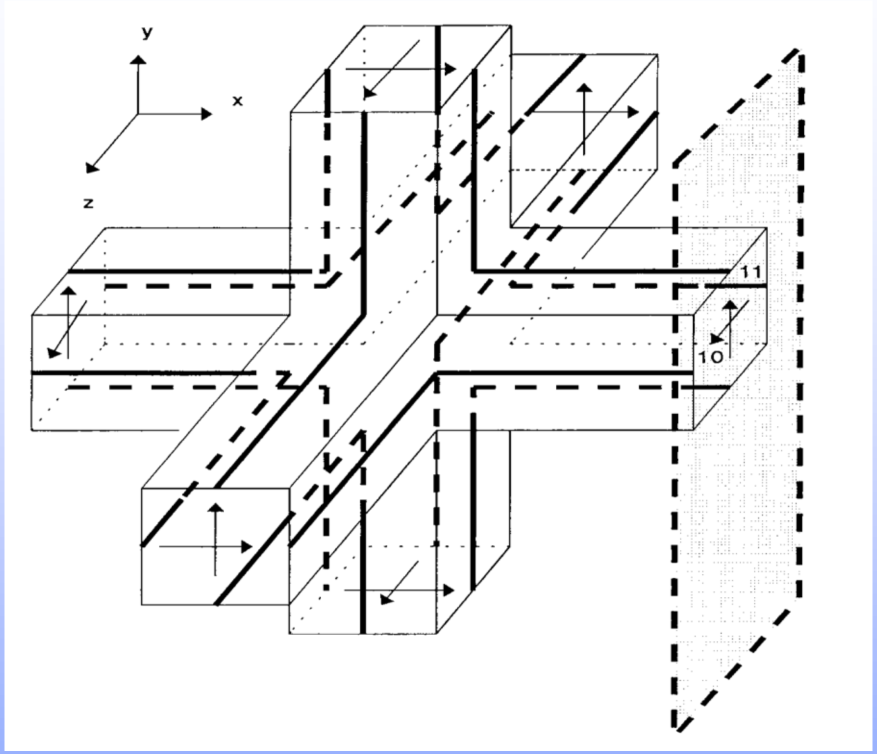


Connection in the SCN



- Reflected impulses become incident on neighboring nodes in the next time step

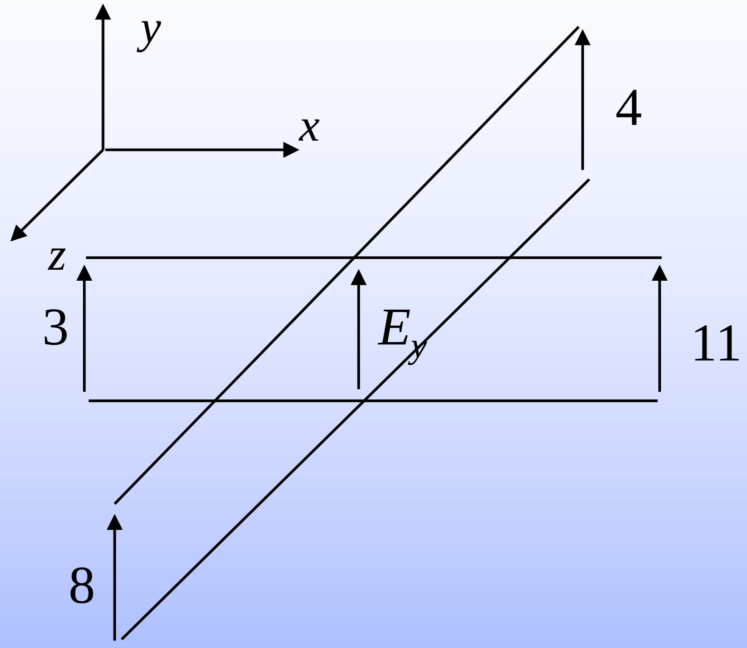
Boundaries in the SCN



$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}, \; Z_o = \sqrt{\frac{L_d}{C_d}} = \sqrt{\frac{\mu}{\varepsilon}}$$

- Johns' matrix may be used to simulate wideband ABCs

TLM Network Ouput

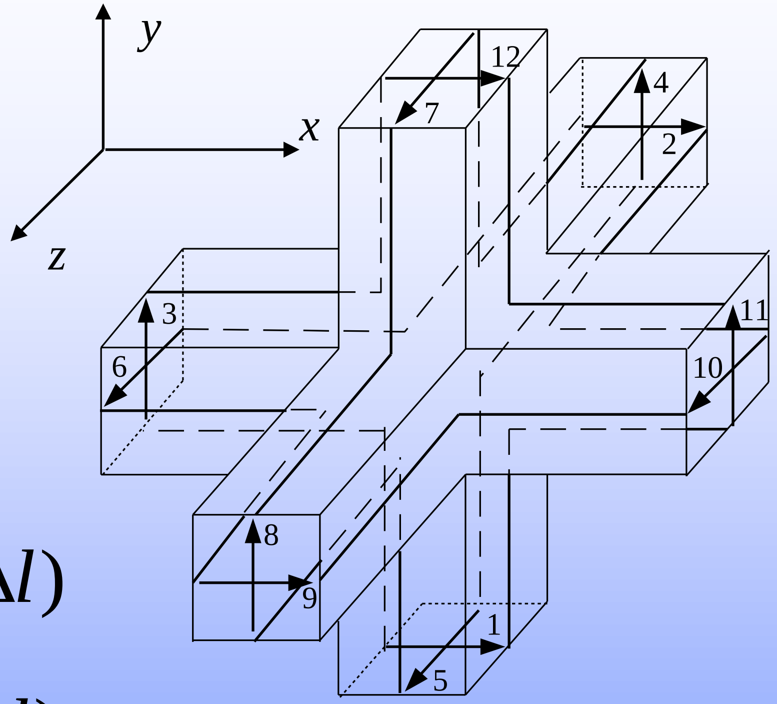


$$E_y = -V_y / \Delta l$$

$$E_y = -0.25(V_3^i + V_3^r + V_4^i + V_4^r + V_8^i + V_8^r + V_{11}^i + V_{11}^r) / \Delta l$$

$$E_y = -(V_3^i + V_4^i + V_8^i + V_{11}^i) / (2\Delta l)$$

TLM Network Output (Cont'd)

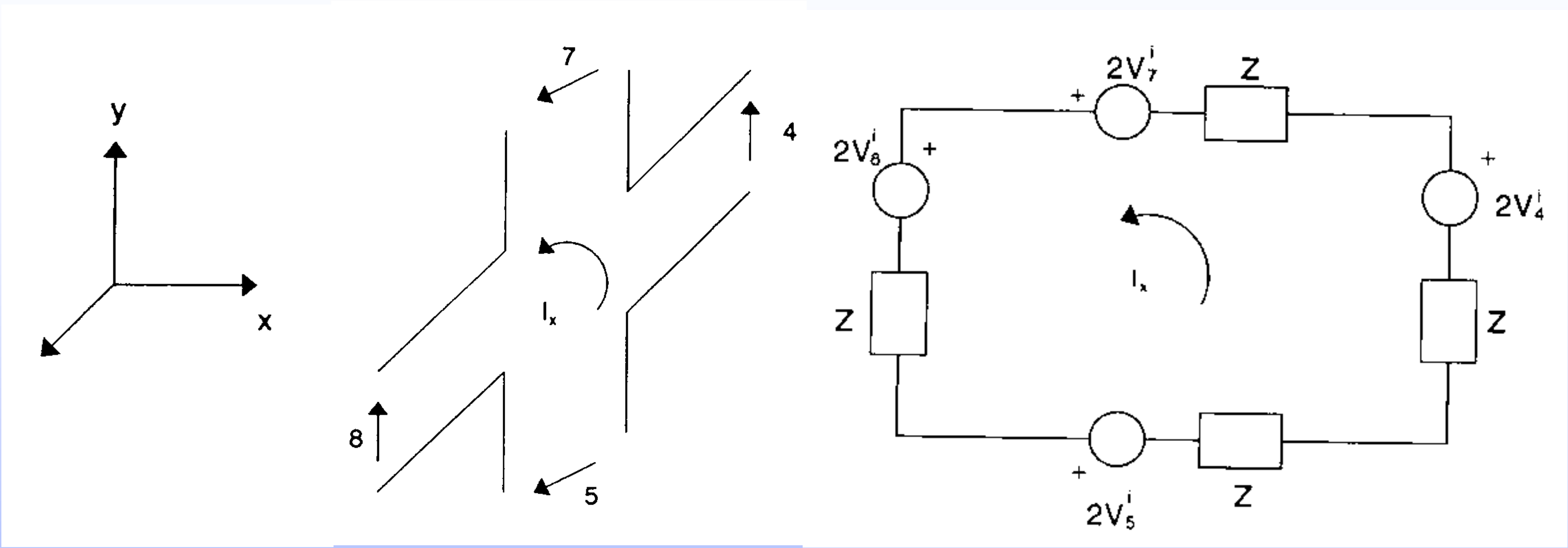


$$E_y = -(V_3^i + V_4^i + V_8^i + V_{11}^i)/(2\Delta l)$$

$$E_x = -(V_1^i + V_2^i + V_9^i + V_{12}^i)/(2\Delta l)$$

$$E_z = -(V_5^i + V_6^i + V_7^i + V_{10}^i)/(2\Delta l)$$

TLM Network Output (Cont'd)

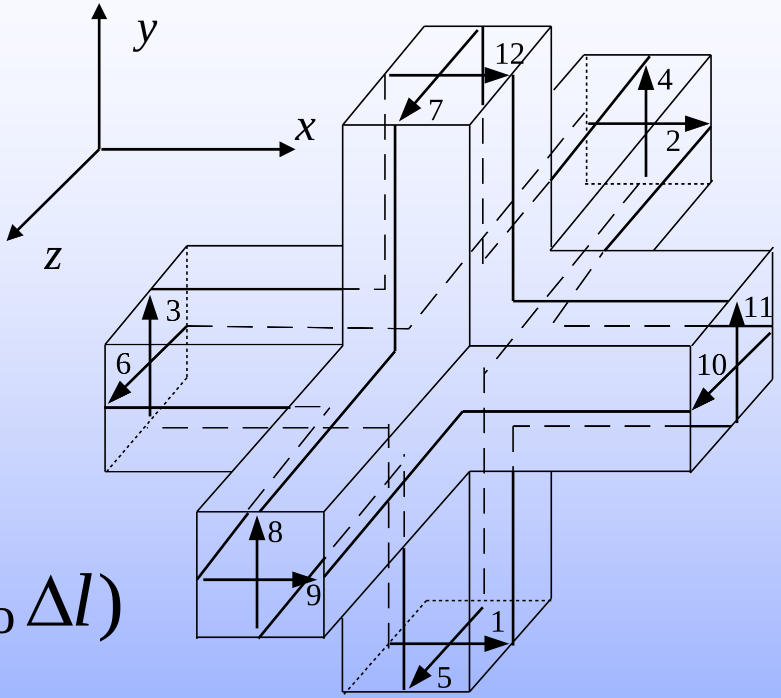


$$H_x = I_x / \Delta l$$

⇓

$$H_x = (V_4^i + V_7^i - V_5^i - V_8^i)/(2 Z_o \Delta l)$$

TLM Network Output (Cont'd)

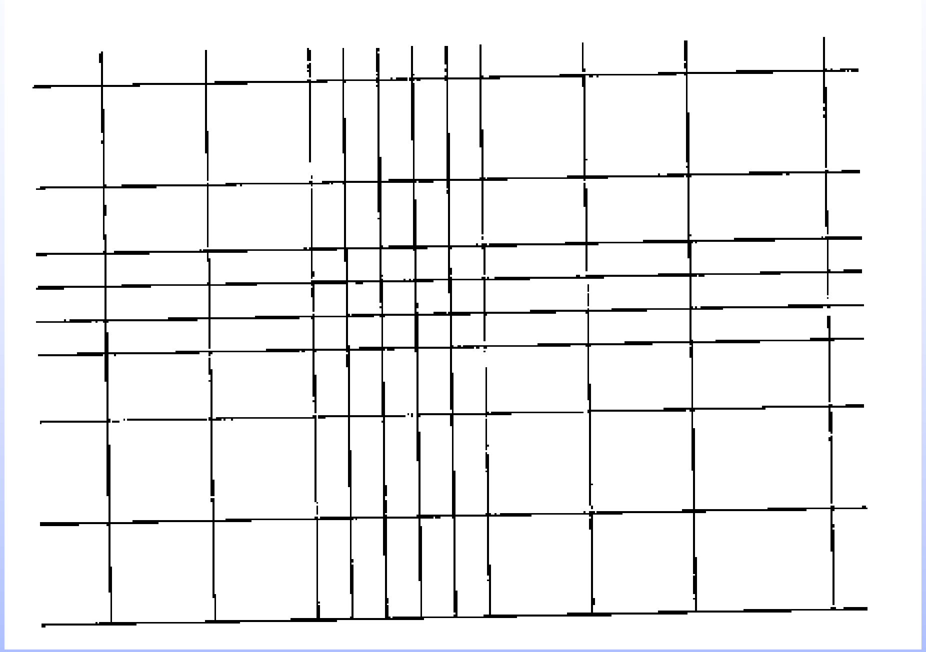


$$H_x = (V_4^i + V_7^i - V_5^i - V_8^i)/(2Z_o \Delta l)$$

$$H_y = (V_6^i + V_9^i - V_{10}^i - V_2^i)/(2Z_o \Delta l)$$

$$H_z = (V_1^i + V_{11}^i - V_3^i - V_{12}^i)/(2Z_o \Delta l)$$

The Variable Mesh SCN



3 open circuit stubs are used to model the permitivitty in the x, y and z directions

3 short circuit stubs are used to model the permeability in the x, y and z directions

Modeling the Extra Capacitance

- We choose the inductance and capacitance of the regular links to model free space

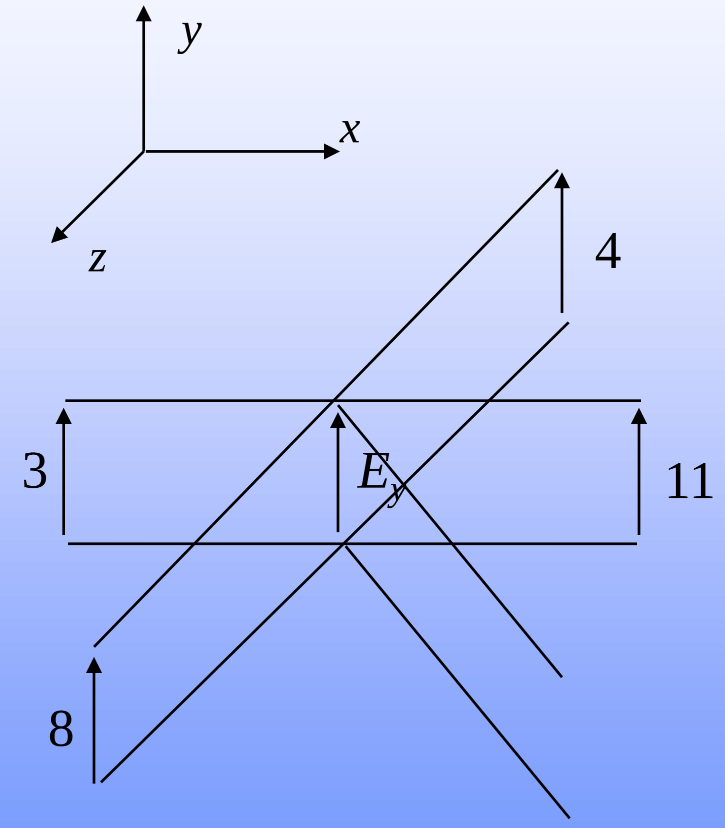
$$C = \frac{\Delta t}{2 Z_o}, \quad L = (\Delta t/2) Z_o$$

$$\Downarrow$$

$$4C + C_y^s = \varepsilon \frac{\Delta x \Delta z}{\Delta y}$$

$$\Downarrow$$

$$Y_y = \frac{2 C_y^s}{\Delta t} = 2\varepsilon \frac{\Delta x \Delta z}{\Delta y \Delta t} - \frac{4}{Z_o}$$

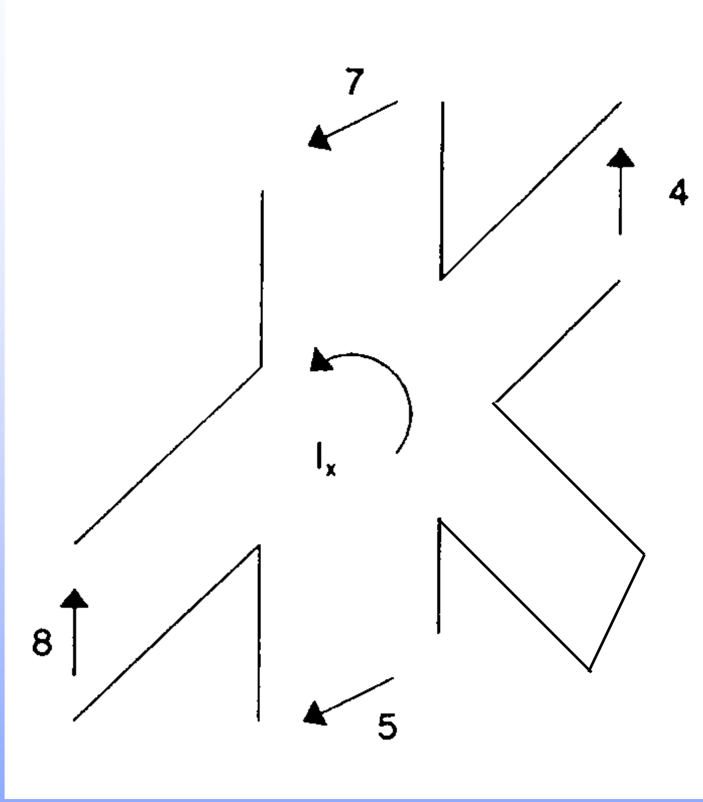


Modeling the Extra Inductance

$$4L + L_x^s = \mu \frac{\Delta y \Delta z}{\Delta x}$$

$$\Downarrow$$

$$Z_x = \frac{2L_x^s}{\Delta t} = 2\mu \frac{\Delta y \Delta z}{\Delta x \Delta t} - 4Z_o$$



The Scattering Matrix

S =

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	*a*	*b*	*d*						*b*		-*d*	*c*	*g*					*i*
2	*b*	*a*				*d*			*c*	-*d*		*b*	*g*				-*i*	
3	*d*		*a*	*b*				*b*			*c*	-*d*		*g*				-*i*
4			*b*	*a*	*d*		-*d*	*c*			*b*			*g*		*i*		
5				*d*	*a*	*b*	*c*	-*d*		*b*					*g*	-*i*		
6		*d*			*b*	*a*	*b*		-*d*	*c*					*g*		*i*	
7				-*d*	*c*	*b*	*a*	*d*		*b*					*g*	*i*		
8			*b*	*c*	-*d*		*d*	*a*			*b*			*g*		-*i*		
9	*b*	*c*				-*d*			*a*	*d*		*b*	*g*				*i*	
10		-*d*			*b*	*c*	*b*		*d*	*a*					*g*		-*i*	
11	-*d*		*c*	*b*				*b*			*a*	*d*		*g*				*i*
12	*c*	*b*	-*d*						*b*		*d*	*a*	*g*					-*i*
13	*e*	*e*							*e*			*e*	*h*					
14			*e*	*e*				*e*			*e*			*h*				
15					*e*	*e*	*e*			*e*					*h*			
16				*f*	-*f*		*f*	-*f*								*j*		
17		-*f*				*f*			*f*	-*f*							*j*	
18	*f*		-*f*								*f*	-*f*						*j*