Lecture 2:Circuit Theory (1)

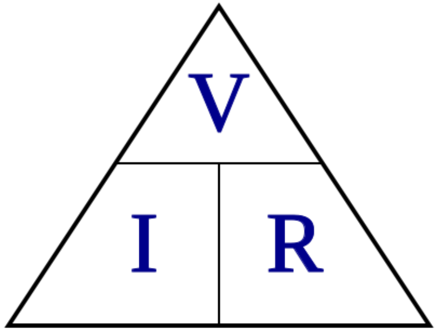
Ohm’s law, Kirchhoff’s voltage law, examples, Kirchhoff’s current law, Mesh Analysis, Voltage Dividers, Current Dividers, Superposition Theorem

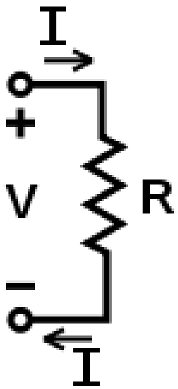
Ohm's Law

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. The constant of proportionality is the resistance

wikipedia

$I=V/R$ or $V=IR$

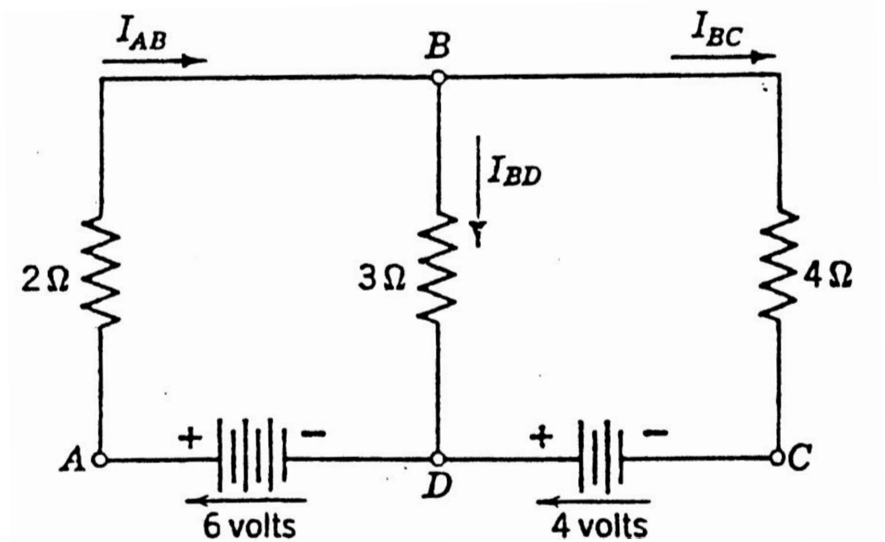


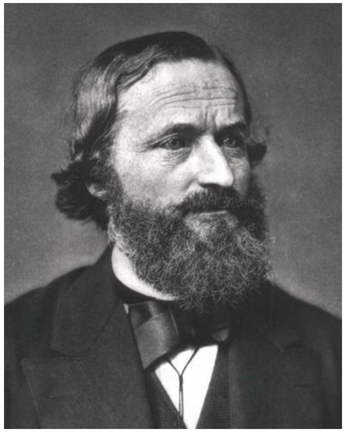


What is the origin of this law in electrmagnetics?

Kirchhoff's Current Law

For any junction: $\sum I_{\text{in}} = \sum I_{\text{out}}$





wikipedia

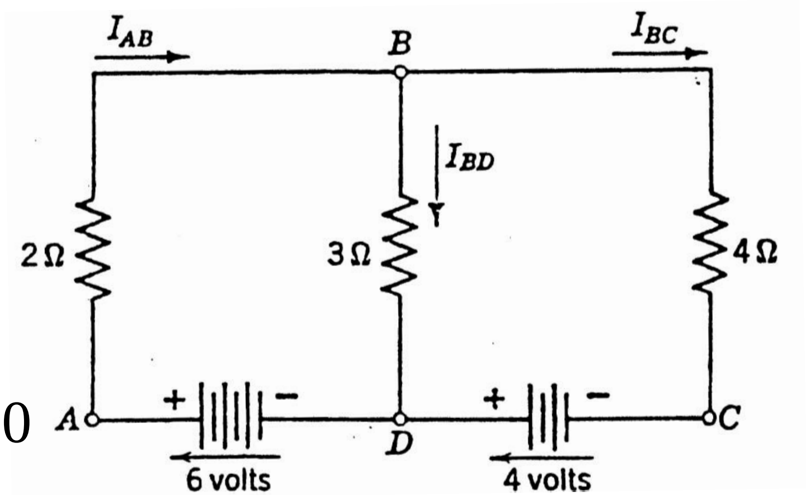
- For junction B: $I_{AB} = I_{BD} + I_{BC}$
- For junction D: $I_{BD} + I_{BC} = I_{AB}$ (dependent)
- Number of independent KCL equations = number of junctions -1

Analogy with water flow in pipes?

Kirchhoff's Voltage Law

For any closed loop: $\sum \vec{V}_{\text{rise}} = 0$,

or $\sum \overleftarrow{V}_{\text{rise}} = 0$



For the left-hand loop: $6 - 2I_{AB} - 3I_{BD} = 0$

For the right-hand loop: $4 + 3I_{BD} - 4I_{BC} = 0$

For the outter loop: $4 + 6 - 2I_{AB} - 4I_{BC} = 0$ (dependent)

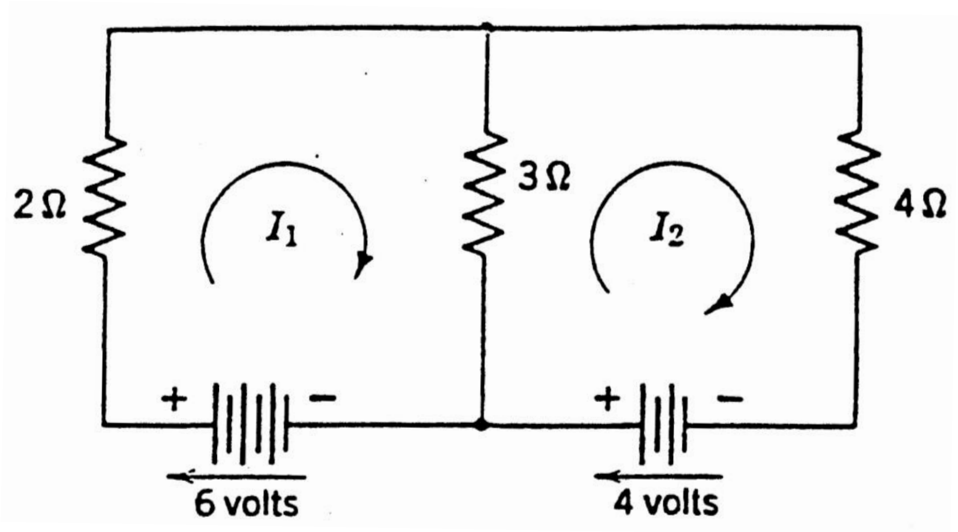
A new loop equation is independent on the previous if it contains new current(s) and/or battery(s).

origin of this law in electrmagnetics?

Mesh-Current Analysis

every loop is assigned one current flowing in the clockwise directions

Kirchhoff's voltage and current laws are then used to solve for these currents



Writing Kirchhoff's voltage laws:

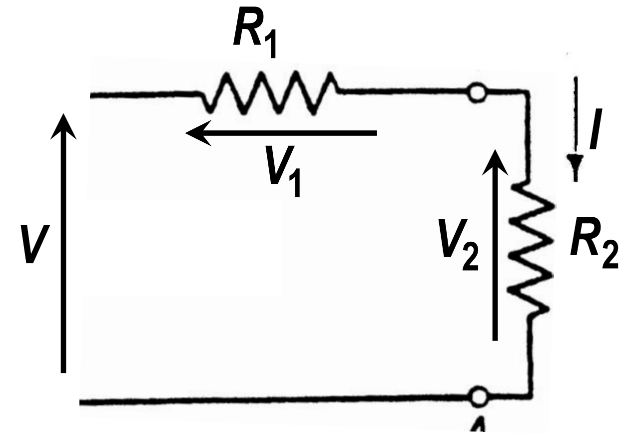
$$\begin{aligned} 6-2I_1-3I_1+3I_2&=0 \\ 4+3I_1-3I_2-4I_2&=0 \end{aligned} \Rightarrow \begin{bmatrix} -5 & 3 \\ 3 & -7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 3 & -7 \end{bmatrix}^{-1} \begin{bmatrix} -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 2.077 \\ 1.462 \end{bmatrix} \text{A}$$

Voltage Divider

The voltage divider is a <u>*series*</u> combination of two resistors.

The total voltage *V* is divided between these resistors according to the <u>*direct*</u> ratio of their resistances:



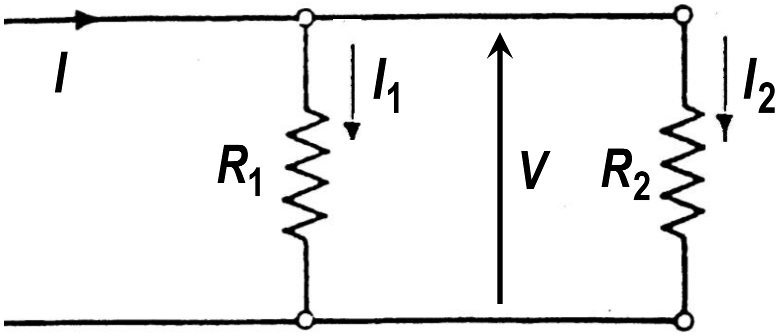
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = I\ R_1 = \frac{V\ R_1}{R_1 + R_2},\ V_2 = I\ R_2 = \frac{V\ R_2}{R_1 + R_2},\ \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Current Divider

The current divider is a *parallel* combination of two resistors.

The total current *I* is divided between these resistors according to the *inverse* ratio of their resistances:

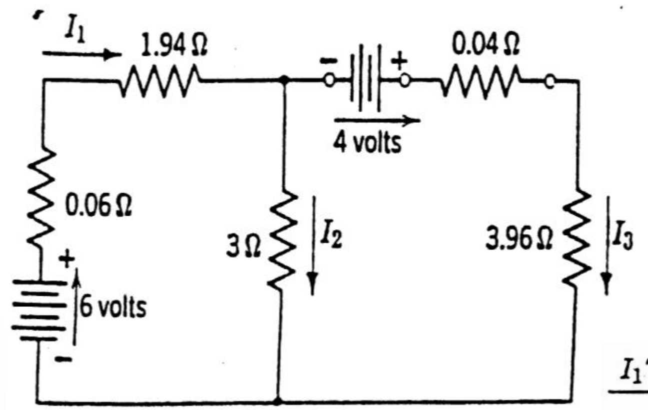


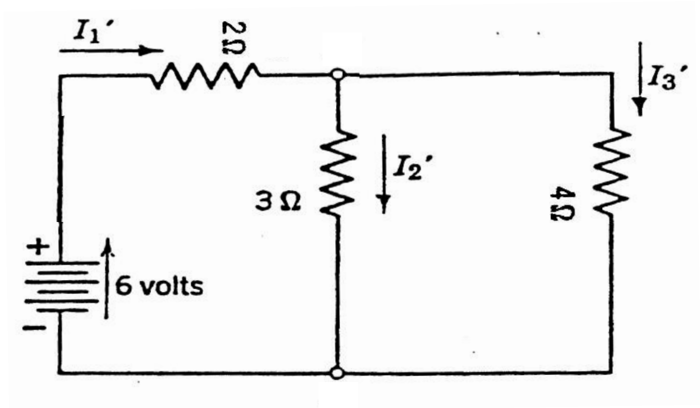
$$V = I \frac{R_1 R_2}{R_1 + R_2}$$

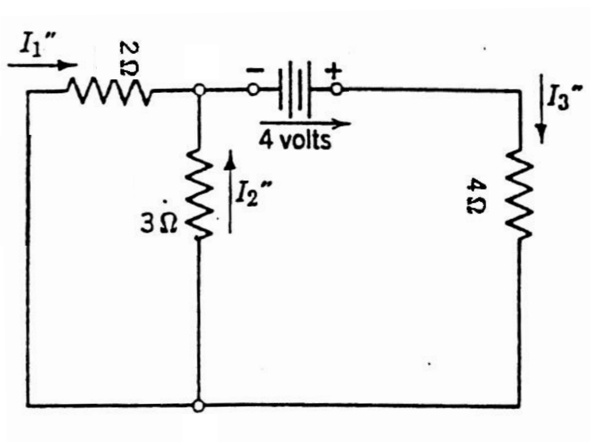
$$I_1 = \frac{V}{R_1} = \frac{I R_2}{R_1 + R_2}, \quad I_2 = \frac{V}{R_2} = \frac{I R_1}{R_1 + R_2}, \quad \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

Superposition Theorem

A circuit which contains more than one voltage source can be considered as the *superposition* of a number of circuits, each has only *one* voltage source only, while the rest of the voltage sources are replaced by *short circuits*.







Superposition (Cont'd)

<u>Original Circuit</u>

This circuit has been solved using Kirchhoff's laws:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2.077 \\ 0.615 \\ 1.462 \end{bmatrix} \text{ A}$$

<u>First Sub-Circuit</u>

$$I_1' = \frac{6}{2+(3\|4)} = 1.615 \text{ A}$$

$$I_2' = I_1' \frac{4}{3+4} = 0.923 \text{ A}$$

$$I_3' = I_1' - I_2' = 0.692 \text{ A}$$

<u>Second Sub-Circuit</u>

$$I_3'' = \frac{4}{4+(3\|2)} = 0.769 \text{ A}$$

$$I_2'' = I_3'' \frac{2}{3+2} = 0.308 \text{ A}$$

$$I_1'' = I_3'' - I_2'' = 0.461 \text{ A}$$

$$I_1 = I_1' + I_1'' = 1.615 + 0.461 = 2.076 \text{ A}$$

$$I_2 = I_2' - I_2'' = 0.923 - 0.308 = 0.615 \text{ A}$$

$$I_3 = I_3' + I_3'' = 0.692 + 0.769 = 1.461 \text{ A}$$