Lecture 3:Circuit Theory (3)

Thevenin's theorem, Norton's theorem, Maximum Power Transfer in DC circuits, transient responses of RC circuits

Thevenin's Theorem

any single port network can be replaced by its ***Thevenin's equivalent***, which consists of a ***voltage*** source V_{TH} in ***series*** with a resistance R_{TH}.

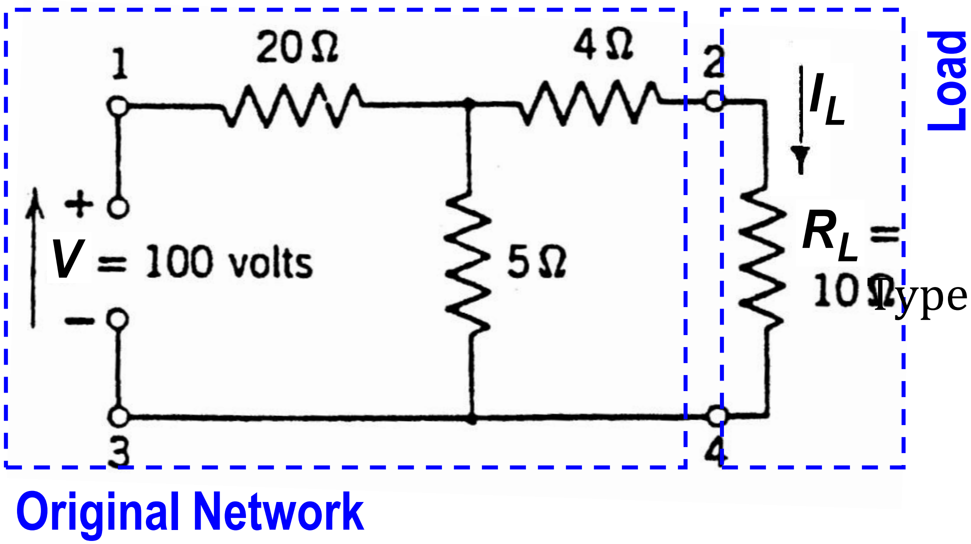
the voltage source V_{TH} is the ***open-circuit voltage*** seen between the network's terminals.

the resistance R_{TH} is the resistance of the network, when all ***voltage sources are replaced by short circuits and all current sources are open circuited***.

this theorem is very useful in analyzing electronic circuits

Thevenin's Theorem

Original Circuit



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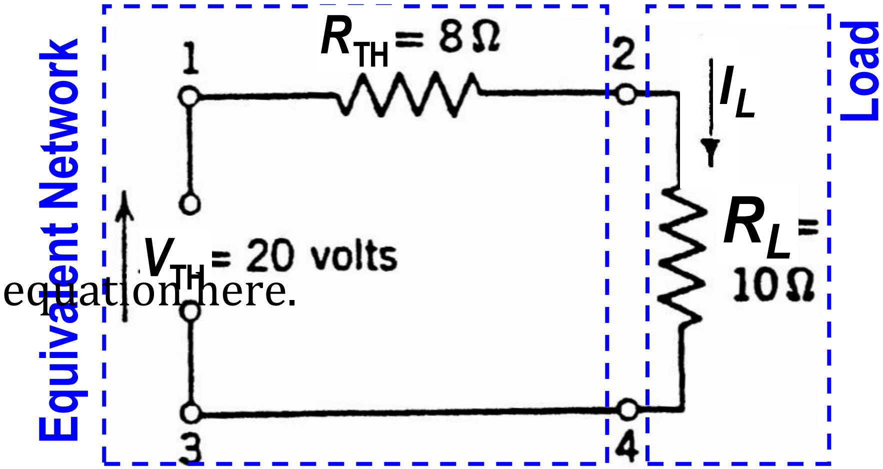
Thevenin's Equivalent



$$I_L = \frac{100}{20+\left(5\|14\right)} \times \frac{5}{14+5} = 1.11 \text{ A}$$

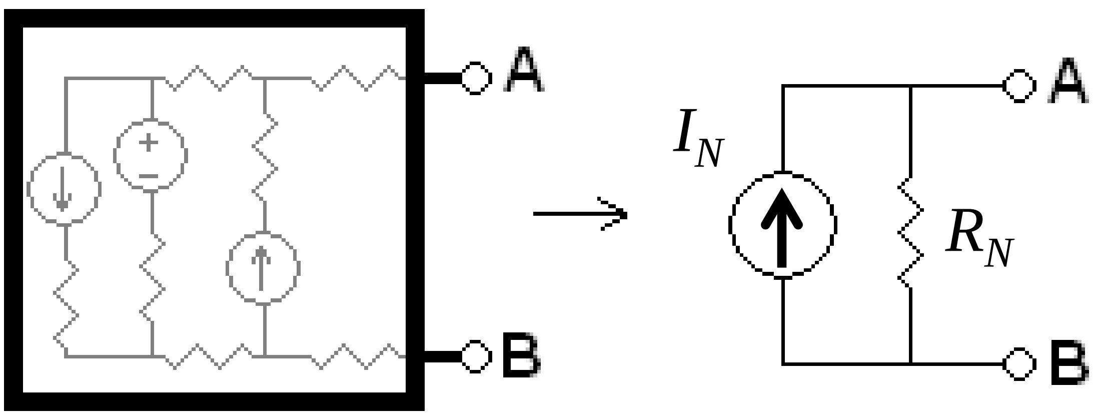
$$V_{\text{TH}} = V_{24}\Big|_{R_L \text{ is replaced by o.c.}} = 100\frac{5}{20+5} = 20 \text{ V}$$

$$R_{\text{TH}} = 4+\left(5\|20\right) = 8\ \Omega$$

$$I_L = \frac{V_{\text{TH}}}{R_{\text{TH}}+R_L} = \frac{20}{8+10} = 1.11 \text{ A}$$

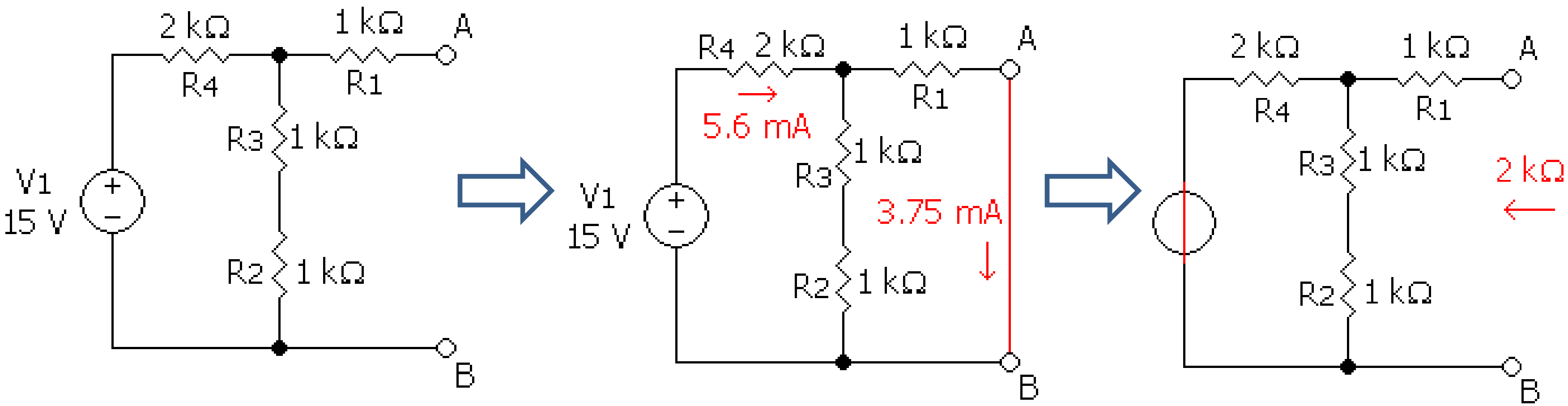
Norton's Theorem

any linear electrical network with voltage and current sources and resistances can be replaced by an equivalent current source I_N in parallel connection with an equivalent resistance R_N.



I_N is evaluated by short-circuiting the output
R_N is the input resistance when all sources are removed
(Short circuit voltage sources, open circuit current sources)

Norton's Theorem (Cont'd)



wikipedia

$$I_{tot} = \frac{15}{2K\Omega + (1K\Omega // (1K\Omega + 1K\Omega))} = 5.625 \text{ mA}$$

$$I_{TH} = I_{tot} \frac{1.0K\Omega + 1.0K\Omega}{1.0K\Omega + 1.0K\Omega + 1.0K\Omega} = 3.75 \text{ mA}$$

$$R_{TH} = 1.0K\Omega + (2.0K\Omega // (1.0K\Omega + 1.0K\Omega)) = 2K\Omega$$

Power

For a resistive load, the power us defined by

$$P = VI = I^2 R = V^2 / R$$

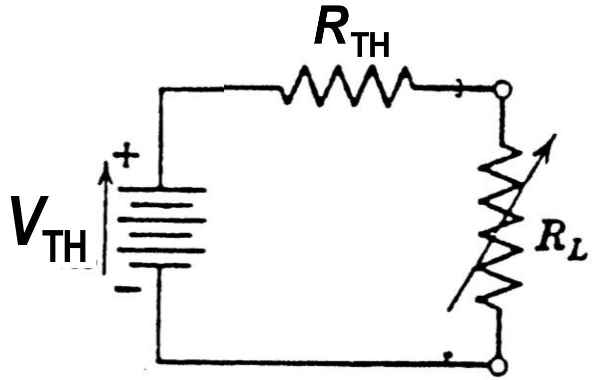
This power is dissipated in the form of heat in the resistor

This heat results from the collision of the flowing electronics in the resistor with the atoms of the resistor

A commercial grade resistor mentions in addition to its value, its maximum power dissipation

Maximum Power Transfer for DC Circuits

It is required to obtain the value of R_L at which maximum power is transferred from the source



$$V_L = V_{\mathrm{TH}} \frac{R_L}{R_{\mathrm{TH}} + R_L}, \quad P_L = \frac{V_L^2}{R_L} = \frac{V_{\mathrm{TH}}^2 \; R_L^2}{\left(R_{\mathrm{TH}} + R_L\right)^2} \; \frac{1}{R_L} = \frac{V_{\mathrm{TH}}^2 \; R_L}{\left(R_{\mathrm{TH}} + R_L\right)^2}$$

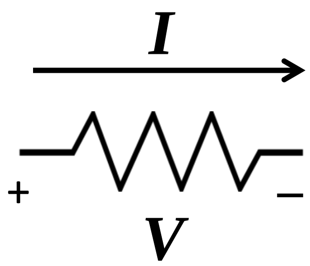
$$\frac{dP_L}{dR_L} = V_{\mathrm{TH}}^2 \frac{\left(R_{\mathrm{TH}} + R_L\right)^2 - 2R_L\left(R_{\mathrm{TH}} + R_L\right)}{\left(R_{\mathrm{TH}} + R_L\right)^4} = 0$$

$$\Downarrow$$

$$\left(R_{\mathrm{TH}} + R_L\right) - 2R_L = 0 \quad \Rightarrow \quad R_L\Big|_{\text{max power}} = R_{\mathrm{TH}}$$

Basic Electric Components

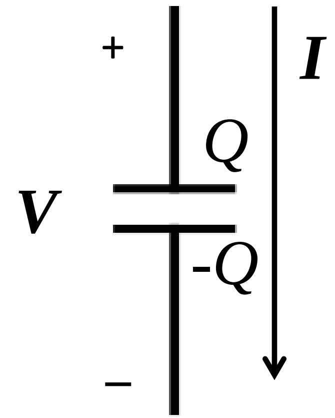
Resistor: $V=IR$, $I=V/R$ (no memory)



Capacitor: stores electric energy (has a memory)

$Q=CV \Rightarrow I=dQ/dt = CdV/dt$

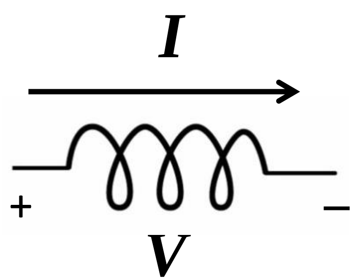
$$V(t) = \frac{1}{C}\int_{-\infty}^{t} I(\tau)d\tau = \frac{Q(t)}{C}$$



Inductor: stores magnetic energy (has a memory)

$\phi=LI \Rightarrow V=d\phi/dt=LdI/dt$

$$I(t) = \frac{1}{L}\int_{-\infty}^{t} V(\tau)d\tau = \frac{\phi(t)}{L}$$



Transient Response in RC Circuits (Charging)

$$E = \frac{Q}{C} + I\,R$$

$$I \equiv \frac{dQ}{dt} = \frac{E}{R} - \frac{Q}{RC} = -\frac{Q-CE}{RC}$$

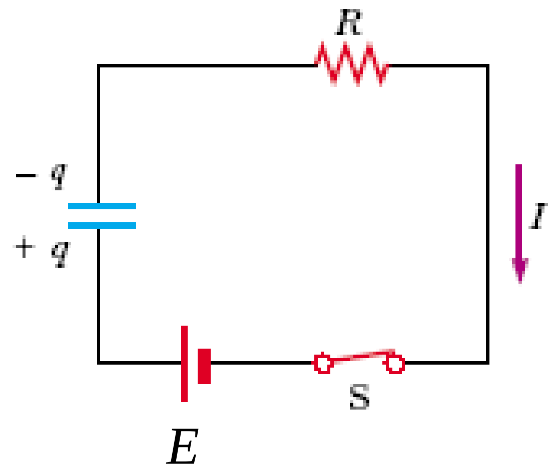
$$\int_0^Q \frac{dQ}{Q-CE} = -\frac{1}{RC}\int_0^t dt$$

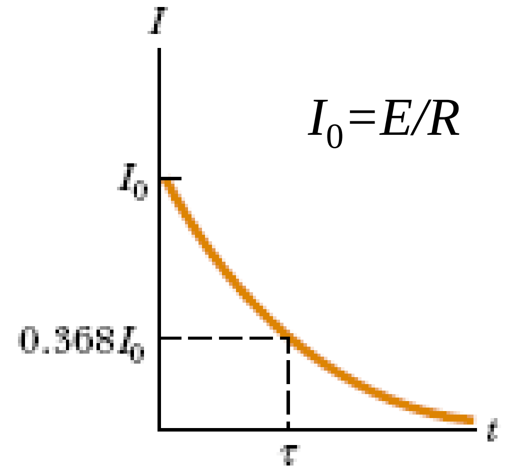
$$\ln\left(\frac{Q-CE}{-CE}\right) = -\frac{t}{RC}$$

$$Q = CE\left(1-e^{-t/RC}\right)$$

$$I(t) \equiv \frac{dQ}{dt} = \frac{E}{R}e^{-t/RC} = I_0\ e^{-t/\tau}$$

$$I\ \Big|_{t=\tau\equiv\text{time constant}=RC} = \frac{I_0}{e} = 0.368\ I_0$$





Transient Response in RC Circuits (Discharging)

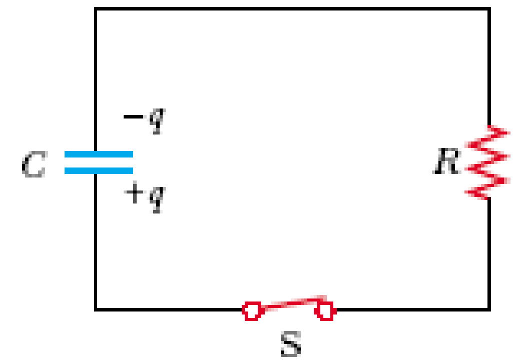
$$\frac{Q}{C} + I\ R = 0$$

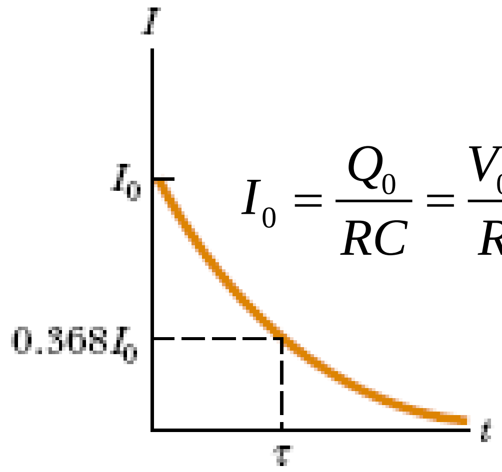
$$I \equiv \frac{dQ}{dt} = -\frac{Q}{RC} \;\Rightarrow \int_{Q_0}^{Q} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC} \;\Rightarrow\; Q = Q_0\ e^{-t/RC}$$

$$I(t) \equiv \frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-t/RC} = I_0\ e^{-t/\tau}$$

$$I\ \Big|_{t=\tau\equiv\text{time constant}=RC} = \frac{I_0}{e} = 0.368\ I_0$$





$$I_0 = \frac{Q_0}{RC} = \frac{V_0}{R}$$