Lecture 5:Circuit Theory (5)

Combination of elements, resonance circuits, filters, power in AC circuits

Combined AC Impedances

$$Z \equiv \begin{array}{c}\text{impedance of} \\ \text{combined elements } (\Omega)\end{array}$$

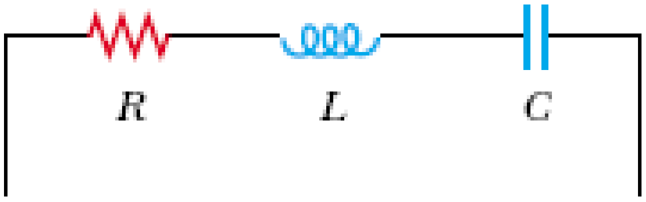
$$= \underbrace{R}_{\text{resistance }(\Omega)} + j \underbrace{X}_{\text{reactance }(\Omega)} \Rightarrow \begin{cases} \underline{\text{Resistor:}}\ R > 0,\ X = 0 \\ \underline{\text{Inductor:}}\ R = 0,\ X = \omega L \\ \underline{\text{Capacitor:}}\ R = 0,\ X = -1/\omega C \end{cases}$$

$$Y \equiv \begin{array}{c}\text{admittance of} \\ \text{combined elements } (1/\Omega)\end{array}$$

$$= \underbrace{G}_{\text{conductance }(1/\Omega)} + j \underbrace{B}_{\text{succeptance }(1/\Omega)} \Rightarrow \begin{cases} \underline{\text{Resistor:}}\ G = 1/R,\ B = 0 \\ \underline{\text{Inductor:}}\ G = 0,\ B = -1/\omega L \\ \underline{\text{Capacitor:}}\ G = 0,\ B = \omega C \end{cases}$$

Series Combination

$$Z_T = Z_1 + Z_2 + \cdots + Z_N, \quad Y_T = 1/Z_T$$



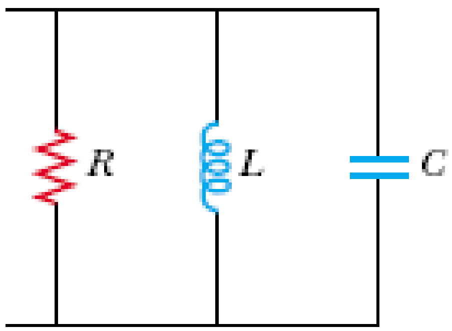
$$Z_T = Z_R + Z_L + Z_C = R + j\omega L + (1/j\omega C)$$

$$Y_T = \frac{1}{Z_T} = \frac{1}{R + j(\omega L - 1/\omega C)}$$

Parallel Combination

$$Y_T = Y_1 + Y_2 + \cdots + Y_N,$$

$$Z_T = 1/Y_T$$



$$Y_T = Y_R + Y_L + Y_C = (1/R) + (1/j\omega L) + j\omega C$$

$$Z_T = \frac{1}{Y_T} = \frac{1}{(1/R) + j(\omega C - 1/\omega L)}$$

Power in AC Circuits

For DC circuits, power consumption is constant and does not change with time!

In general, $p(t) \equiv \text{instantaneous power}$

$$= v\,i = V_{\max}\; I_{\max}\; \cos\left(\omega t + \varphi_V\right)\, \cos\left(\omega t + \varphi_I\right)$$

$$\overline{p} \equiv \text{time-average power} = \frac{1}{T}\int_0^T V_{\max}\; I_{\max}\; \cos\left(\underbrace{\omega t + \varphi_V}_{\theta_1}\right)\, \cos\left(\underbrace{\omega t + \varphi_I}_{\theta_2}\right) dt$$

$$= \frac{V_{\max}\; I_{\max}}{2T}\int_0^T \left[\cos\left(\underbrace{2\omega t + \varphi_V + \varphi_I}_{\theta_1+\theta_2}\right) + \cos\left(\underbrace{\varphi_V - \varphi_I}_{\theta_1-\theta_2}\right)\right] dt$$

$$= \frac{V_{\max}\; I_{\max}}{2T}\underbrace{\int_0^T \cos\left(2\omega t + \varphi_V + \varphi_I\right) dt}_{0} + \frac{V_{\max}\; I_{\max}}{2T}\int_0^T \cos\left(\varphi_V - \varphi_I\right) dt$$

Power in AC Circuits (Cont'd)

$$\text{time-average power} = \frac{V_{\max}\ I_{\max}}{2T}\ \cos\left(\varphi_V - \varphi_I\right)\int_0^T dt$$

$$T$$

$$\therefore\ \overline{p} \equiv \text{time-average power (active power)}$$

$$= \frac{1}{2}V_{\max}\ I_{\max}\ \underbrace{\cos\left(\varphi_V - \varphi_I\right)}_{\text{power factor}} = \underbrace{V_{\text{rms}}\ I_{\text{rms}}}_{\text{apparent power}}\ \underbrace{\cos\left(\varphi_V - \varphi_I\right)}_{\text{power factor}}$$

$$\text{power factor} \equiv \cos\left(\varphi_V - \varphi_I\right) \underset{Z=V/I}{=} \cos\left(\varphi_Z\right)$$

$$\underset{Z=R+jX}{=} \frac{R}{\sqrt{R^2 + X^2}} \underset{\substack{R\text{ is }+\\ X\text{ is }\pm}}{\Rightarrow} 0 \le \text{power factor} \le 1$$

What is the time-average power for resistors, capacitors, and inductors?

Alternative Expression for AC Power

The same value of time-average power can be obtained using the following formula:

$$\overline{p} \equiv \text{time-average power} = \frac{1}{2}\ \Re\text{e}\ \left(V\ I^*\right)$$

<u>Proof</u>: $\frac{1}{2}\Re\text{e}\ \left(V\ I^*\right) = \frac{1}{2}\Re\text{e}\ \left(V_{\max}\ e^{j\varphi_V}\ I_{\max}\ e^{-j\varphi_I}\right)$

$$= \frac{1}{2}V_{\max}\ I_{\max}\ \Re\text{e}\left[\ e^{j(\varphi_V - \varphi_I)}\ \right]$$

$$= \frac{1}{2}V_{\max}\ I_{\max}\ \cos\ \left(\varphi_V - \varphi_I\right) = \overline{p}$$

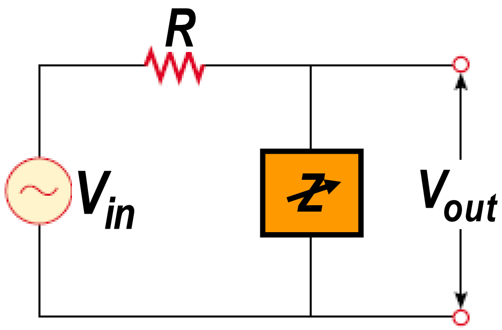
Filters

The filter is a ***frequency-selective*** two-port network, which allows certain frequency band(s) to pass from input to output port, and stops other frequency band(s).

If a *frequency-dependent impedance* (Z) is combined with a resistor (R) in voltage divider configurations, different filters can be realized. Other frequency-selective configurations are also used.

Filters find wide applications in different applications in electrical engineering

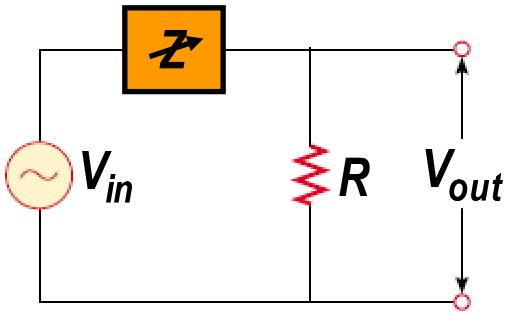
Voltage Divider as a Filter



$$A_v \equiv \text{voltage gain} \equiv \frac{V_{out}}{V_{in}} = \frac{Z}{Z+R}$$

$$|A_v| = \left|\frac{Z}{Z+R}\right| = \left|\frac{1}{1+R/Z}\right| = \frac{1}{|1+R/Z|} \propto |Z|$$

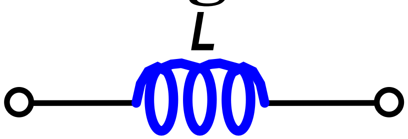
Voltage Divider (Cont'd)



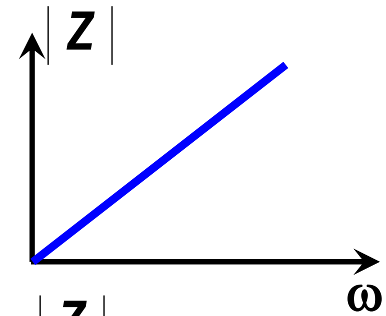
$$A_v \equiv \text{voltage gain} \equiv \frac{V_{out}}{V_{in}} = \frac{R}{R+Z}$$

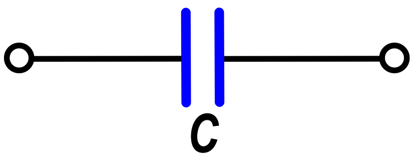
$$|A_v| = \left|\frac{R}{R+Z}\right| = \left|\frac{1}{1+Z/R}\right| = \frac{1}{|1+Z/R|} \propto \frac{1}{|Z|}$$

Voltage Divider (Cont'd)

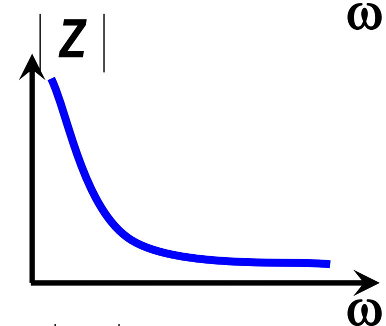


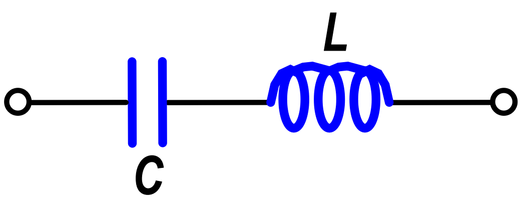
$$|Z| = \omega L = \begin{cases} 0 \text{ (s.c.)}, & \omega = 0 \\ \infty \text{ (o.c.)}, & \omega = \infty \end{cases}$$



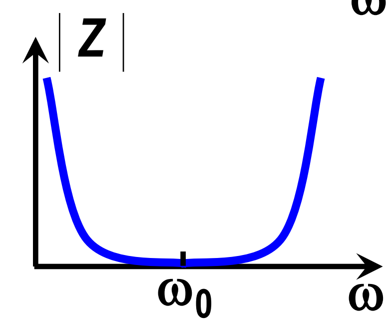


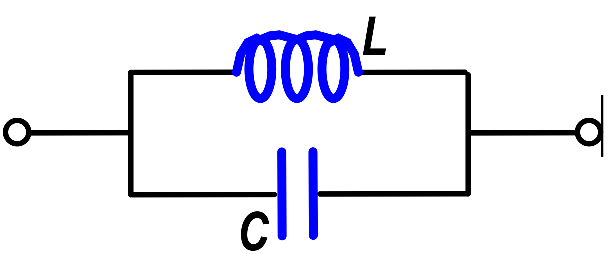
$$|Z| = \frac{1}{\omega C} = \begin{cases} \infty \text{ (o.c.)}, & \omega = 0 \\ 0 \text{ (s.c.)}, & \omega = \infty \end{cases}$$





$$|Z| = \left| \omega L - \frac{1}{\omega C} \right| = \begin{cases} \infty \text{ (o.c.)}, & \omega = 0 \\ 0 \text{ (s.c.)}, & \omega = \omega_0 \\ \infty \text{ (o.c.)}, & \omega = \infty \end{cases}$$





$$|Z| = \frac{1}{\left| \omega C - \dfrac{1}{\omega L} \right|} = \begin{cases} 0 \text{ (s.c.)}, & \omega = 0 \\ \infty \text{ (o.c.)}, & \omega = \omega_0 \\ 0 \text{ (s.c.)}, & \omega = \infty \end{cases}$$