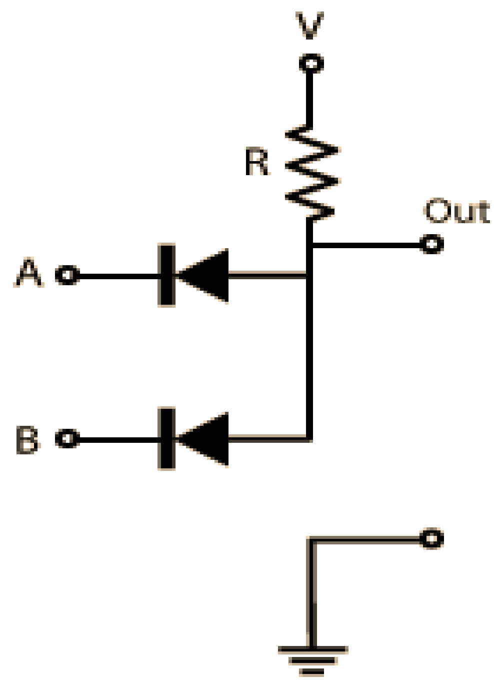
Lecture 20: Boolean Algebra

Diode Logic, De Morgan's Theorems, Examples

Diode AND Gate

if any of the inputs is connected to group (logic “0”), the output will be logic “0”

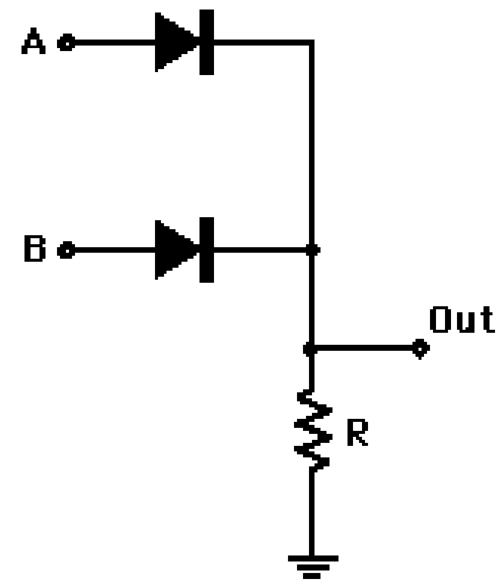
only when both inputs are high (logic “1”), both diodes will be off, and the output is logic “1”



Diode OR Gate

if any of the two inputs is high “logic 1” the corresponding diode turns on and the output is logic “1”

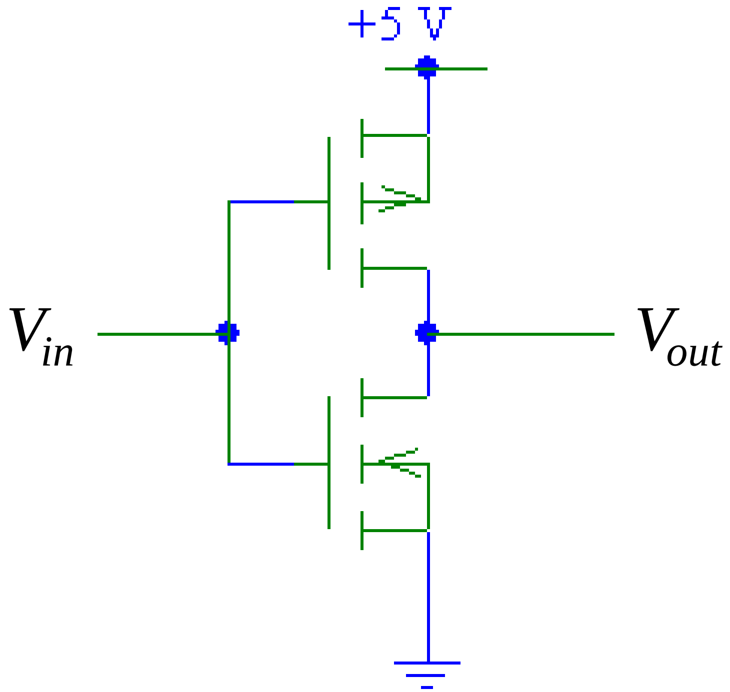
only when both inputs are low “logic 0”, both diodes are off and the output is low “logic 0”



CMOS Inverter

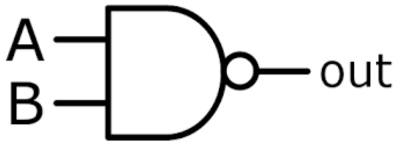
when the input is high (logic "1"), the NMOS transistor is on, the PMOS transistor is off, and the output is connected to ground (logic "0")

when the input is low (logic "0"), the NMOS transistor is off, the PMOS transistor is on, and the output is connected to the supply voltage (logic "1")



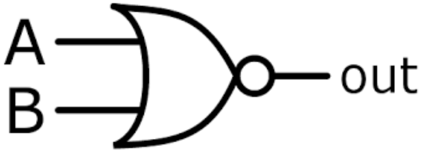
Other Useful Gates

NAND gate



A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate



A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0

Axioms of Boolean Algebra

identity $x + 0 = x1 = x$

commutative $x + y = y + x, \; x\,y = y\,x$

distributive $x\,(y + z) = xy + xz$

$x + (y\,z) = (x + y)(x + z)$

$xy + x\overline{y} = x, \quad x + xy = x$

complement $x + \overline{x} = 1, \quad x\overline{x} = 0$

DE MORGAN's Theorems

$$\overline{x+y} = \overline{x}.\overline{y}$$

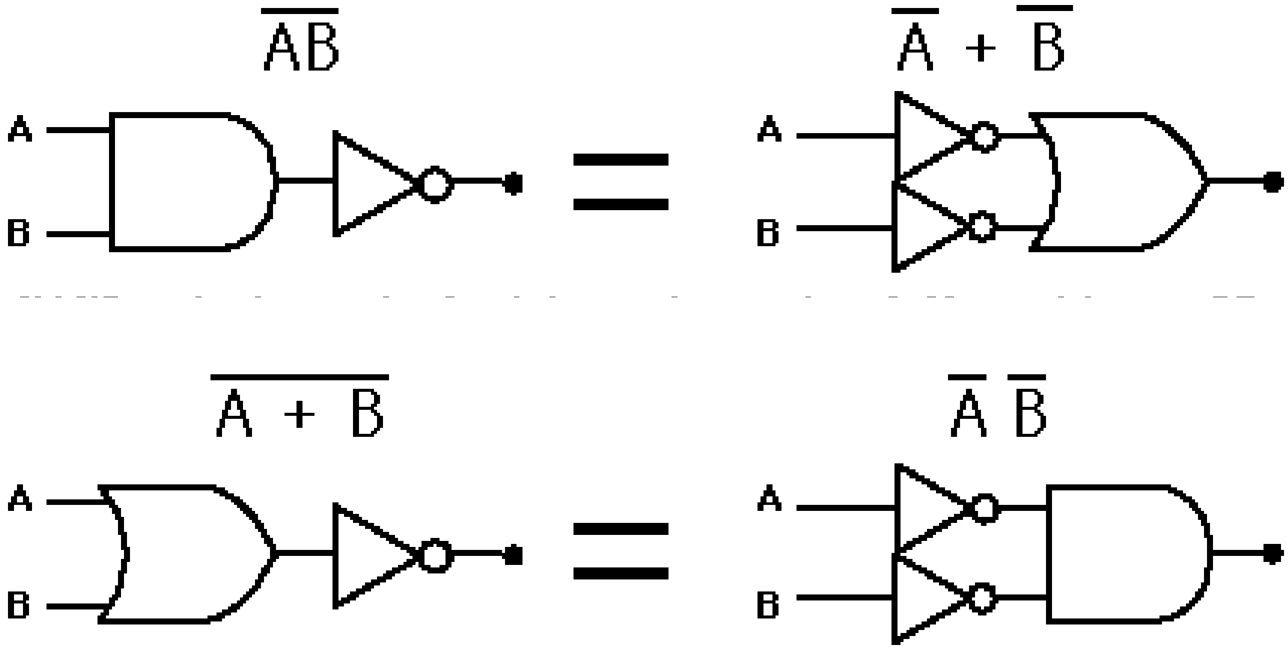
$$\overline{x.y} = \overline{x} + \overline{y}$$

negating the result of an OR logic function is equivalent to ANDing of the negated two inputs

negating the result of an AND logic function is equivalent to ORing of the negated two inputs

this theorems are very useful in Boolean algebra and in synthesizing truth tables

Illustration



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Some Useful Theorems

Table 2.1
Postulates and Theorems of Boolean Algebra

Postulate 2 identity	(a)	$x + 0 = x$
Postulate 5 complement	(a)	$x + x' = 1$
Theorem 1 idempotent	(a)	$x + x = x$
Theorem 2 0 and 1 ops	(a)	$x + 1 = 1$
Theorem 3, involution		$(x')' = x$
Postulate 3, commutative	(a)	$x + y = y + x$
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$
Theorem 6, absorption	(a)	$x + xy = x$