

Lecture 21: Combinational Logic Functions

Simplification of Logic Functions, Sum of Products, Karnaugh Maps, Examples

Simplification of Logic Functions

using Boolean algebra rules, logic functions can be simplified
the result can be implemented as a sum of products (cascade of
AND and OR gates)

Example:

$$F = ABC + A\bar{B}C + AB\bar{C}$$

$$F = ABC + ABC + A\bar{B}C + AB\bar{C}$$

$$F = (ABC + A\bar{B}C) + (ABC + AB\bar{C})$$

$$F = AC(B + \bar{B}) + AB(C + \bar{C})$$

$$F = AC + AB$$

Implementation of SOPs

a product implies using AND gates

summation implies using OR gate

any SOP can thus be implemented using inverters, AND gates, and OR gates

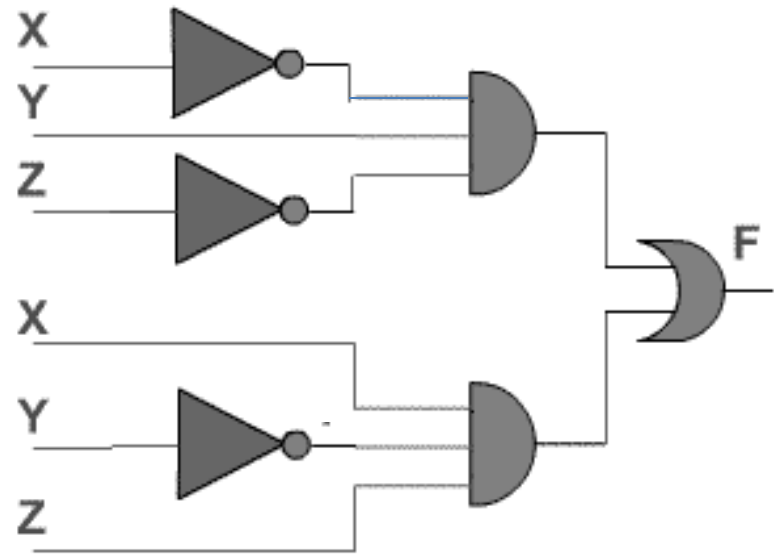
$$F = \bar{X}Y\bar{Z} + X\bar{Y}Z$$

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Implementation of SoPs(Cont'd)

the more compact the expression is, the fewer gates are needed to implement its expression

complex logic expression can not be simplified by inspection only!



Karnaugh Maps

a Karnaugh map is a way to simplify logic functions of up to six variables

the map is derived from the truth table by combining variables in groups of 1, 2, or 4

only one logical variable is allowed to change when moving horizontally or vertically in the map

the map is inspected to get the most compact logical expression

Karnaugh Maps (Continued)

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

		YZ			
		00	01	11	10
X	0	0	0	1	0
	1	0	1	1	1

we then combine the expressions that would give a logical output of “1” along the rows and along the columns but not diagonally

$$F = XZ + YZ + XY$$

Karnaugh Maps (Continued)

$$F = XZ + YZ + XY$$

the implementation is?

Rules for Grouping in Karnaugh Maps

		X	
		0	1
Y	0	0	0
	1	1	0

		X	
		0	1
Y	0	0	0
	1	1	1

		X	
		0	1
Y	0	0	1
	1	1	0

		X	
		0	1
Y	0	0	1
	1	1	1

Rules for Grouping (Cont'd)

YZ

	00	01	11	10
X ⁰	0	0	1	0
X ¹	0	1	1	1

YZ

	00	01	11	10
X ⁰	1	1	1	1
X ¹	0	0	0	0

YZ

	00	01	11	10
X ⁰	1	1	1	1
X ¹	0	0	1	1

YZ

	00	01	11	10
X ⁰	1	1	1	1
X ¹	0	0	1	1

Rules for Grouping (Cont'd)

		YZ			
		00	01	11	10
X	0	1	1	1	1
	1	1	0	0	1

Rules Summary

no zeros allowed

no diagonals (only horizontal and vertical grouping)

only power of 2 number of cells in each group

groups as large as possible

every “1” in the map must be within a group

overlapping OK

wrapping around OK

smallest number of groups possible