3 Procedure

3.1 2nd Order Circuits

The idea of this exercise is to compare values obtained by theory and measurement, relating to the *transient* behaviour of RLC circuits. Using the component values listed above, construct a variety of circuits and calculate values of the pertinent parameters, such as natural frequency ω_o and damping factor ξ. Compare the calculated values with those you obtain by measurement. In your report, give a detailed explanation of the theoretical method you used in your calculations.

Note that the the damping factor for the *parallel* RLC circuit is given by

$$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

whereas for the *series* case it is given by

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

There are a couple of hints you will need in measuring ω_o and ξ. It is suggested you measure these only for the underdamped case.

Underdamped case: The equation for the underdamped response $v(t)$ is

$$v(t) = K_1 e^{-\xi\omega_o t}\cos\left(\sqrt{(1-\xi^2)}\omega_o t\right) + K_2 e^{-\xi\omega_o t}\sin\left(\sqrt{(1-\xi^2)}\omega_o t\right) \quad (1)$$

For the purposes of this experiment, we can ignore the effect of the second term above, since it only serves to change the phase of the response.

It is easiest if you choose a value of $\xi \leq 0.2$. This is because the natural frequency ω_n of the damped oscillation from (1) is $\omega_n = \sqrt{(1-\xi^2)}\omega_o$. This is the frequency of the waveform you will see displayed on the scope. If $\xi \leq 0.2$, then $\sqrt{(1-\xi^2)} \geq 0.98$ and $\omega_n \approx \omega_o$. Then, ω_o can be approximately measured as shown in figure 1. Measure the period τ_o of the oscillation as shown in the figure. Then, $\omega_o \approx 2\pi/\tau_o$.

To measure ξ, we consider the ratio of measured values v_1 and v_2 at times t_1 and t_2 respectively in (1), as shown in figure 1. Because we are taking the two measurements at the same point on the $\cos(\cdot)$ waveform, and because we ignore the second term in (1), we obtain

$$\begin{aligned}\frac{v_2}{v_1} &= e^{\left(-\omega_o(t_1-t_2)\xi\right)} \\ &= e^{\left(-2\pi\xi\right)}. \end{aligned} \quad (2)$$

Thus, ξ can be determined after measuring the ratio of v_2 to v_1.

Critically damped case: In this case, it is difficult to make quantitative measurements. You can verify that a circuit is close to critically damped in the following way. Use the resistor value corresponding to a critically

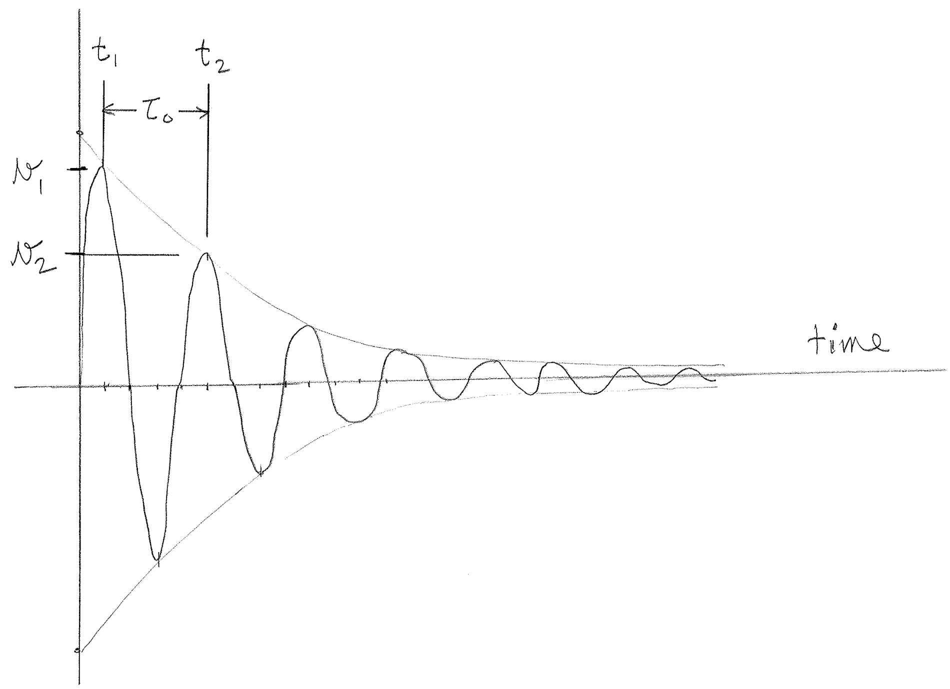

Figure 1: Plot of the voltage waveform of an underdamped RLC circuit.

damped condition. Then you should see the response in red in figure 2. The unfortunate part is that there doesn't seem to be a way of measuring whether this response is indeed critically damped (on the scope, it looks almost indistinguishable from the overdamped case). However, if you adjust the resistor so that the corresponding damping factor is about 20% smaller, then you should observe the "slightly underdamped", or solid blue response shown in figure 2, which has a slight overshoot. Similarly, by adjusting the resistor so that the damping factor increases by about 20%, you should observe the dashed blue "slightly overdamped" response in figure 2. This way, you can verify that your original circuit is close to being critically damped.

Overdamped case: It is not necessary to do detailed measurements for this case. Try resistor values corresponding to an overdamped value of ξ and observe the response.

3.2 Response of a Simple RC circuit to a Sinusoidal Input

The idea of this exercise is to get a feeling for how circuits respond to sinusoidal inputs. Hook up the following circuit, and measure the magnitude and phase of the output phasor $\tilde{v}_{\text{out}}$, relative to the input phasor $\tilde{v}$, vs. frequency.

Measure your circuit over the range $0.1 f_o \leq f \leq 20 f_o$, where f_o is the so-called *cutoff* frequency, defined as

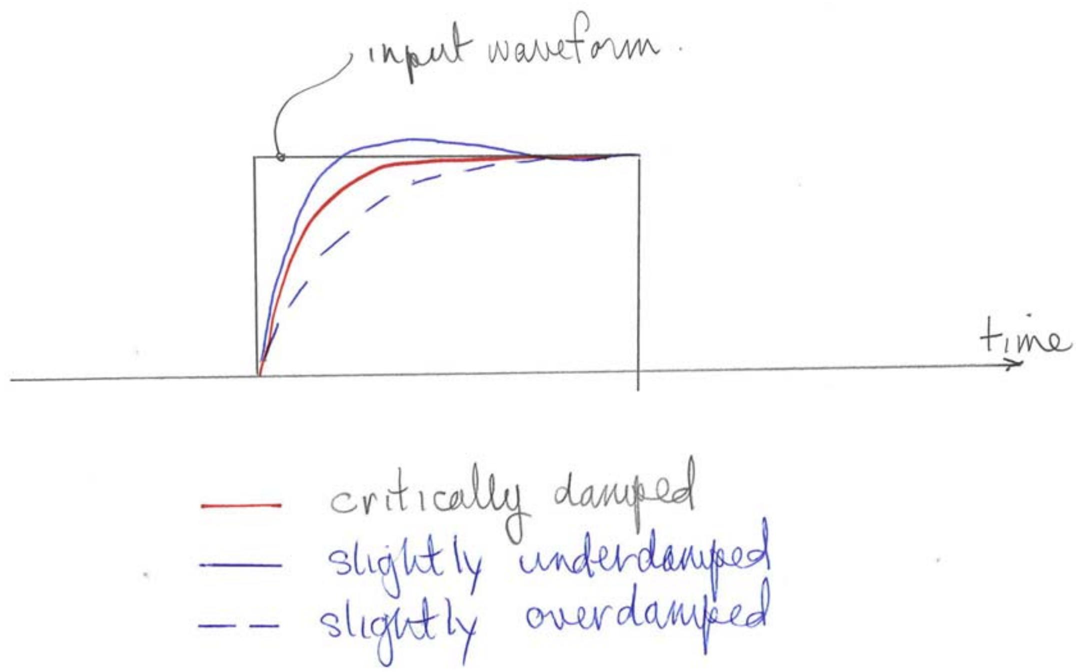

Figure 2: Slightly underdamped, critically damped, and slightly overdamped responses.

$f = 1/(2\pi RC)$, and plot both magnitude and phase vs. frequency.

What is the output phasor (relative to the input phasor) at $f = f_o$?