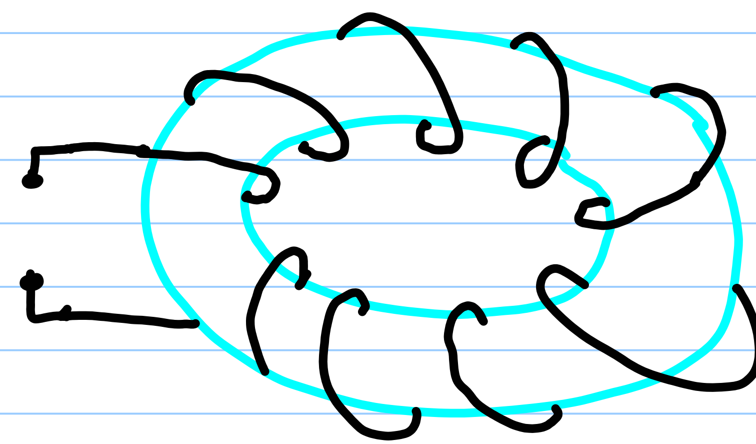
Lecture 16

From section 6.2 of Textbook

Solve E6.4, 6.21, 6.23, 6.24, 6.28,
6.32, 6.35, 6.42

Inductors

* An Inductor is a two terminal device that stores energy in its magnetic field.

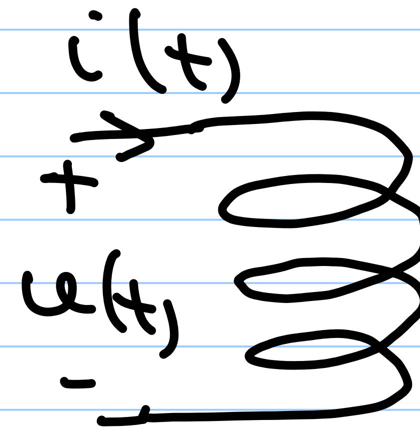


Inductors (Cont'd)

* Curent in inductor creates a magnetic flux Φ.
* For a LINEAR inductor $\Phi = Li$, where L is the inductance (Henries)
* From Faraday's Law, voltage is equal to rate of change of flux

$$v(t) = \frac{d\Phi}{dt} = L\frac{di}{dt}$$ (Short Circuit in DC)

Voltage and Current



$$v(t) = L\frac{di}{dt}$$

$$\Rightarrow \quad di = \frac{1}{L} v(t)\, dt$$

$$\int_{-\infty}^{t} di(\tau) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau$$

$i(-\infty) = 0$

$$\Rightarrow \quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau$$

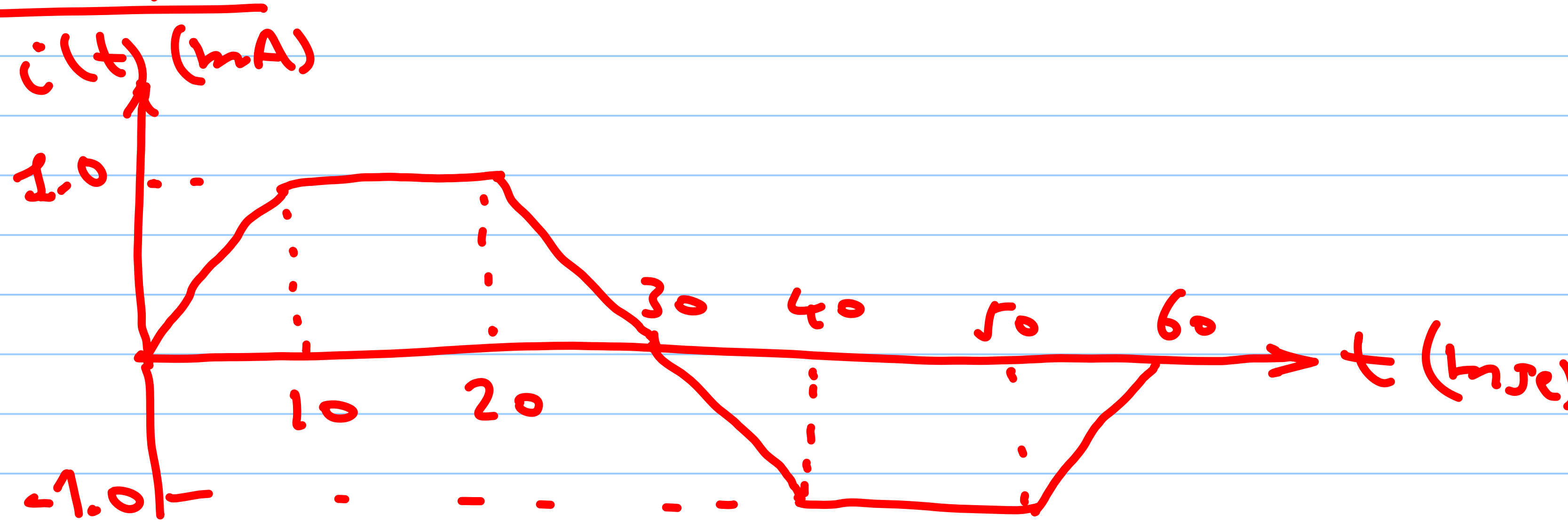
Voltage and Curent (Cont'd)

* Assume we have $i(t_0) = i_0$

$$i(t) = \frac{1}{L}\left(\int_{-\infty}^{t_0} v(\tau)\, d\tau + \int_{t_0}^{t} v(\tau)\, d\tau \right)$$

$$\Rightarrow i(t) = i(t_0) + \underbrace{\frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau}_{i(t_0 \to t)}$$

Example

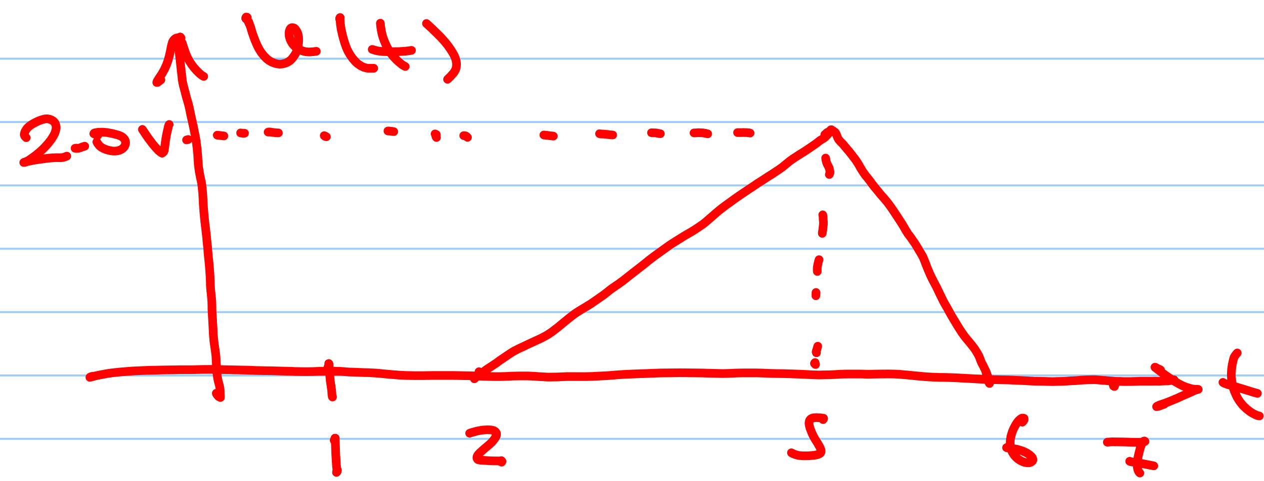


Find the voltage across an Inductor $L = 0.5\,H$ for the shown current waveform

Example

If the current through an inductor $L = 0.1\,H$ is $i(t) = 0.1 \sin(200\pi t)$, find the inductor voltage as a function of time

Example



Find the current waveform of an inductor $L = 2H$. Take $i(2) = 0A$.

Example

Find the current waveform in an inductor $L = 4H$ for the shown voltage. Take $v(t) = 0$ for $t < 0$

Power and Energy

* Power supplied to the inductor is given by $p(t) = v(t)\,i(t)$

$$p(t) = i(t) * L\frac{di(t)}{dt}$$

* Notice that current must change in a continuous way in an inductor.

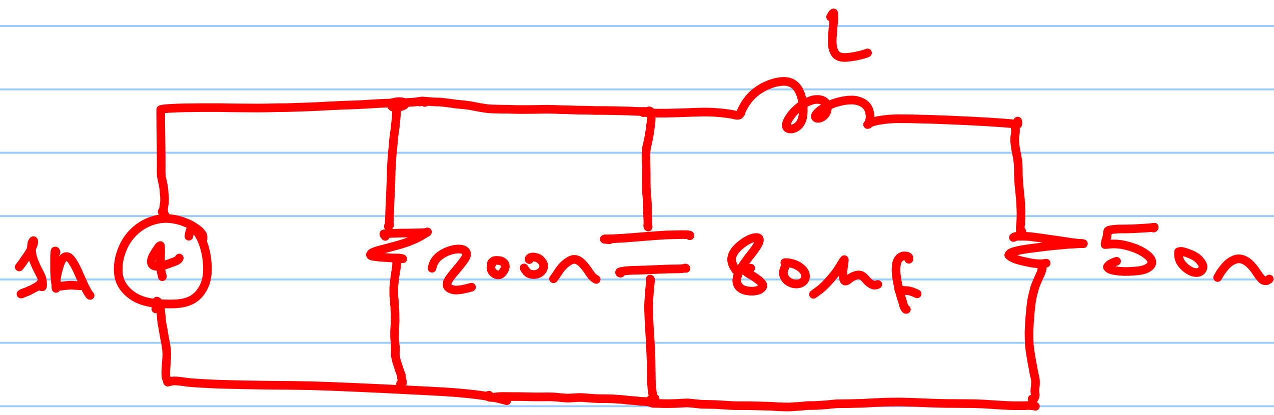
Power and Energy (Cont'd)

$$w(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = \int_{-\infty}^{t} i(\tau) * L \frac{di(\tau)}{d\tau}\, d\tau$$

$$w(t) = L \int_{-\infty}^{t} i(\tau)\, d\tau$$

$$w(t) = \frac{1}{2} L\, i^2(t) \quad J$$

Example



If the total energy stored in this circuit is 80 mJ, what is the value of L?