Lecture 20

From section 7.3 of Textbook

Solve E7.7 - E7.11, 7.75, 7.80, 7.83, 7.87, 7.90, 7.92

Second order Circuit

* A second order circuit has 2 energy storage elements
* Its responses (current or voltage) are represented by a 2nd order Differential equation
* This circuits include LC series resonance circuits and parallel resonance circuits.

Mathematical Background

The D.E. $\frac{d^2x(t)}{dt^2} + a_1 \frac{dx}{dt} + a_2 x(t) = f(t)$

has two solutions, the homogenous solution and the particular integral solution

$$x(t) = x_h(t) + x_{P.I}(t)$$

Mathematical Background (Cont'd)

* The homogenous solution $x_h(t)$ is a solution of the system

$$\frac{d^2x(t)}{dt^2} + a_1\frac{dx}{dt} + a_2 x(t) = 0$$

* rewritting the D.E.

$$\frac{d^2x(t)}{dt^2} + 2\xi\omega_0\frac{dx(t)}{dt} + \omega_0^2 x(t) = 0$$

* This D.E. has a solution of the form

Homogenous Solution

* $x(t) = Ke^{st}$

Substituting into the D.E, we get

$$Ks^2e^{st} + 2K\zeta\omega_0 s e^{st} + \omega_0^2 * Ke^{st} = 0$$

$$\Rightarrow \quad s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

the Characteristic equation

* The natural frequencies are

$s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$ and $s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$

Homogenous Solution (Cont'd)

* Overdamped Case $(\zeta > 1)$

$x_h(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$, $s_1 \neq s_2$ and both are real

* Critically damped Case $\zeta = 1$

$x_h(t) = K_1 e^{s_1 t} + K_2 t e^{s_1 t}$ $(s_1 = s_2)$ and both are real

Homogenous Solution

* Underdamped Case $\zeta < 1$

$\Rightarrow$ roots are complex

$$s_1 = -\zeta \omega_0 + j\omega_0\sqrt{1-\zeta^2} = -\sigma + j\omega_d$$

$$s_2 = -\zeta \omega_0 - j\omega_0\sqrt{1-\zeta^2} = -\sigma - j\omega_d$$

$$\Rightarrow x_h(t) = e^{-\sigma t}\left(A_1 \cos \omega_d t + A_2 \sin \omega_d t\right)$$

The P.I Case

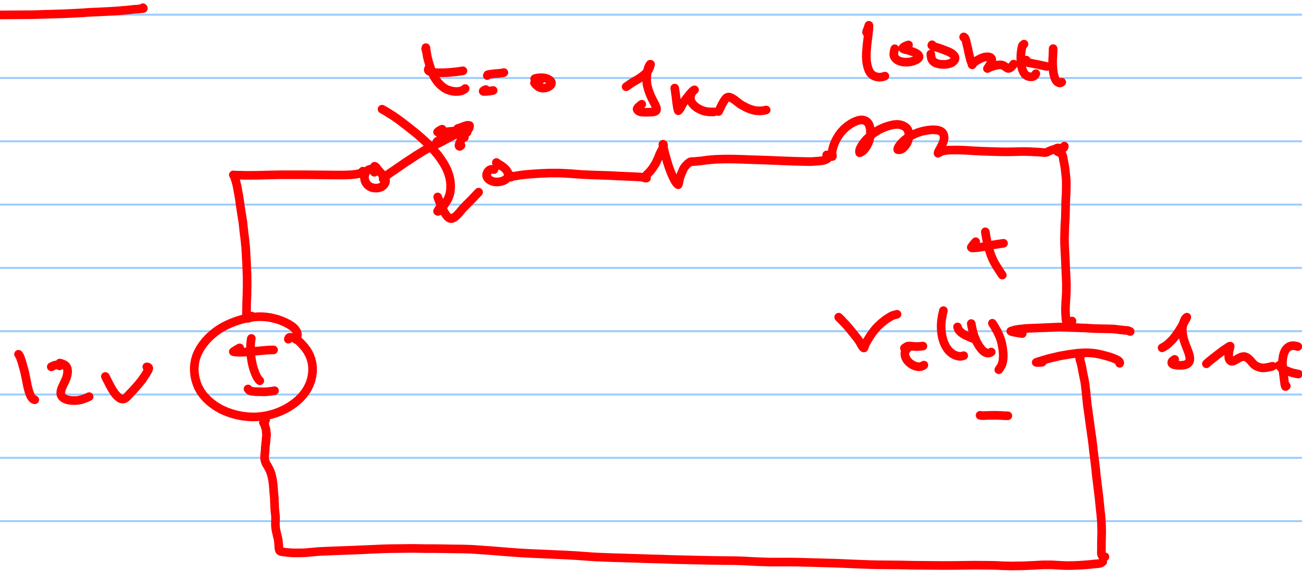
* The particular Integral Solution is a solution to $\frac{d^2x}{dt^2} + a_1\frac{dx}{dt} + a_2x(t) = f(t)$
* For the case $f(t) = A$, we have

$$x_{P.I}(t) = \frac{A}{a_2}$$

* It follows that

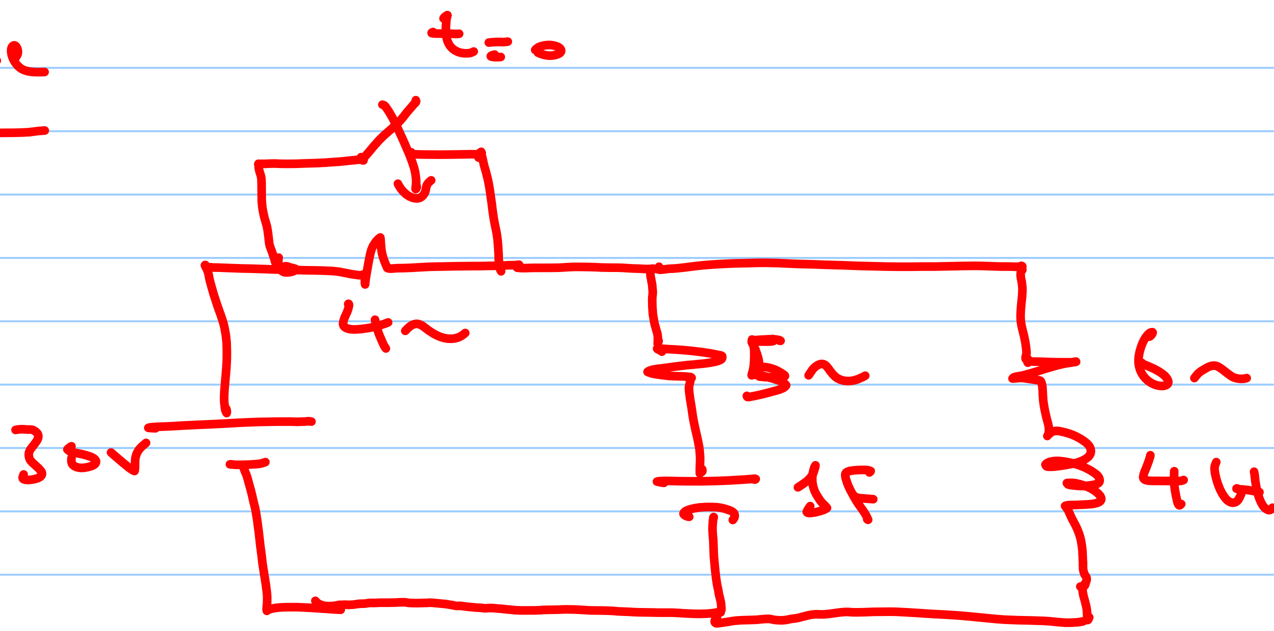
$$x(t) = x_h(t) + x_{P.I}(t)$$

<u>Example</u>



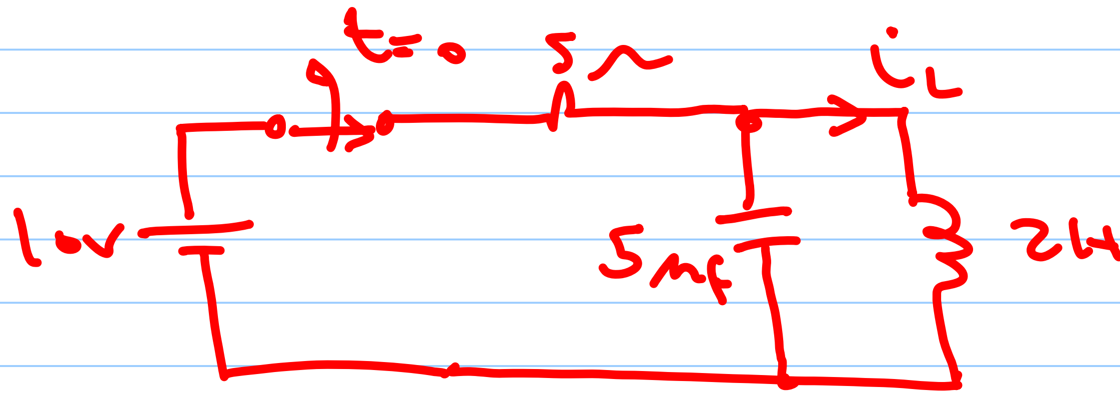
Find $v_c(t)$ for $t>0$, if $v_c(0)=0$.

Example



* obtain the initial conditions for i_C and i_L. Find $i_C(t)$ for $t > 0$

Example



find the natural frequencies of the current i_L