

# Lecture 21

From Sections 8.1 & 8.2

Solve: E8.1, E8.2, 8.2, 8.4,

8.7

## Sinusoidal Signals

\* A sinusoid is defined by 3 parameters, amplitude, frequency, and phase

$$v(t) = A \cos(\omega t + \phi)$$

$\swarrow$   $\omega$   $\swarrow$   $\swarrow$   
rad/s    rad/s    rad

## Example

Plot the sinusoidal waveforms

$$v_1(t) = 35 \sin(120\pi t + 60^\circ) \text{ V}$$

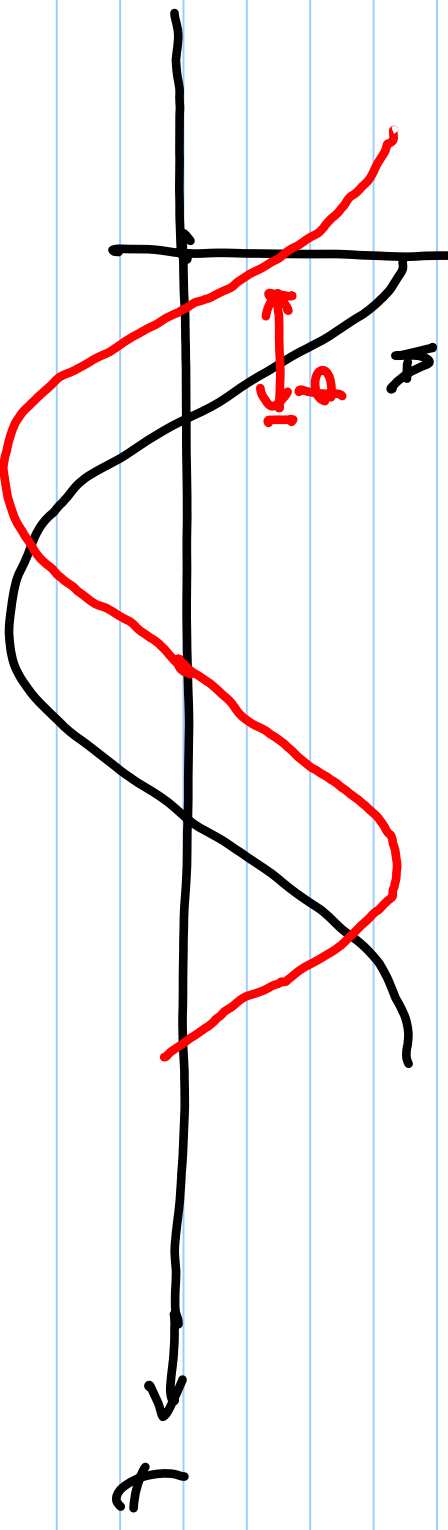
$$v_2(t) = 10 \cos(150t - 120^\circ) \text{ V}$$

$$i_1(t) = 0.5 \cos(120\pi t + 270^\circ) \text{ V}$$

## Lead-Lag Relationships

\* The signal  $u_2(t) = A \cos(\omega t + \phi)$   
leads the signal  $u_1(t) = A \cos(\omega t)$

by  $\phi$

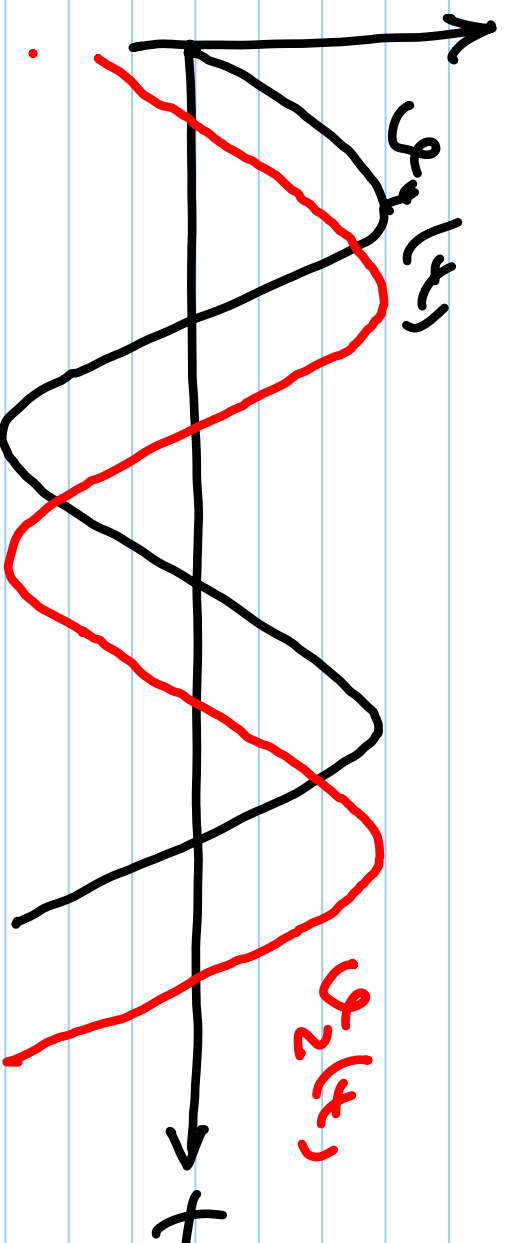


## Lead-Lag Relations (Cont'd)

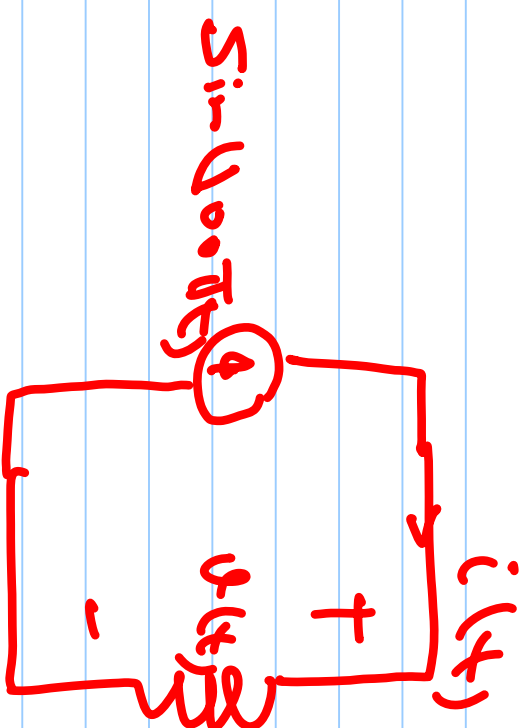
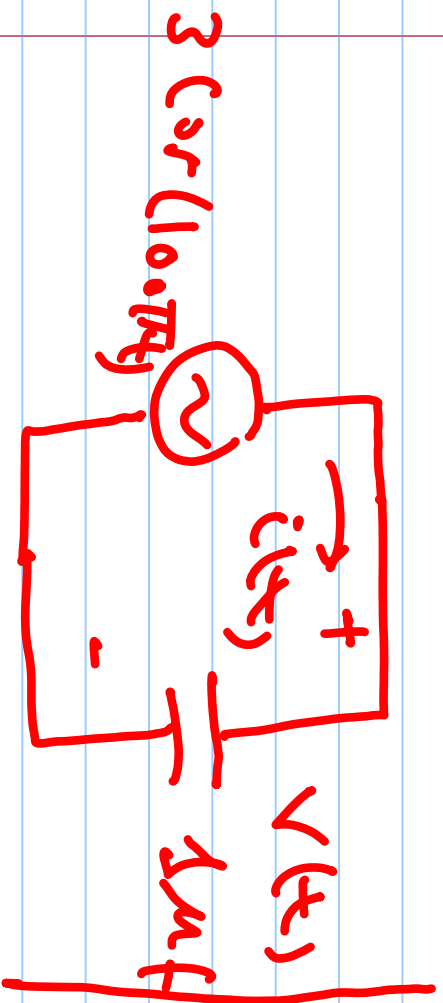
\* The signal  $y_2(t) = A \sin(\omega t - \phi)$

Lags the signal  $y_1(t) = A \sin(\omega t)$

by an angle  $\phi$



# Example



For these two circuit plot both  $v(t)$  and  $i(t)$

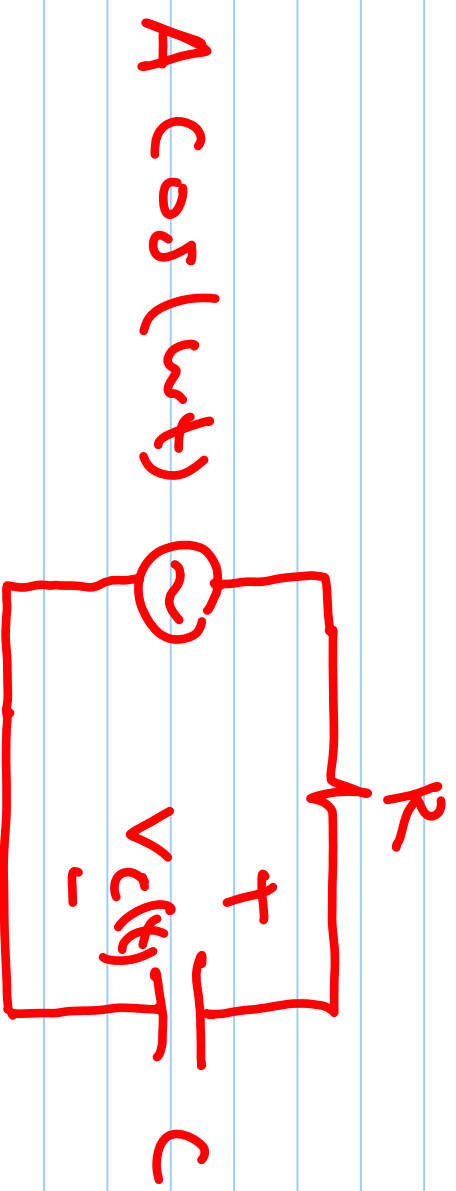
## Linear Circuits

\* A linear circuit does not change frequency.

\* All steady state responses (voltage or current) of such circuit are sinusoidal

\* They only differ in amplitude and phase

## Example



\* Write the differential equation of this circuit

\* By assuming a steady state solution of the form  $Q_C(t) = A_C \cos(\omega t + \phi_C)$   
Find  $Q_C(t)$ .



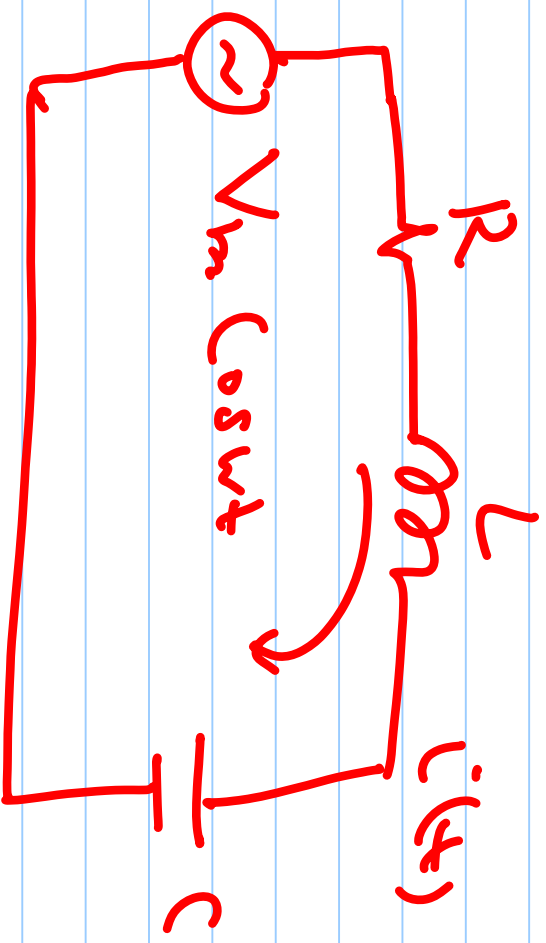
## Complex Forcing Function

\* Euler's Equation  $e^{j\omega t} = \cos \omega t + j \sin \omega t$

$$\Rightarrow \cos \omega t = \operatorname{Re}(e^{j\omega t})$$

\* A linear circuit can be analyzed using the complex response and then take the real part of the response only!

## EXamp 1e



\* Solve for the steady state current using complex functions