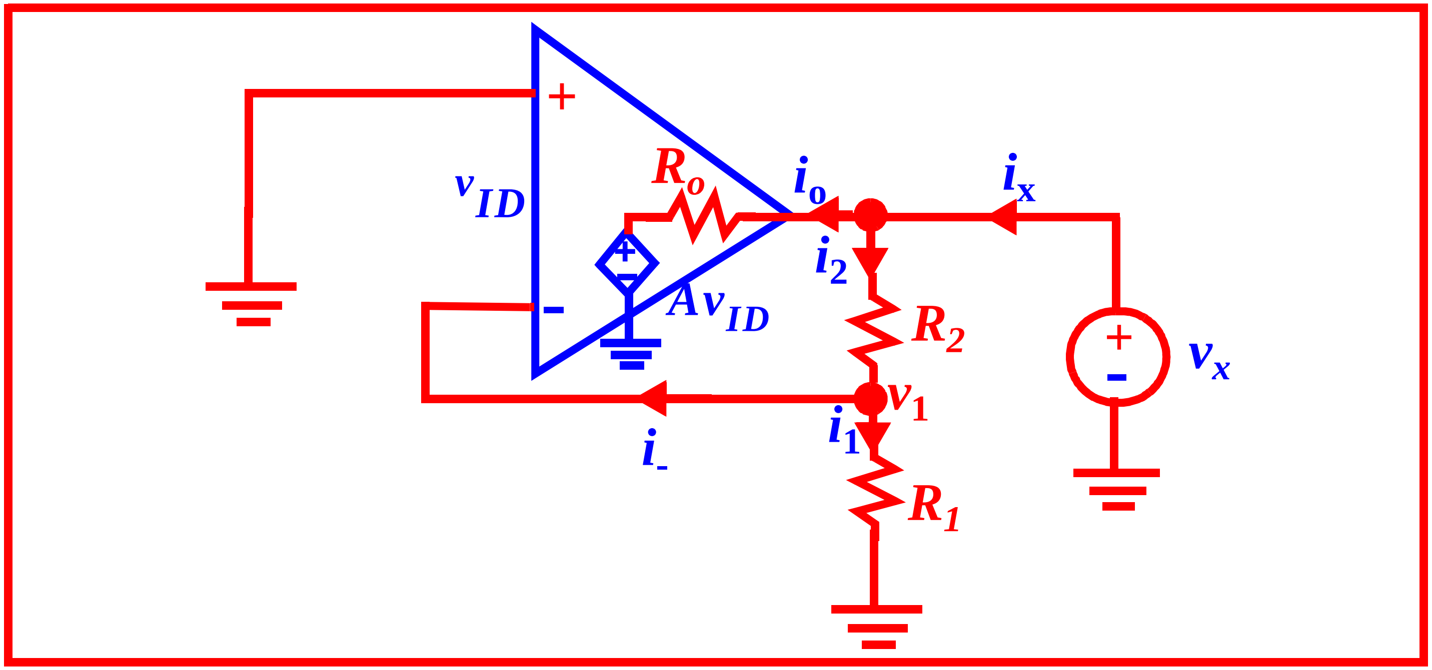
ECE 2EI4 - Lecture 8

Non-Zero Output Resistance - Non-Inv. Amplifier

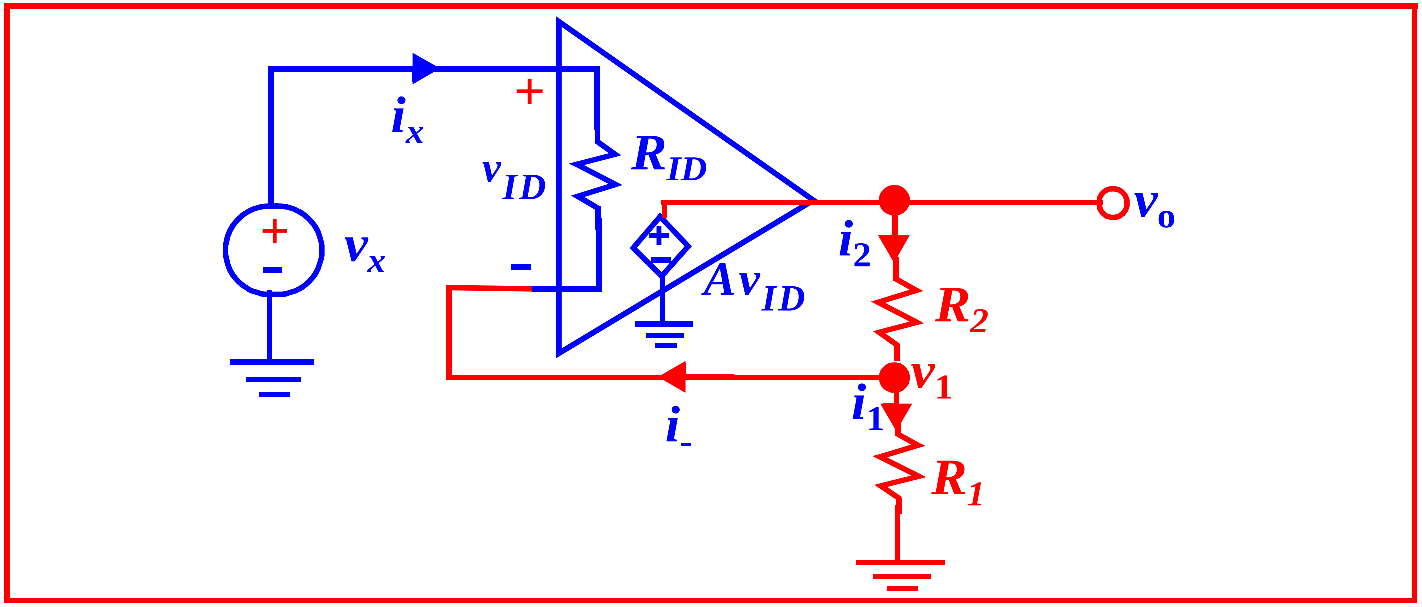


$$v_1 = \frac{R_1}{R_1 + R_2} \cdot v_x = \beta v_x; \quad i_x = i_o + i_2 = \frac{v_x - Av_{ID}}{R_O} + \frac{v_x}{R_1 + R_2};$$

$v_1 = -v_{ID} = \beta v_x$. Therefore, $\frac{1}{R_{OUT}} = \frac{i_x}{v_x} = \frac{1 + A\beta}{R_O} + \frac{1}{R_1 + R_2}$.

$$R_{OUT} = \frac{R_O}{1 + A\beta} \parallel (R_1 + R_2) \approx \frac{R_O}{1 + A\beta} \approx \frac{R_O}{A\beta}.$$

Finite Input Resistance- Non-Inv. Amplifier

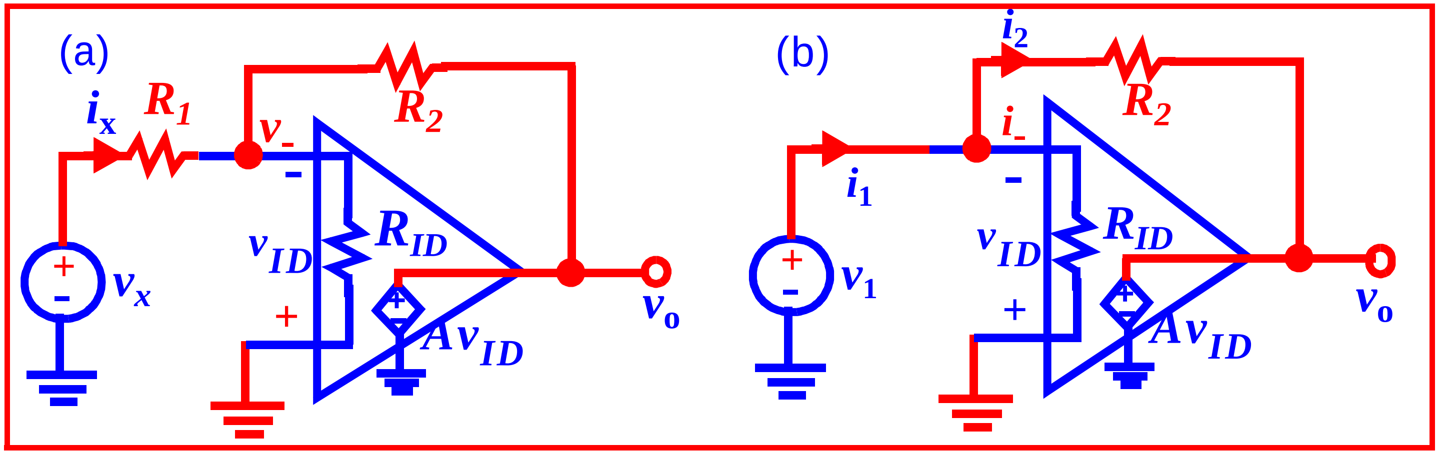


$$v_1 \approx \frac{R_1}{R_1 + R_2} \cdot v_o = \beta v_o = \beta A v_{ID} = A\beta(v_x - v_1).$$

$$v_1 = \frac{A\beta}{1 + A\beta} v_x \text{ and } i_x = \frac{v_x - \dfrac{A\beta}{1 + A\beta} v_x}{R_{ID}} = \frac{v_x}{(1 + A\beta) R_{ID}}$$

Since $\boxed{R_{IN} = v_x / i_x}$, then $\boxed{R_{IN} = R_{ID} \cdot (1 + A\beta) \approx A\beta R_{ID}}$.

Input Resistance for Inverting Amplifier (Read carefully)



$R_{IN} = \frac{v_x}{i_x}$; $v_x = i_x R_1 + v_-$ so, we get that $R_{IN} = R_1 + \frac{v_-}{i_x}$.

Total input resistance R_{IN} = R_1 + Resistance looking into the inverting terminal of the amplifier (use second circuit (b) above)

$$i_1 = i_- + i_2 = \frac{v_1}{R_{ID}} + \frac{v_1 - v_0}{R_2} = \frac{v_1}{R_{ID}} + \frac{v_1(1+A)}{R_2} \text{ since } v_0 = -Av_1.$$

$$G_{1,IP} = \frac{i_1}{v_1} = \frac{1}{R_{ID}} + \frac{1+A}{R_2} \quad \text{or} \quad R_{1,IP} = R_{ID} \| \frac{R_2}{1+A}.$$

In ckt (a), overall R_{IN} is $R_{IN} = R_1 + R_{ID} \| \frac{R_2}{1+A} \approx R_1 + \frac{R_2}{1+A} \approx R_1$.

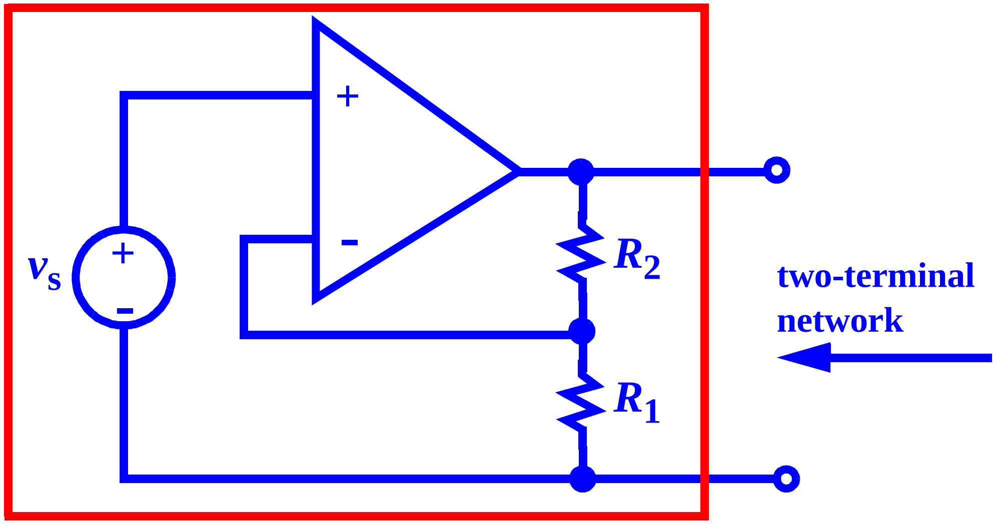
Comparison of Inv. and Non-Inv. Amplifier

(Read carefully)

	Inverting Amplifier	Non-Inverting Amplifier
Voltage Gain A_V	$-\frac{R_2}{R_1}\left(\frac{A\beta}{1+A\beta}\right) \approx -\frac{R_2}{R_1}$	$\left(\frac{A}{1+A\beta}\right) \approx \frac{1}{\beta} = 1+\frac{R_2}{R_1}$
Input Resistance R_{IN}	$R_1 + R_{ID} \parallel \frac{R_2}{1+A} \approx R_1$	$R_{ID} \cdot (1+A\beta)$
Output Resistance R_{OUT}	$\frac{R_O}{1+A\beta} \approx \frac{R_O}{A\beta}$	$\frac{R_O}{1+A\beta} \approx \frac{R_O}{A\beta}$

Example:

The following circuit is a two-terminal network with $R_1 = 400\Omega$ and $R_2 = 60k\Omega$. What is its Thevenin equivalent circuit if the operational amplifier has $A = 2\times10^4$, $R_{ID} = 200k\Omega$, and $R_O = 100\Omega$?

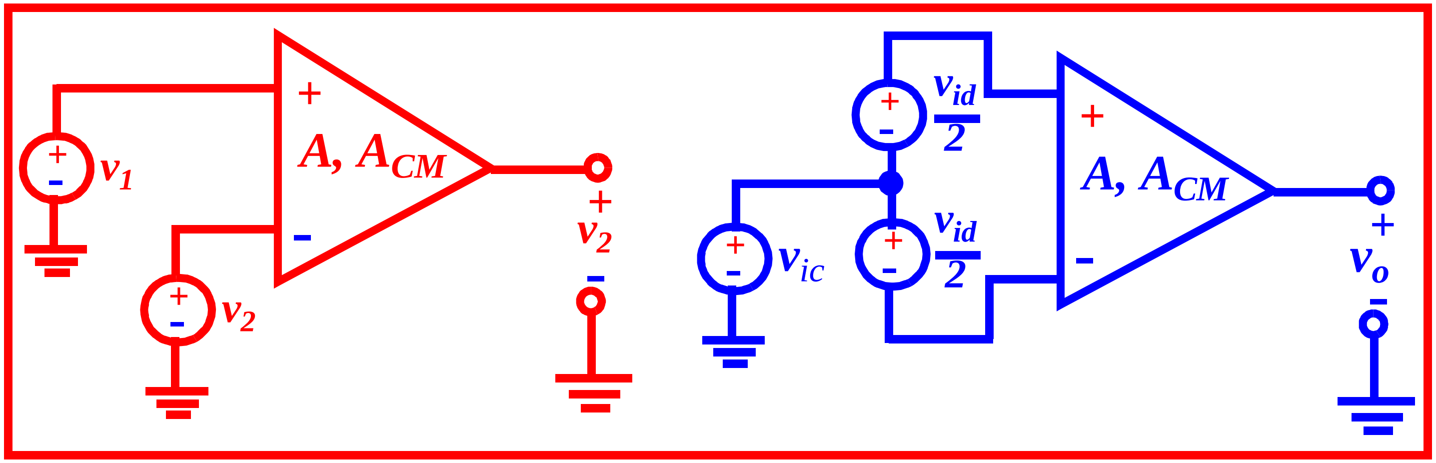


$$A\beta = (2\times10^4)\left(\frac{400}{400 + 60k}\right) = 132$$

The open circuit voltage is $v_{th} = \frac{A}{1 + A\beta}v_s = \frac{2\times10^4}{1 + 132}v_s = 150v_s$.

$$R_{th} = R_{OUT} = \frac{R_O}{1 + A\beta} = \frac{100}{1 + 132} = 0.749\Omega$$

Finite Common Mode Rejection Ratio

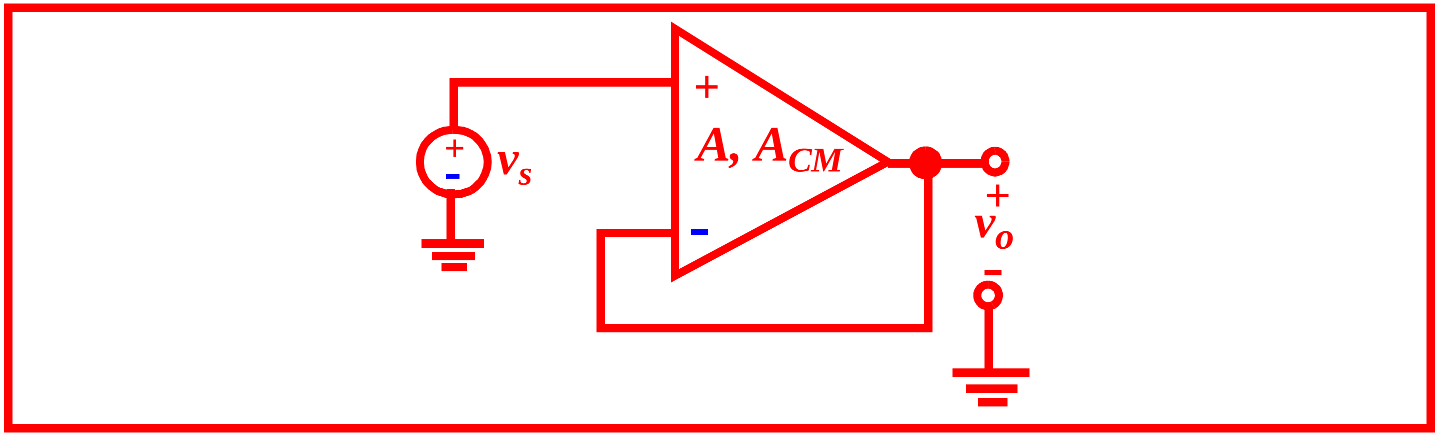


$$v_{ic} = \frac{v_1 + v_2}{2} \;;\; v_o = A(v_1 - v_2) + A_{CM}\left(\frac{v_1 + v_2}{2}\right) = Av_{id} + A_{CM}v_{ic}$$

$$v_1 = v_{ic} + \frac{v_{id}}{2} \;;\; v_2 = v_{ic} - \frac{v_{id}}{2}$$

$$\boxed{v_o = A\left(v_{id} + \frac{A_{CM}v_{ic}}{A}\right) = A\left(v_{id} + \frac{v_{ic}}{CMRR}\right)}.$$

Voltage-Follower Gain Error Due to CMRR (Read carefully)



$$v_{id} = v_s - v_0 \text{ and } v_{ic} = \frac{v_s + v_0}{2}.$$

$$v_0 = A\left(v_s - v_0 + \frac{v_s + v_0}{2 \cdot CMRR}\right) .$$

$$A_V = \frac{v_0}{v_s} = \frac{A(1 + \{2 \cdot CMRR\}^{-1})}{1 + A(1 - \{2 \cdot CMRR\}^{-1})}$$ and the Gain Error is $A_{v,ideal} - A_{v,actual}$

Gain Error Due to CMRR (Read carefully)

$$GE = 1 - \frac{A(1+\{2 \cdot CMRR\}^{-1})}{1+A(1-\{2 \cdot CMRR\}^{-1})} = \frac{(1 - A \cdot CMRR^{-1})}{1+A(1-\{2 \cdot CMRR\}^{-1})}.$$

$$GE \approx A^{-1} - CMRR^{-1}$$

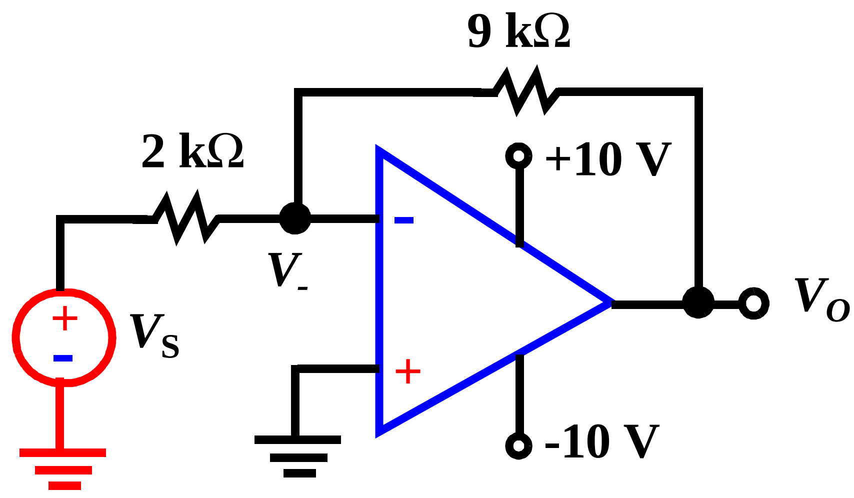
A^{-1} => error due to finite gain

$CMRR^{-1}$ => error due to finite CMRR

PSRR

PSRR is a parameter closely related to CMRR and is a measure of the ability of the Op Amp to reject variations in the power supply. It is the change in IP voltage per unit change in power supply voltage. It is $PSRR = \frac{\Delta V_{input}}{\Delta V_{supply}}$ in dB or µV/V. Its values are similar to CMRR.

Example: The output-voltage range of the op amp in the following figure is equal to the power supply voltages. What are the values of V_o and V_- for the amplifier if the dc input V_S is (a) -2 V and (b) 4 V?



Inverting Amplifier: $v_o = A_V v_S = -\frac{9k}{2k} v_S = -4.5 v_S$ **as long as** $|v_o| \le 10V$ **as restricted by the op amp power supply voltages.**

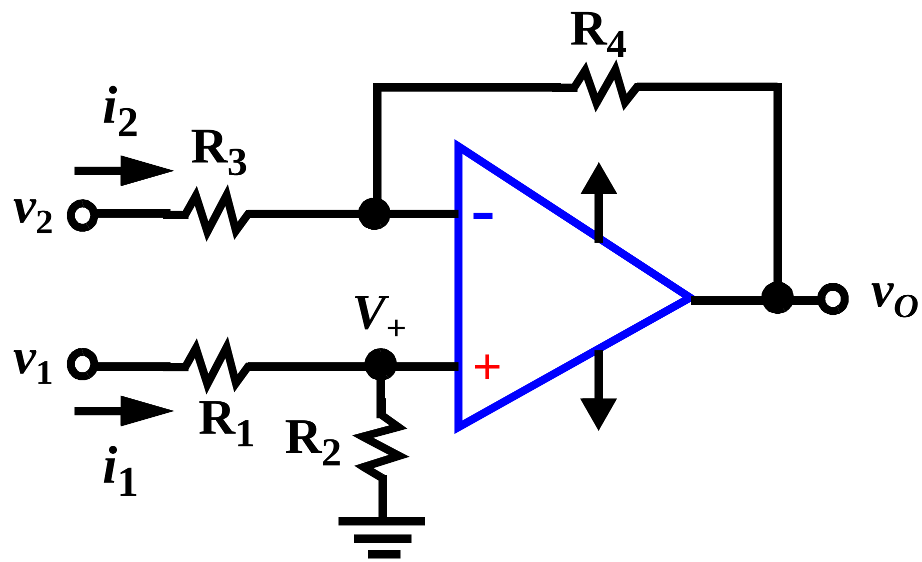
(a) $V_o = -4.5(-2) = 9V$**, feedback loop is working and** $V_- = 0V$

(b) $V_o = -4.5(4) = -18V$ **; V_o saturates at** $V_o = -10V$.

The feedback loop is broken since the open-loop gain is now 0.
The output voltage does not change when the input changes so A = 0.

By superposition, $$V_- = 4\left(\frac{9k}{2k+9k}\right) - 10\left(\frac{2k}{2k+9k}\right) = 1.45V$$

Example: (a) Use ideal op amp assumptions to derive an expression for the output voltage of the following circuit. (b) Find the input resistance presented to v_1 and v_2.



(a) Inv. Input: $v_O = \left(-\frac{R_4}{R_3}\right)v_2$

Non-inv. Input: $v_O = \left(1 + \frac{R_4}{R_3}\right)v_+ = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)v_1$

Superposition: $v_O = \left(-\frac{R_4}{R_3}\right)v_2 + \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right)v_1$

(b) for v_1, $R_{IN} = \left.\frac{v_1}{i_1}\right|_{v_2 = 0} = R_1 + R_2$

for v_2, $R_{IN} = \left.\frac{v_2}{i_2}\right|_{v_1 = 0} = R_3$

(Verify Results using PSpice)