

Model Simplifications for Forward-Active Region

For $v_{BE} > 4 \frac{kT}{q} = 0.1 \text{ V}$ and $v_{BC} < -4 \frac{kT}{q} = -0.1 \text{ V}$

$$I_C = I_S \left\{ \exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right) \right\} - \frac{I_S}{\beta_R} \left\{ \exp\left(\frac{v_{BC}}{V_T}\right) - 1 \right\} \cong I_S \exp\left(\frac{v_{BE}}{V_T}\right).$$

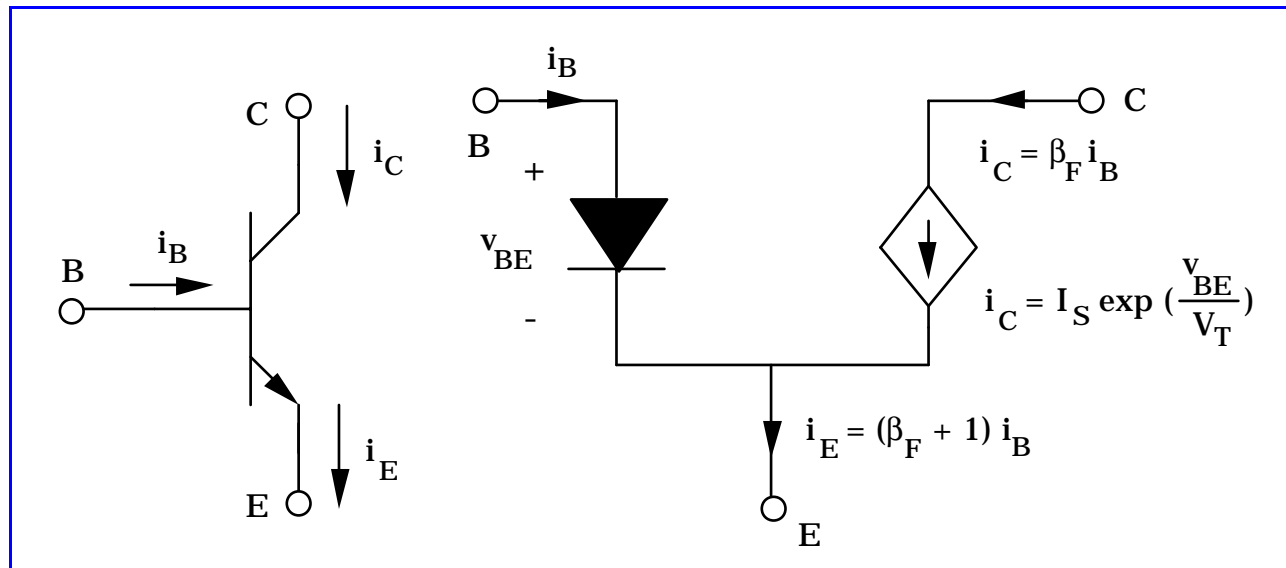
$$I_E = I_S \left\{ \exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right) \right\} + \frac{I_S}{\beta_F} \left\{ \exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right\} \cong \frac{I_S}{\alpha_F} \exp\left(\frac{v_{BE}}{V_T}\right).$$

$$I_B = \frac{I_S}{\beta_F} \left\{ \exp\left(\frac{v_{BE}}{V_T}\right) - 1 \right\} + \frac{I_S}{\beta_R} \left\{ \exp\left(\frac{v_{BC}}{V_T}\right) - 1 \right\} \cong \frac{I_S}{\beta_F} \exp\left(\frac{v_{BE}}{V_T}\right).$$

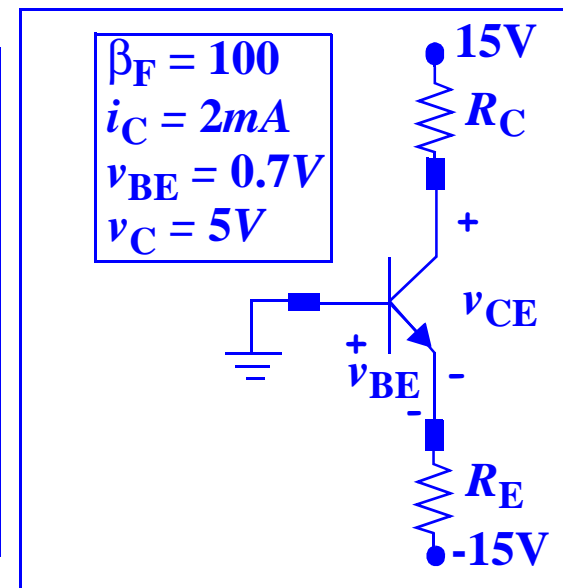
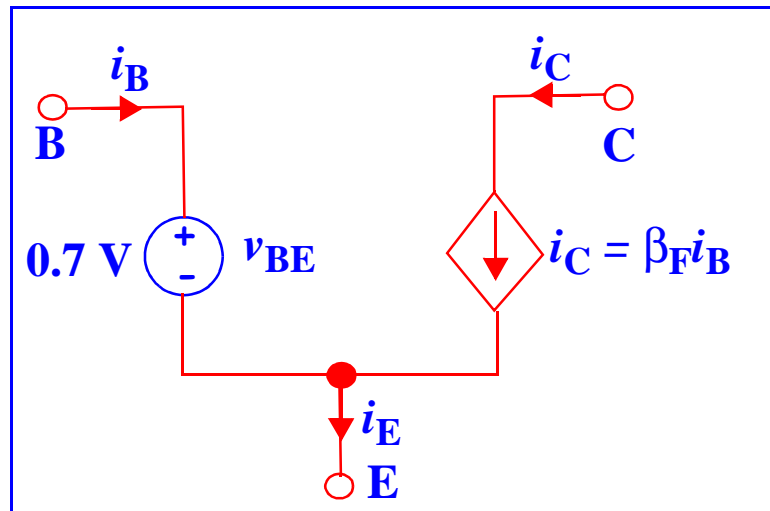
Also, $i_C = \beta_F i_B$, $i_E = i_B + i_C = (\beta_F + 1)i_B$ and $i_C = \alpha_F i_E$.

$$\alpha_F = \beta_F / (1 + \beta_F) \quad \text{and} \quad \beta_F = \alpha_F / (1 - \alpha_F).$$

Simplified model for npn transistor in forward active region

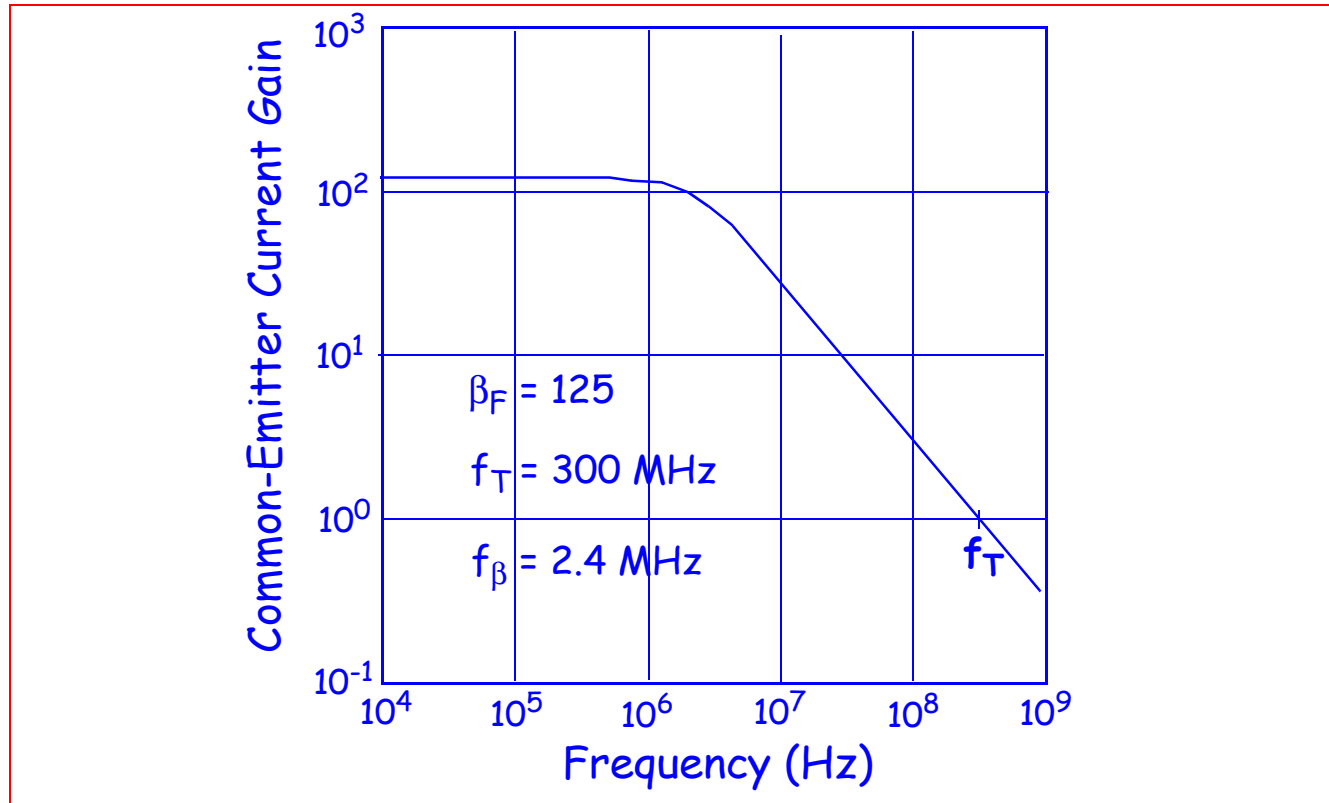


Further simplifications for the forward-active mode.



Frequency Dependence of Common-Emitter β

$$\beta(f) = \frac{\beta_F}{\sqrt{1 + [\beta_F f / f_T]^2}} = \frac{\beta_F}{\sqrt{1 + [f / f_\beta]^2}}$$



Transconductance

$$g_m = \left. \frac{di_C}{dv_{BE}} \right|_{Q\text{-point}} = \frac{d}{dv_{BE}} \left(I_S \exp \frac{v_{BE}}{V_T} \right) = \frac{1}{V_T} I_S \exp \frac{v_{BE}}{V_T} = \frac{I_C}{V_T} .$$

Modeling in the Saturation Region

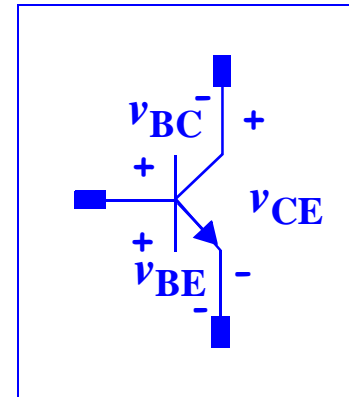
Both BE and BC are forward biased and there is a small voltage between C and E - v_{CESAT}

$$i_C = I_S \exp \left(\frac{v_{BE}}{V_T} \right) - \frac{I_S}{\alpha_R} \exp \left(\frac{v_{BC}}{V_T} \right) ; \quad i_B = \frac{I_S}{\beta_F} \exp \left(\frac{v_{BE}}{V_T} \right) + \frac{I_S}{\beta_R} \exp \left(\frac{v_{BC}}{V_T} \right) .$$

Using $\beta_R = \frac{\alpha_R}{1 - \alpha_R}$, we solve to get

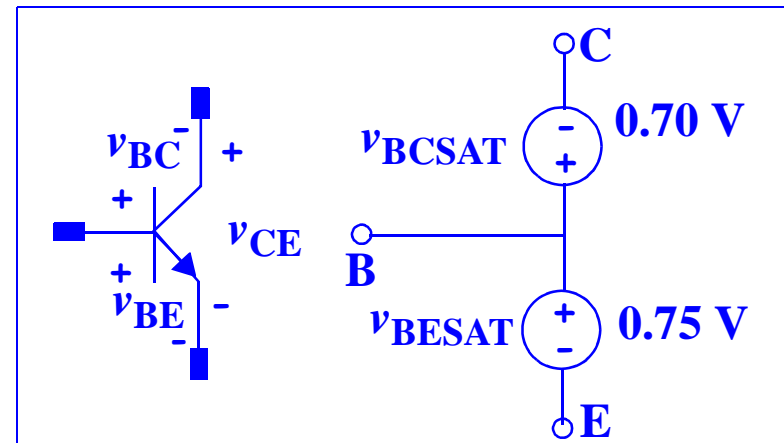
$$v_{BE} = V_T \ln \left(\frac{i_B + (1 - \alpha_R) i_C}{I_S \left[\frac{1}{\beta_F} + (1 - \alpha_R) \right]} \right); \quad v_{BC} = V_T \ln \left(\frac{i_B - i_C / \beta_F}{I_S \left\{ \frac{1}{\alpha_R} \right\} \left[\frac{1}{\beta_F} + (1 - \alpha_R) \right]} \right).$$

$$v_{CESAT} = V_T \ln \left[\left(\frac{1}{\alpha_R} \right) \frac{1 + \frac{i_C}{(\beta_R + 1) i_B}}{1 - \frac{i_C}{\beta_F i_B}} \right] \quad \text{for } i_B > i_C / \beta_F$$



Given i_C , determine i_B for a desired v_{CESAT}

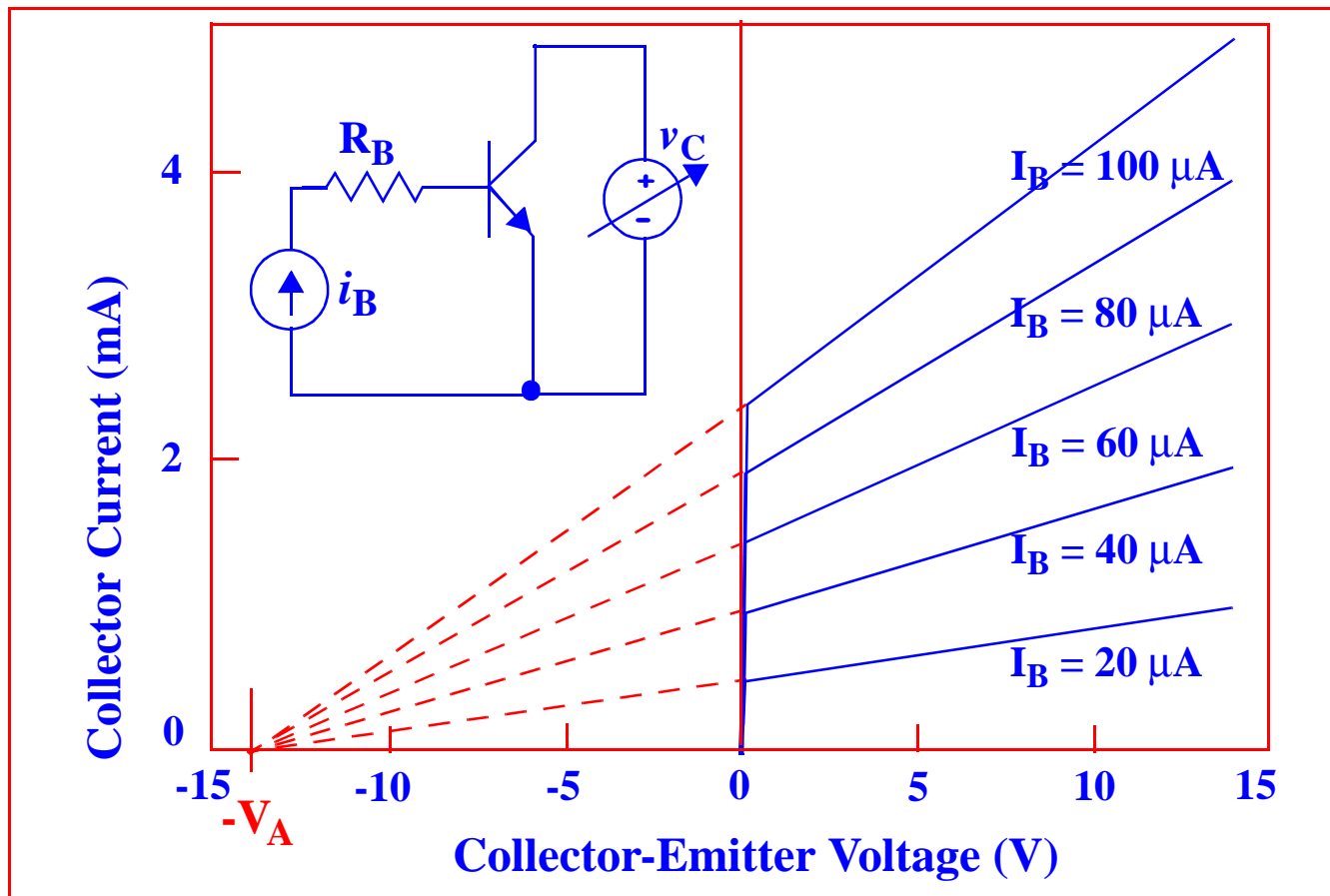
Simplified model for npn transistor in saturation



Early Effect and Early Voltage

When the output characteristic curves are extrapolated back to zero, they all intersect at a common point, $v_{CE} = -V_A$, a phenomenon called the **Early Effect**. Values of V_A are

$$25V \leq V_A \leq 150V$$



Modeling the Early Effect

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right) \left[1 + \frac{v_{CE}}{V_A}\right]; \quad \beta_F = \beta_{F0} \left[1 + \frac{v_{CE}}{V_A}\right]; \quad i_B = \frac{I_S}{\beta_{F0}} \exp\left(\frac{v_{BE}}{V_T}\right).$$

β_{F0} is the value of β_F extrapolated to $v_{CE} = 0$.

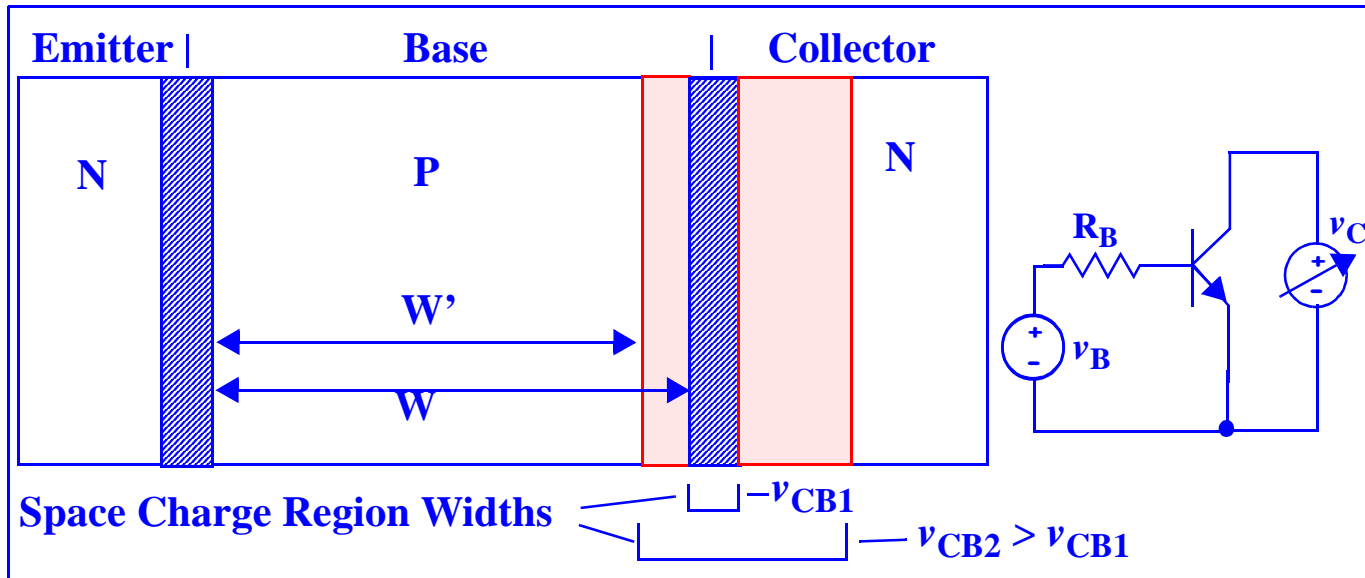
Origin of the Early Effect

Modulation of W_B by the CB bias (called **base-width modulation**) is the cause of the **Early Effect**.

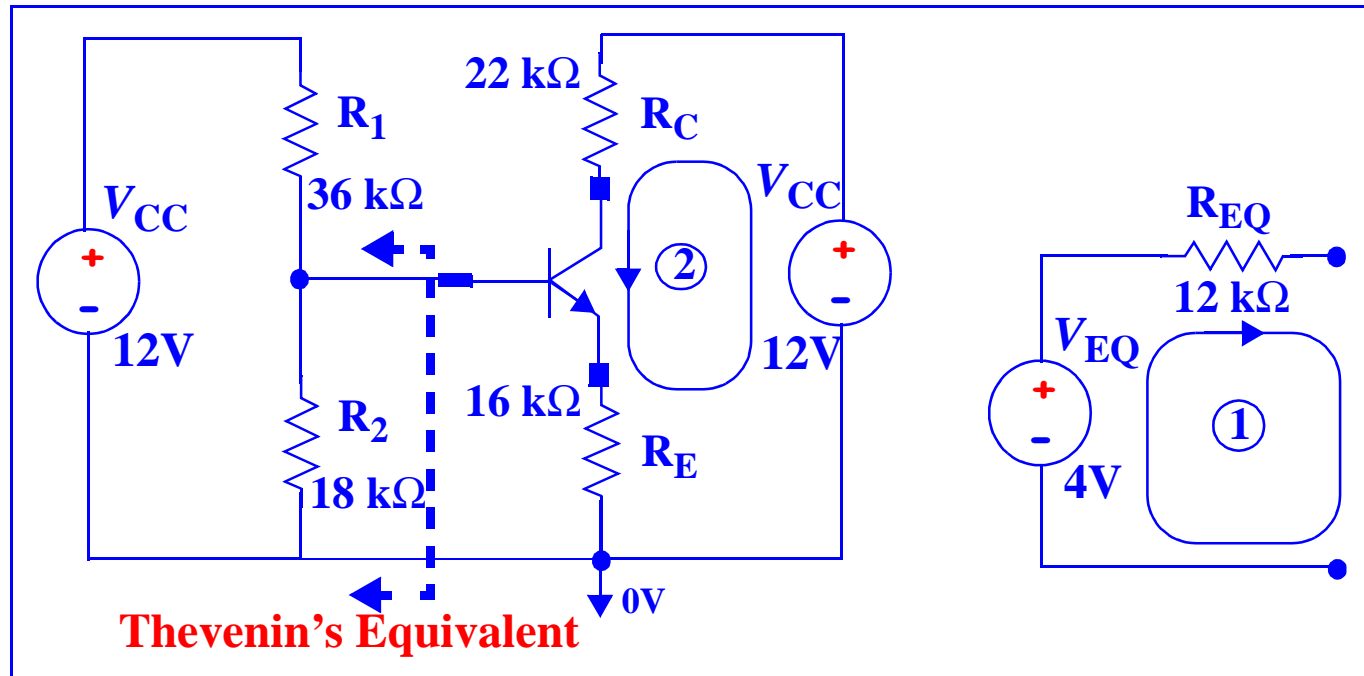
Recall
$$i_T = \left| qAD_n \frac{dn}{dx} \right| = \left| -qAD_n \frac{n_{bo}}{W_B} \left\{ \exp\left(\frac{v_{BE}}{V_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right) \right\} \right|.$$

Therefore, a decrease in W_B results in an increase in transport current i_T .

The **Early Effect** decreases the **output resistance** and places an important limit on the amplification factor of the BJT.



Biasing the BJT



$$V_{EQ} = V_{CC} \frac{R_2}{R_1 + R_2} \quad \text{and} \quad R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

Loop 1: $V_{EQ} = R_{EQ} I_B + V_{BE} + R_E I_E$ or $4 = 12 I_B + V_{BE} + 16 I_E$ with I_B and I_E in mA.

Assume forward active region of operation, $I_E = (\beta_F + 1) I_B$.

$$4 = 12 I_B + V_{BE} + 16 (\beta_F + 1) I_B.$$

For $\beta_F = 75$, $I_B = 0.00269\text{mA} = 2.69\mu\text{A}$. Also, $I_C = 201.5\mu\text{A}$, $I_E = 204\mu\text{A}$

Loop 2:
$$V_{CE} = V_{CC} - R_C I_C - R_E I_E = V_{CC} - \left(\frac{R_E}{\alpha_F} + R_C \right) I_C.$$

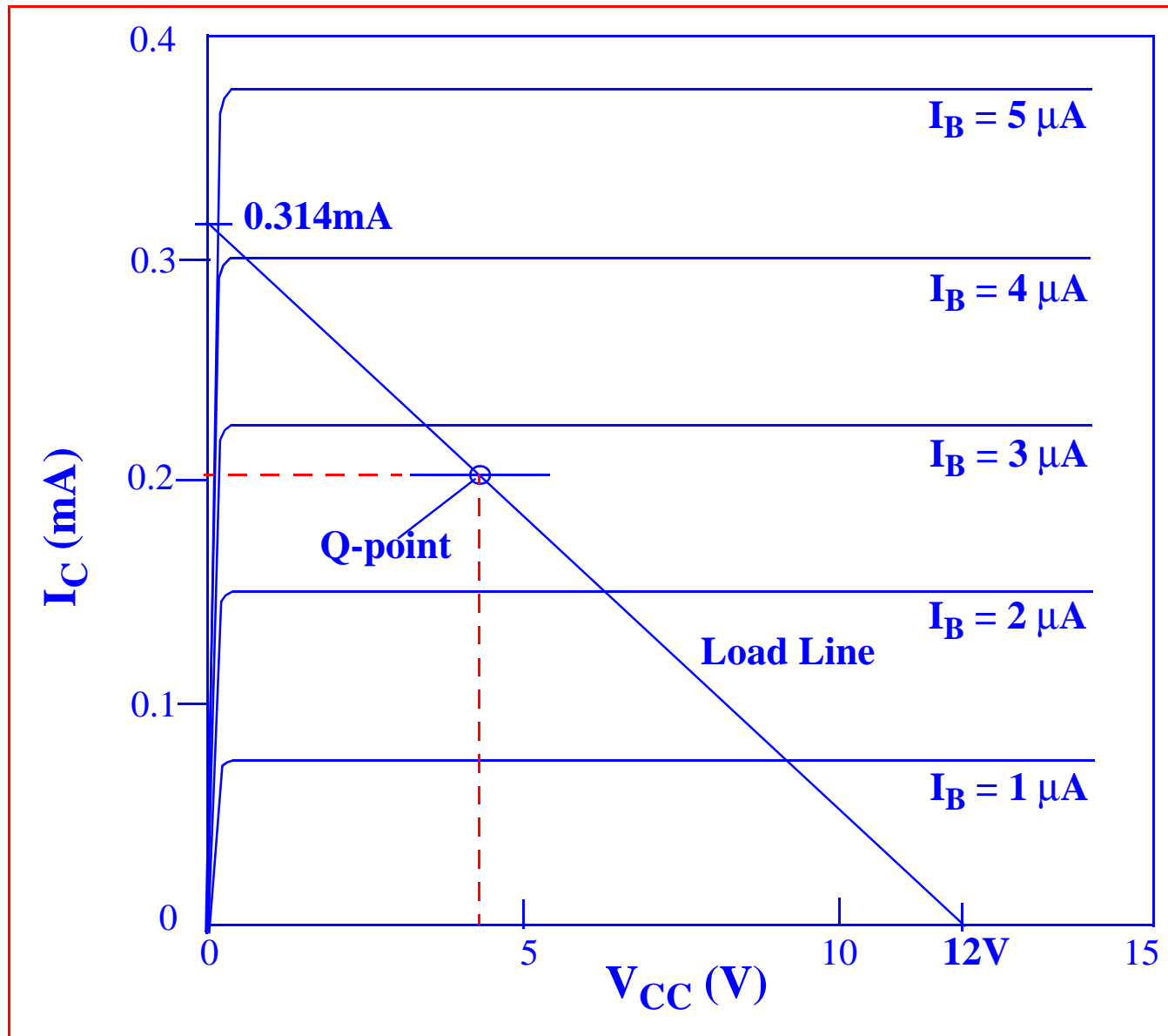
$$V_{CE} = 12 - 38.2I_C = 12 - 7.70 = 4.30\text{V} \text{ with } I_C \text{ in mA.}$$

Check for forward active region of operation.

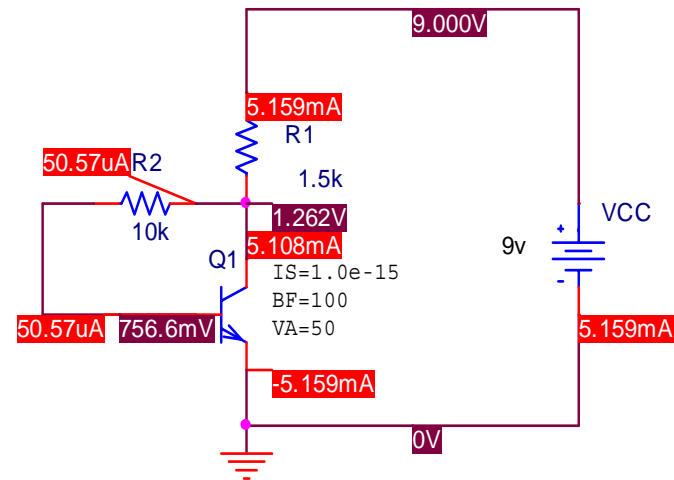
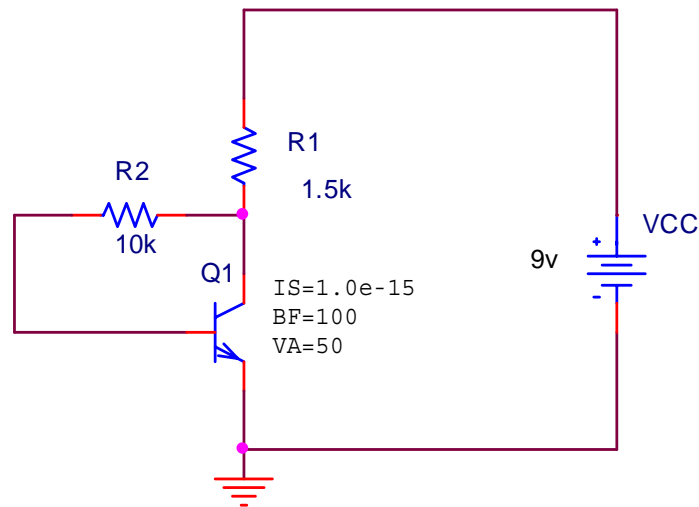
$$V_{BC} = V_{BE} - V_{CE} = 0.7 - 4.30 = -3.60\text{V}.$$

The Q-point is $I_C = 201.5\mu\text{A}$, $V_{CE} = 4.30\text{V}$.

Load Line Analysis: $V_{CE} = 12 - 38.2I_C$ with I_C in mA.



PSPICE EXAMPLE



*Libraries:

* Local Libraries :

.LIB ".\example11.lib"

* From [PSPICE NETLIST] section of C:\Program Files\OrCAD\PSpice\PSpice.ini file:

.lib "nom.lib"

*Analysis directives:

.OP

.PROBE V(*) I(*) W(*) D(*) NOISE(*)

PSPICE EXAMPLE (Cont'd)

```
.INC ".\example11-SCHEMATIC1.net"
```

```
**** INCLUDING example11-SCHEMATIC1.net ****
```

```
* source EXAMPLE11
```

```
V_VCC      N00025 0 9v
```

```
R_R1       N00162 N00025 1.5k
```

```
Q_Q1       N00162 N00208 0 Qbreakn
```

```
R_R2       N00208 N00162 10k
```

```
**** RESUMING example11-SCHEMATIC1-Example11Profile.sim.cir ****
```

```
.END
```

```
****      BJT MODEL PARAMETERS
```

```
*****
```

```
      Qbreakn  
      NPN  
      IS  1.000000E-15  
      BF  100  
      NF  1
```

PSPICE EXAMPLE (Cont'd)

VAF 50

BR 1

NR 1

CN 2.42

D .87

**** SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
(N00025)	9.0000	(N00162)	1.2623	(N00208)	.7566		

VOLTAGE SOURCE CURRENTS

NAME	CURRENT
------	---------

V_VCC	-5.159E-03
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TOTAL POWER DISSIPATION 4.64E-02 WATTS

**** 01/01/04 13:10:55 ***** PSpice Lite (Mar 2000) *****

PSPICE EXAMPLE (Cont'd)

**** OPERATING POINT INFORMATION TEMPERATURE = 27.000 DEG C

**** BIPOLAR JUNCTION TRANSISTORS

NAME	Q_Q1
MODEL	Qbreakn
IB	5.06E-05
IC	5.11E-03
VBE	7.57E-01
VBC	-5.06E-01
VCE	1.26E+00
BETADC	1.01E+02
GM	1.97E-01
RPI	5.11E+02
RX	0.00E+00

PSPICE EXAMPLE (Cont'd)

RO	9.89E+03	
CBE	0.00E+00	
CBC	0.00E+00	
CJS	0.00E+00	
BETAAC	1.01E+02	
CBX/CBX2	0.00E+00	
FT/FT2	3.14E+18	
JOB CONCLUDED		
TOTAL JOB TIME		.10