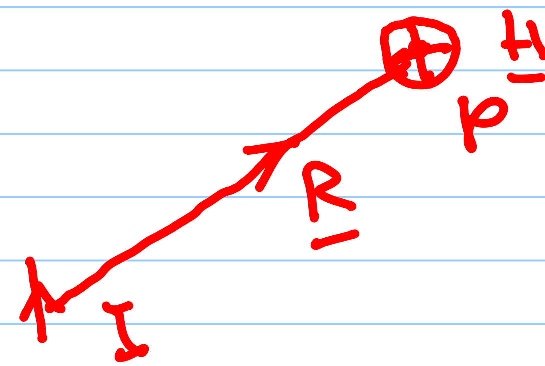
Lecture 5

from sections 7.1 - 7.4

Solve 7.2, 7.5, 7.10, 7.13, 7.21,
7.23, 7.26, 7.27, 7.30

Electrostatics vs. Magnetostatics

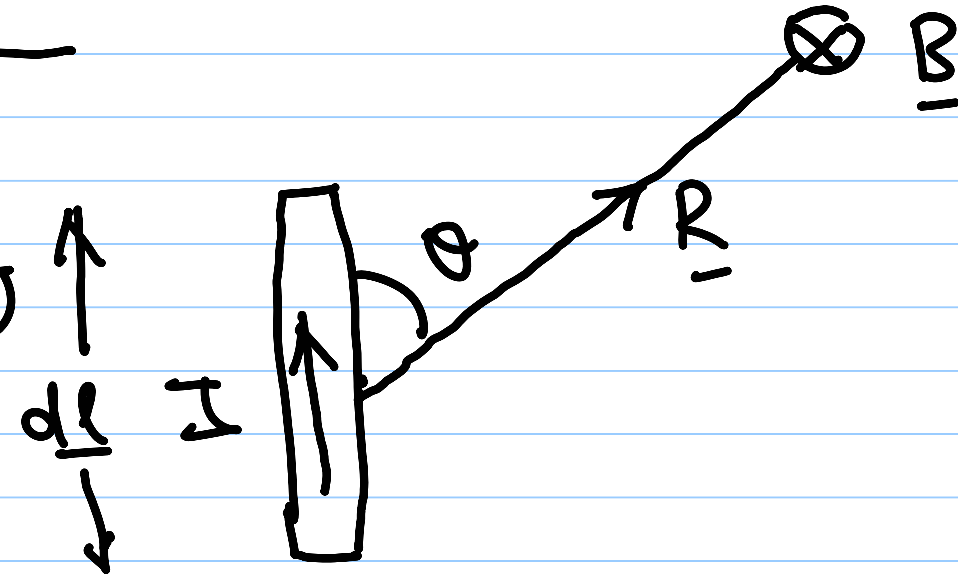
Electrostatics	Magnetostatics
• q (C)	I
$\underline{E}$ V/m	$\underline{H}$ A/m
$\underline{D} = \varepsilon \underline{E}$ C/m^2	$\underline{B} = \mu \underline{H}$ Weber/m^2



Biot-Savart Law

$$dB = \frac{\mu}{4\pi} \frac{I\,dl}{R^2} \sin\theta \quad (I)$$

$$d\underline{B} = \frac{\mu}{4\pi} \frac{I\,(d\underline{l} \times \hat{R})}{R^2}$$

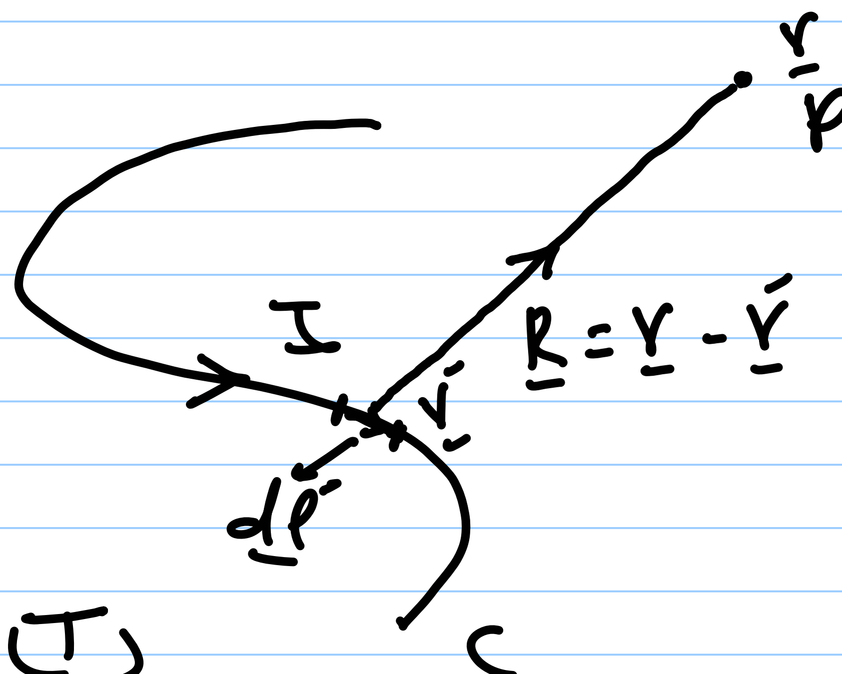


* Biot-Savart Law is the fundamental law in magnetostatics

Superposition for Line Currents

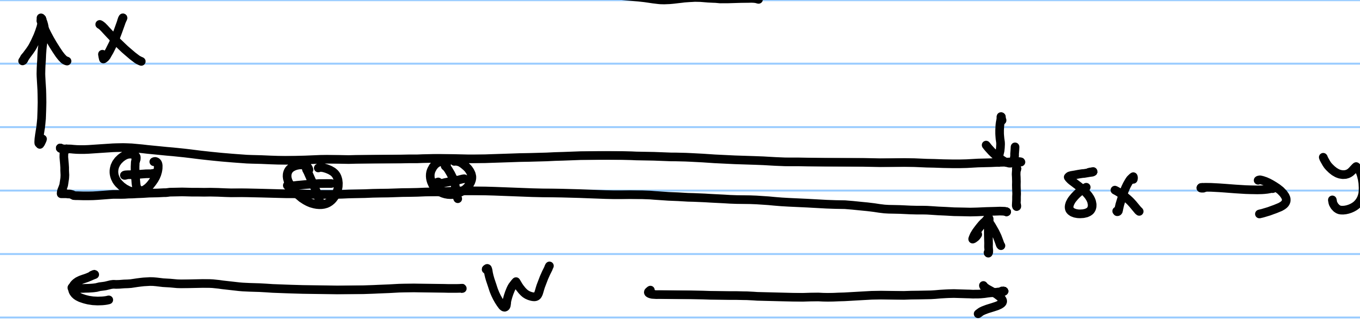
$$d\underline{B}' = \frac{\mu}{4\pi}\,\frac{I\, d\underline{l}' \times \hat{\underline{R}}}{R^2}$$

$$\underline{B} = \frac{\mu}{4\pi}\int_C \frac{I\, d\underline{l}' \times \hat{\underline{R}}}{R^2} \quad (T)$$



* Notice that magnetic field rotates around the current

Surface Current Density

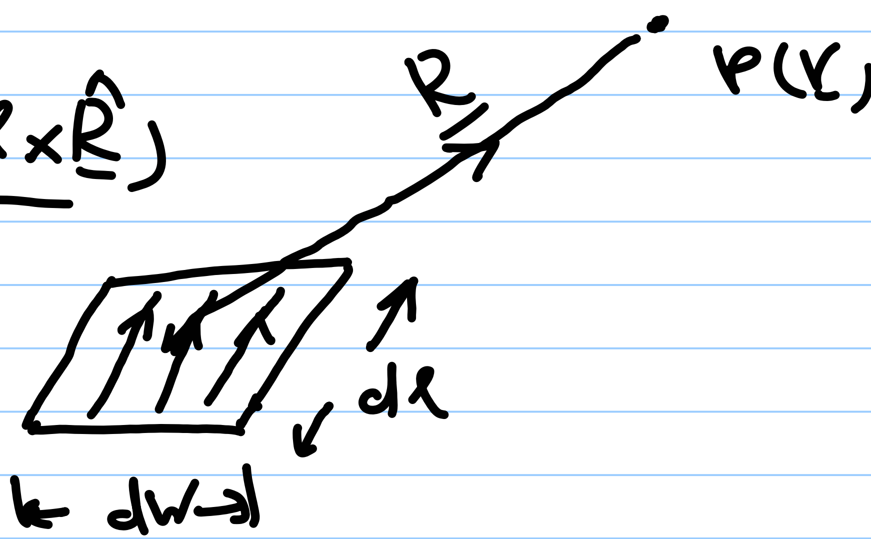


$$I = \iint_S \underline{J} \cdot d\underline{s} = \int_0^W \underbrace{\left(\int_{x=0}^{\delta x} J \, dx \right)}_{J_s \ A/m} dy$$

$$I = \int_0^W J_s \, dy$$

Superposition Using Surface Currents

$$d\underline{B} = \frac{\mu}{4\pi}\,\frac{\overset{I}{J_s\,dw}\,(d\underline{l}\times\underline{\hat{R}})}{R^2}$$



$ds = dl\,dw$, $d\underline{l} = dl\,\underline{a}_l$

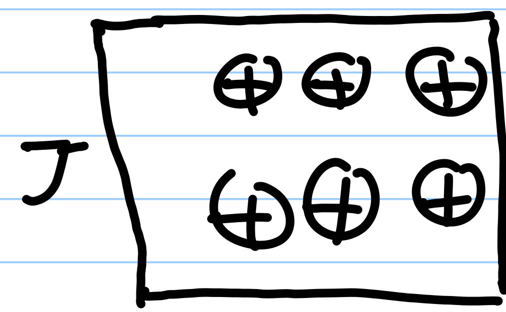
$\underline{J}_s = J_s\,\underline{a}_l$

$$d\underline{B} = \frac{\mu}{4\pi}\,\frac{(\underline{J}_s \times \underline{\hat{R}})}{R^2}\,ds$$

$$\underline{B} = \frac{\mu}{4\pi}\iint_{s'} \frac{\underline{J}_s(\underline{r}')\times\underline{\hat{R}}\;ds'}{R^2} \qquad (T)$$

Superposition for Volumetric Current

Similarly, for a volumetric current density, we have



$$d\underline{B} = \frac{\mu}{4\pi} \left(\frac{\underline{J} \times \hat{\underline{R}}}{R^2} \right) dv \qquad \underline{J} = \lim_{\Delta s \to 0} \frac{\Delta I}{\Delta s} \ \mathrm{A/m^2}$$

$$\underline{B} = \frac{\mu}{4\pi} \iiint_V \frac{\underline{J}(\underline{r}') \times \hat{\underline{R}}}{R^2} \, dv'$$

Ampère Circuital Law

$$\oint \underline{H} \cdot d\underline{l} = I_{enc}$$

* Surface Integral of $\underline{D}$ gives enclosed charge

* Line Integral of $\underline{H}$ gives enclosed current

Differential Form of Ampére's Law

$$\oint \underline{H} \cdot d\underline{l} = \iint_S (\nabla \times \underline{H}) \cdot d\underline{s}$$

$$\iint_S (\nabla \times \underline{H}) \cdot d\underline{s} = \iint_S \underline{J} \cdot d\underline{s}$$

$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$\Rightarrow \nabla \times \underline{H} = \underline{J}$$