Lecture 16

From section 11.4

Self read sections 11.7 and 11.8

Voltage Standing Wave Ratio (VSWR)

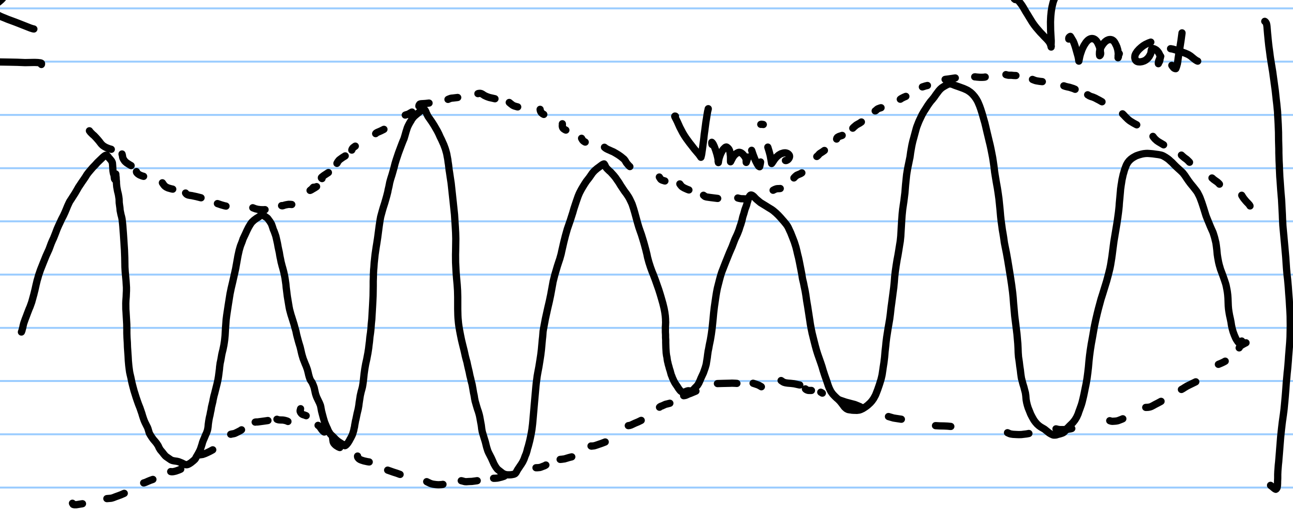
$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$V(z) = V_0^+ \left(1 + \frac{V_0^-}{V_0^+} e^{2\gamma z}\right)$$

$$V(z) = V_0^+ \left(1 + \Gamma_L e^{-2\gamma(\ell - z)}\right)$$

$$\left.\begin{aligned} V_{Max} &= V_0^+ (1 + |\Gamma_L|) \\ V_{Min} &= V_0^+ (1 - |\Gamma_L|) \end{aligned}\right\} \quad VSWR = \frac{V_{Max}}{V_{MIN}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

VSWR



* Our target is $\Gamma_\ell = 0 \Rightarrow VSWR = 1$
* Worst performance $VSWR = \infty$ (standing wave)

power Transmission

* If there is noreflection

$$v(z,t) = |V_0| \cos(\omega t - \beta z)\, e^{-\alpha z}$$

$$I(z,t) = \frac{|V_0|}{|Z_0|} \cos(\omega t - \beta z - \theta)\, e^{-\alpha z} \quad (Z_0 = |Z_0| e^{j\theta})$$

$$p(z,t) = v(z,t)\, I(z,t) = \frac{|V_0|^2}{2|Z_0|}\left[\cos(2\omega t - 2\beta z - \theta) + \cos\theta\right] e^{-2\alpha z}$$

$$p_{av} = \frac{1}{T}\int_0^T p(z,t)\, dt = \frac{|V_0|^2}{2|Z_0|} \cos\theta\, e^{-2\alpha z}$$

Power Transmission (Cont'd)

* Same result can be obtained in frequency domain: $\tilde{V}(z) = V_0 e^{-\gamma z} = V_0 e^{-\alpha z} e^{-j\beta z}$

$$\tilde{I}(z) = \frac{\tilde{V}(z)}{Z_0} = \frac{|V_0|}{|Z_0|} e^{-\alpha z} e^{-j\beta z - j\theta}$$

$$P_{av} = \frac{1}{2} \mathrm{Re}(\tilde{V}\, \tilde{I}^*) = \frac{1}{2} \mathrm{Re}\left(\frac{|V_0|^2}{|Z_0|} e^{-2\alpha z} e^{j\theta}\right)$$

$$P_{av} = \frac{|V_0|^2}{2|Z_0|} e^{-2\alpha z} \cos\theta = P_{av}(0)\, e^{-2\alpha z}$$

Power Transmission with Reflection

$$\tilde{V}_i(z) = V_{oi}\, e^{-\alpha z}\, e^{-j\beta z}$$

$$\tilde{V}_r(z) = V_{or}\, e^{\alpha z}\, e^{j\beta z}$$

$$\tilde{V}(z) = V_{oi}\, e^{-\alpha z}\, e^{-j\beta z} + V_{or}\, e^{\alpha z}\, e^{j\beta z}$$

$$= V_{oi}\, e^{-\alpha z}\, e^{-j\beta z} + \Gamma_\ell V_{oi}\, e^{-\alpha(\ell - z)}\, e^{-j\beta(\ell - z)}$$

Reflected Power

$$P_r = \frac{1}{2} \mathrm{Re}\left(\tilde{V}_r \tilde{I}_r^*\right)$$

$$= \frac{1}{2} \mathrm{Re}\Big(\Gamma_\ell V_{oi}\, e^{-\alpha(\ell - z)}\, e^{j\beta(\ell - z)}$$

$$* \frac{\Gamma_\ell^* V_{oi}^*}{|Z_0|}\, e^{-\alpha(\ell - z)}\, e^{-j\beta(\ell - z)}\, e^{-j\theta}\Big)$$

$$P_r = \frac{1}{2} \mathrm{Re}\left(|\Gamma_\ell|^2 \frac{|V_{oi}|^2}{|Z_0|} e^{-2\alpha(\ell - z)}\, e^{-j\theta}\right)$$

Reflected Power ((Cont'd)

at $z = \ell$ (load side), we have

$$P_r(0) = \frac{1}{2} \mathrm{Re}\left(|\Gamma_\ell|^2 \frac{|V_{0i}|^2}{|Z_0|} e^{-j\theta} \right)$$

$$P_r(0) = \frac{|\Gamma_\ell|^2}{2} \frac{|V_{0i}|^2}{|Z_0|} \cos\theta$$

$$P_r(z) = P_r(0)\, e^{-2\alpha(\ell - z)}$$