ECE3FI4
Theory and Applications in Electromagnetic

Term II, January – April 2007

MATLAB Example and Exercises (Set 16 solution)

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Exercise: a toroid whose axis is the z axis carries a current of 5.0 A and has 200 turns. The inner radius is 1.5 cm while the outer radius is 2.5 cm. Write a MATLAB program that compute and plots the magnetic field in the x-y plane in the region $-4.0 \text{ cm} \le x \le 4.0 \text{ cm}$ and $-4.0 \text{ cm} \le y \le 4.0 \text{ cm}$.

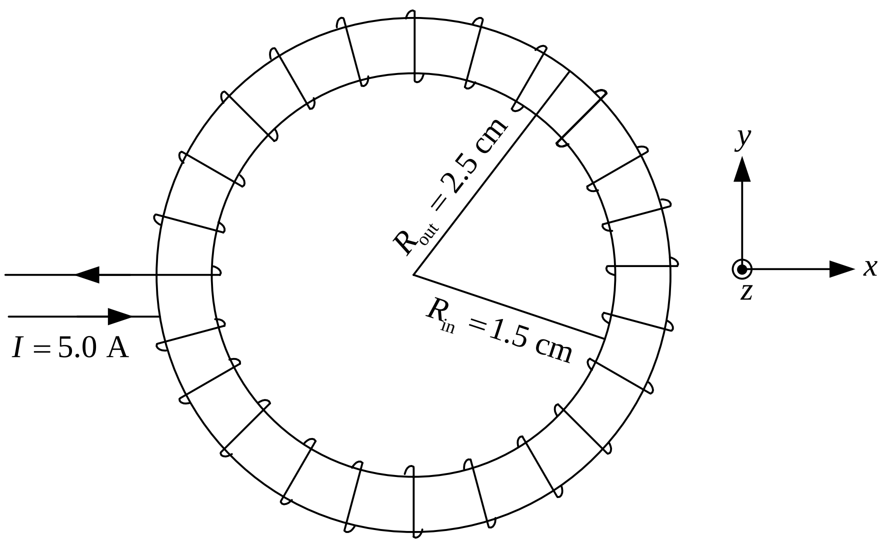

Figure E16.1 See the exercise problem

Analytical solution:

The cross section of the toriod is shown in Figure E16.2. Similar to the example problem, we can assign each point on the winding an angle value. For instance, if a point has a θ (θ is the angle shown in Figure E16.2) of $\pi/3$, and it is on the third turn, then the angle value we assign to this point is $\phi' = \pi/3 + 2\pi \times (3-1) = 13\pi/3$. general, $\phi' = \theta + 2(k-1)\pi$, where $k \in Z,\ 1 \le k \le$ number of turns and $0 \le \theta < 2\pi$. Knowing the value of ϕ', we can find the cylindrical coordinates of a point on the k-th turn :

$$\boxed{R = R_1 + R_2 \cos\theta = R_1 + R_2 \cos\phi'} \quad (1)$$

$$\boxed{\phi = \frac{2(k-1)\pi}{\text{Number of turns}}} \quad (2)$$

$$\boxed{z = R_2 \sin\theta = R_2 \sin\phi'} \quad (3)$$

Then we want to divide the winding to many segment along the direction of the current I in order to allow MATLAB program to calculate the magnetic field. Assume we want to divide the winding to n segments, then we have to pick up n+1 points on the winding. And for the i-th point, the angle is given by

$$\phi'_i = \phi'_{\min} + \frac{\phi'_{\max} - \phi'_{\min}}{n}(i-1)\text{, where } \phi'_{\min} = 0 \text{ and } \phi'_{\max} = 2\pi \times \left(\text{Number of turns}\right)$$

by plugging this equation into (1) (2) and (3), we can find R_i, ϕ_i, and z_i. Also, we can find the rectangular coordinate x_i, y_i, and z_i by using the formula

$$x_i = R_i \cos\phi_i$$

$$y_i = R \sin\phi_i$$

$$z_i = z_i$$

Note the i-th segment is a vector given by

$$\Delta\mathbf{L}_i = (x_{i+1} - x_i)\mathbf{a}_x + (y_{i+1} - y_i)\mathbf{a}_y + (z_{i+1} - z_i)\mathbf{a}_y$$

and the vector $\mathbf{R}_i$ (pointing from the center of the i-th segment to the observation point) is given by

$$\mathbf{R}_i = P - C_i = \left(x, y, z\right) - \left(\frac{x_{i+1} - x_i}{2}, \frac{y_{i+1} - y_i}{2}, \frac{z_{i+1} - z_i}{2}\right)$$

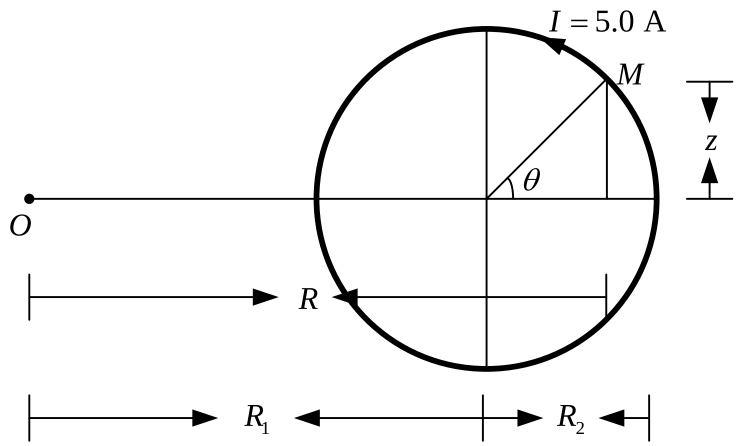

Figure E16.2 Cross section view of the toroid in the example problem. *R* and *z* are cylindrical coordinates of point *M*. It can be shown that $R = R_1 + R_2 \cos\theta$, $z = R_2 \sin\theta$. And if M is on the *k*-th turn, $\phi = \dfrac{2(k-1)\pi}{\text{Number of turns}}$.