CHAPTER 8

P.E. 8.1

(a) $\boldsymbol{F} = m\dfrac{\partial \boldsymbol{u}}{\partial t} = Q\boldsymbol{E} = \underline{\underline{6\boldsymbol{a}_z N}}$

(b) $\dfrac{\partial \boldsymbol{u}}{\partial t} = 6\boldsymbol{a}_z = \dfrac{\partial}{\partial t}(u_x, u_y, u_z) \Rightarrow$

$$\frac{\partial u_x}{\partial t} = 0 \to u_x = A$$

$$\frac{\partial u_y}{\partial t} = 0 \to u_y = B$$

$$\frac{\partial u_z}{\partial t} = 6 \to u_z = 6t + C$$

Since $\boldsymbol{u}(t=0) = 0$, $A = B = C = 0$

$u_x = 0 = u_y, \quad u_z = 6t$

$$u_x = \frac{\partial x}{\partial t} = 0 \to x = A$$

$$u_y = \frac{\partial y}{\partial t} = 0 \to y = B$$

$$u_z = \frac{\partial z}{\partial t} = 6t \to z = 3t^2 + C_1$$

At $t = 0$, $(x,y,z) = (0,0,0) \to A_1 = 0 = B_1 = C_1$

Hence , $(x,y,z) = (0,0,3t^2)$,

$\boldsymbol{u} = 6t\boldsymbol{a}_z$ at any time. At P(0,0,12), $z = 12 = 3t^2 \to t = 2s$

$\underline{\underline{t = 2s}}$

(c) $\boldsymbol{u} = 6t\boldsymbol{a}_z = 12\boldsymbol{a}_z$ m/s .

$$\boldsymbol{a} = \frac{\partial \boldsymbol{u}}{\partial t} = \underline{\underline{6\boldsymbol{a}_z \, m/s^2}}$$

(d) $K.E = \dfrac{1}{2}m|\boldsymbol{u}|^2 = \dfrac{1}{2}(1)(144) = \underline{\underline{72J}}$

P.E. 8.2

(a) $m\boldsymbol{a} = e\boldsymbol{u}\times\boldsymbol{B} = (eB_o u_y, -eB_o u_x, 0)$

$$\frac{d^2x}{dt^2} = \frac{eB_o}{m}\frac{dy}{dt} = \omega\frac{dy}{dt} \qquad (1)$$

$$\frac{d^2y}{dt^2} = -\frac{eBo}{m}\frac{dx}{dt} = -\omega\frac{dx}{dt} \qquad (2)$$

$$\frac{d^2z}{dt^2} = 0; \Rightarrow \frac{dz}{dt} = C_1 \qquad (3)$$

From (1) and (2),

$$\frac{d^3x}{dt^3} = \omega\frac{d^2y}{dt^2} = -\omega^2\frac{dx}{dt}$$

$(D^2 + w^2 D)x = 0 \to Dx = (0, \pm j\omega)x$

$x = c_2 + c_3\cos\omega t + c_4\sin\omega t$

$$\frac{dy}{dt} = \frac{1}{\omega}\frac{d^2x}{dt^2} = -c_3\omega\cos\omega t - c_4\omega\sin\omega t$$

At $t = 0$, $\boldsymbol{u} = (\alpha, 0, \beta)$. Hence,

$$c_1 = \beta, c_3 = 0, c_4 = \frac{\alpha}{\omega}$$

$$\underline{\underline{\frac{dx}{dt} = \alpha\cos\omega t, \frac{dy}{dt} = -\alpha\sin\omega t, \frac{dz}{dt} = \beta}}$$

(b) Solving these yields

$$x = \frac{a}{\omega}\sin\omega t, y = \frac{\alpha}{\omega}\cos\omega t, z = \beta t$$

The starting point of the particle is $(0, \dfrac{\alpha}{\omega}, 0)$

(c) $x^2 + y^2 = \dfrac{\alpha^2}{\omega^2}$, $z = \beta t$

showing that the particles move along a helix of radius α/ω placed along the z-axis.

P.E. 8.3

(a) From Example 8.3, QuB = QE regardless of the sign of the charge.

$E = uB = 8 \times 10^6 \times 0.5 \times 10^{-3} = \underline{4 \text{ kV/m}}$

(b) Yes, since QuB = QE holds for any Q and m.

P.E. 8.4

By Newton's 3rd law, $F_{12} = F_{21}$, the force on the infinitely long wire is:

$$F_l = -F = \frac{\mu_o I_1 I_2 b}{2\pi}\left(\frac{1}{\rho_o} - \frac{1}{\rho_o + a}\right)a_\rho$$

$$= \frac{4\pi\times10^{-7}\times50\times3}{2\pi}\left(\frac{1}{2}-\frac{1}{3}\right)a_\rho = \underline{\underline{5a_\rho \ \ \mu N}}$$

P.E. 8.5

$$m = ISa_n = 10\times10^{-4}\times50\frac{(2,6,-3)}{7}$$

$= 7.143 \times 10^{-3}$ (2, 6, -3)

$$= \underline{\underline{(1.429a_x + 4.286a_y - 2.143a_z)\times10^{-2} \text{ A-m}^2}}$$

P.E. 8.6

(a)

$$T = m\times B = \frac{10\times10^{-4}\times50}{7\times10}\begin{vmatrix}2 & 6 & -3\\ 6 & 4 & 5\end{vmatrix}$$

$$= \underline{\underline{0.03a_x - 0.02a_y - 0.02a_z \text{ N-m}}}$$

(b) $|T| = ISB\sin\theta \rightarrow |T|_{max} = ISB$

$$|T|_{max} = \frac{50\times10^{-3}}{10}|6a_x + 4a_y + 5a_z| = \underline{\underline{0.04387 \text{ Nm}}}$$

P.E. 8.7

(a) $\mu_r = \frac{\mu}{\mu_o} = 4.6,\ \chi_m = \mu_r - 1 = \underline{\underline{3.6}}$

(b) $H = \frac{B}{\mu} = \frac{10\times10^{-3}e^{-y}}{4\pi\times10^{-7}\times4.6}a_z\ A/m \quad = \underline{\underline{1730e^{-y}a_z}}$ A/m

(c) $M = \chi_m H = \underline{\underline{6228e^{-y}a_z}}$ A/m

P.E. 8.8

$$a_n = \frac{3a_x + 4a_y}{5} = \frac{6a_x + 8a_y}{10}$$

$$B_{1n} = (B_1 \bullet a_n)a_n = \frac{(6+32)(6a_x + 8a_y)}{1000}$$

$$= 0.228a_x + 0.304a_y = B_{2n}$$

$$B_{1t} = B_1 - B_{1n} = -0.128a_x + 0.096a_y + 0.2a_z$$

$$B_{2t} = \frac{\mu_2}{\mu_1}B_{1t} = 10B_{1t} = -1.28a_x + 0.96a_y + 2a_z$$

$$B_2 = B_{2n} + B_{2t} = \underline{\underline{-1.052a_x + 1.264a_y + 2a_z}} \text{ Wb/m}^2$$

P.E. 8.9

(a) $B_{1n} = B_{2n} \rightarrow \mu_1 H_{1n} =_z \mu_2 H_{2n}$

or $\mu_1 H_1 \bullet a_{n21} = \mu_2 H_2 \bullet a_{n21}$

$$\mu_o\frac{(60+2-36)}{7} = 2\mu_o\frac{(6H_{2x}-10-12)}{7}$$

$35 = 6H_{2x}$

$\underline{\underline{H_{2x} = 5.833 \text{ A/m}}}$

(b) $K = (H_1 - H_2)\times a_{n12} = a_{n21}\times(H_1 - H_2)$

$$= a_{n21}\times\left[(10,1,12) - (35/6, -5, 4)\right]$$

$$= \frac{1}{7}\begin{vmatrix}6 & 2 & -3\\ 25/6 & 6 & 8\end{vmatrix}$$

$K = \underline{\underline{4.86a_x - 8.64a_y + 3.95a_z}}$ A/m

(c) Since $B = \mu H$, B_1 and H_1 are parallel, i.e. they make the same angle with the normal to the interface.

$$\cos\theta_1 = \frac{H_1 \bullet a_{n21}}{|H_1|} = \frac{26}{7\sqrt{100+1+144}} = 0.2373$$

$\underline{\underline{\theta_1 = 76.27^o}}$

$$\cos\theta_2 = \frac{H_2 \bullet a_{n21}}{|H_2|} = \frac{13}{7\sqrt{(5.833)^2 + 25 + 16}} = 0.2144$$

$\underline{\underline{\theta_2 = 77.62^o}}$

P.E. 8.10

(a) $L' = \mu_o \mu_r n^2 S = 4\pi\times10^{-7}\times1000\times16\times10^6\times4\times10^{-4}$

$= \underline{8.042 \text{ H/m}}$

(b) $W_m' = \frac{1}{2}L'I^2 = \frac{1}{2}(8.042)(0.5^2) = \underline{\underline{1.005}}$ J/m

P.E. 8.11 From Example 8.11,

$$L_{in} = \frac{\mu_o l}{8\pi}$$

$$L_{ext} = \frac{2w_m}{I^2} = \frac{1}{I^2}\iiint \frac{\mu I^2}{4\pi^2\rho^2}\rho d\rho d\phi dz$$

$$= \frac{1}{4\pi^2}\int_0^l dz \int_0^{2\pi} d\phi \int_a^b \frac{2\mu_o}{(1+\rho)\rho}d\rho$$

$$= \frac{2\mu_o}{4\pi^2} \bullet 2\pi l \int_a^b \left[\frac{1}{\rho} - \frac{1}{(1+\rho)}\right]d\rho$$

$$= \frac{\mu_o l}{\pi}\left[\ln\frac{b}{a} - \ln\frac{1+b}{1+a}\right]$$

$$L = L_{in} + L_{ext} = \underline{\underline{\frac{\mu_o l}{8\pi} + \frac{\mu_o l}{\pi}\left[\ln\frac{b}{a} - \ln\frac{1+b}{1+a}\right]}}$$

P.E. 8.12

(a) $L'_{in} = \frac{\mu_o}{8\pi} = \frac{4\pi\times10^{-7}}{8\pi} = \underline{0.05\ \mu H/m}$

$L'_{ext} = L' - L'_{in} = 1.2 - 0.05 = \underline{1.15\ \mu H/m}$

(b) $L' = \frac{\mu_o}{2\pi}\left[\frac{1}{4} + \ln\frac{d-a}{a}\right]$

$$\ln\frac{d-a}{a} = \frac{2\pi L'}{\mu_o} - 0.25 = \frac{2\pi\times1.2\times10^{-6}}{4\pi x10^{-7}} - 0.25$$

$$= 6 - 0.25 = 5.75$$

$$\frac{d-a}{a} = e^{5.75} = 314.19$$

$$d - a = 314.19a = 314.19\times\frac{2.588\times10^{-3}}{2} = 406.6mm$$

$$d = 407.9mm = \underline{\underline{40.79cm}}$$

P.E. 8.13
This is similar to Example 8.13. In this case, however, h=0 so that

$$A_1 = \frac{\mu_o I_1 a^2 b}{4b^3}\mathbf{a}_\phi$$

$$\phi_{12} = \frac{\mu_o I_1 a^2}{4b^2} \bullet 2\pi b = \frac{\mu_o \pi I_1 a^2}{2b}$$

$$m_{12} = \frac{\phi_{12}}{I_1} = \frac{\mu_o \pi a^2}{2b} = \frac{4\pi\times10^{-7}\times\pi\times4}{2\times3}$$

$$= \underline{2.632\ \mu H}$$

P.E. 8.14

$$L_{in} = \frac{\mu_o}{8\pi}l = \frac{\mu_o 2\pi\rho_o}{8\pi} = \frac{4\pi\times10^{-7}\times10\times10^{-2}}{4}$$

$$= \underline{31.42\ nH}$$

P.E. 8.15

(a) From Example 7.6,

$$B_{ave} = \frac{\mu_o NI}{l} = \frac{\mu_o NI}{2\pi\rho_o}$$

$$\phi = B_{ave} \bullet S = \frac{\mu_o NI}{2\pi\rho_o} \bullet \pi a^2$$

$$\text{or } I = \frac{2\rho_o\phi}{\mu a^2 N} = \frac{2\times10\times10^{-2}\times0.5\times10^{-3}}{4\pi\times10^{-7}\times10^{-4}\times10^3}$$

$$= \underline{795.77A}$$

Alternatively, using circuit approach

$$R = \frac{l}{\mu S} = \frac{2\pi\rho_o}{\mu_o S} = \frac{2\pi\rho_o}{\mu_o\pi a^2}$$

$$\Im = NI = \frac{\phi\Re}{N} = \frac{2\rho_o\phi}{\mu a^2 N}, \quad \text{as obtained before.}$$

$$\Re = \frac{2\rho_o}{\mu a^2} = \frac{2\times10\times10^{-2}}{4\pi\times10^{-7}\times10^{-4}} = 1.591\times10^9$$

$$\Im = \phi\Re = 0.5x10^{-3}x1.591x10^9 = 7.9577x10^5$$

$$I = \frac{\Im}{N} = 795.77A \text{ as obtained before.}$$

(b) If $\mu = 500\mu_o$,

$$I = \frac{795.77}{500} = \underline{\underline{1.592}}\ \text{A}$$

P.E. 8.16

$$\Im = \frac{B^2{}_a S}{2\mu_o} = \frac{(1.5)^2 \times 10 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = \frac{22500}{8\pi} = \underline{\underline{895.25\text{N}}}$$

Prob. 8.1

$$F = q\left(E + u \times B\right)$$

$$\text{If } F = 0, \quad E = -u \times B = B \times u$$

$$= \begin{vmatrix} 10 & 20 & 30 \\ 3 & 12 & -4 \end{vmatrix} \times 10^5 \times 10^{-3}$$

$$E = \underline{\underline{-44\,a_x + 13\,a_y + 6\,a_z\ \text{kV/m}}}$$

Prob. 8.2

$$F = m\omega^2 r = 9.11 \times 10^{-31} \times (2 \times 10^{16})^2 (0.4 \times 10^{-10}) = \underline{\underline{14.576\ \text{nN}}}$$

Prob. 8.3

From Example 8.3, $QuB = QE \longrightarrow u = \frac{E}{B} = \frac{10 \times 10^3}{1}$

$$\text{K.E.} = \frac{1}{2}mu^2 = \frac{1}{2} \times 1.673 \times 10^{-27} \times 10^8 = \underline{\underline{8.365 \times 10^{-20}\ \ \text{J}}}$$

Prob. 8.4

(a) $F = ma = Q(E + u \times B)$

$$\frac{d}{dt}(u_x, u_y, u_z) = 2\left[-4a_y + \begin{vmatrix} u_x & u_y & u_z \\ 5 & 0 & 0 \end{vmatrix}\right] = -8a_y + 10u_z a_y - 10u_y a_z$$

i.e. $\frac{du_x}{dt} = 0 \rightarrow u_x = A_1$ (1)

$\frac{du_y}{dt} = -8 + 10u_z$ (2)

$\frac{du_z}{dt} = -10u_y$ (3)

$$\frac{d^2u_y}{dt^2} = 0 + 10\frac{du_z}{dt} = -100u_y$$

$$\ddot{u}_y + 100u_y = 0 \rightarrow u_y = B_1 \cos 10t + B_2 \sin 10t$$

From (2),

$$10u_z = 8 + \dot{u}_y = 8 - 10B_1 \sin 10t + 10B_2 \cos 10t$$

$u_z = 0.8 - B_1 \sin 10t + B_2 \cos 10t$

At t=0, $u = 0 \rightarrow A_1 = 0, B_1 = 0, B_2 = -0.8$

Hence,

$u = (0,\ -0.8\sin 10t,\ 0.8 - 0.8\cos 10t)$ (4)

$u_x = \frac{dx}{dt} = 0 \rightarrow x = c_1$

$u_y = \frac{dy}{dt} = -0.8\sin 10t \rightarrow y = 0.08\cos 10t + c_2$

$u_z = \frac{dz}{dt} = 0.8 - 0.8\cos 10t \rightarrow z = 0.8t + c_3 - 0.08\sin 10t$

At t=0, (x, y, z) = (2, 3, -4) ⇒ $c_1 = 2$, $c_2 = 2.92$, $c_3 = -4$

Hence (x, y, z) = (2, 2.92 + 0.08cos10t, 0.8t – 0.08sin10t – 4)

At t=1,

(x, y, z) = $\underline{\underline{(2, 2.853, -3.156)}}$

(b) From (4), at t=1, $u = \underline{\underline{(0, -0.435, 1.471)}}$ m/s

$$\text{K.E.} = \frac{1}{2}m|u|^2 = \frac{1}{2}(1)(0.435^2 + 1.471^2) = \underline{\underline{1.177\text{J}}}$$

Prob. 8.5

$ma = Qu \times B$

$$10^{-3} a = -2 \times 10^{-3} \begin{vmatrix} u_x & u_y & u_z \\ 0 & 6 & 0 \end{vmatrix}$$

$$\frac{d}{dt}(u_x, u_y, u_z) = (12u_z, 0, -12u_x)$$

i.e. $\frac{du_x}{dt} = 12u_z$ (1)

$$\frac{du_y}{dt} = 0 \rightarrow u_y = A_1 \qquad (2)$$

$$\frac{du_z}{dt} = -12u_x \qquad (3)$$

From (1) and (3),

$$\ddot{u}_x = 12\dot{u}_z = -144u_x$$

or

$$\ddot{u}_x + 144u_x = 0 \rightarrow u_x = c_1 \cos 12t + c_2 \sin 12t$$

From (1), $u_z = -c_1 \sin 12t + c_2 \cos 12t$

At t=0,

$u_x=5$, $u_y=0$, $u_z=0 \rightarrow A_1=0=c_2$, $c_1=5$

Hence,

$$\boldsymbol{u} = (5\cos 12t, 0, -5\sin 12t)$$

$$\boldsymbol{u}(t=10s) = (5\cos 120, 0, -5\sin 120) = \underline{\underline{4.071\boldsymbol{a}_x - 2.903\boldsymbol{a}_z}} \text{ m/s}$$

$$u_x = \frac{dx}{dt} = 5\cos 12t \rightarrow x = \frac{5}{12}\sin 12t + B_1$$

$$u_y = \frac{dy}{dt} = 0 \rightarrow y = B_2$$

$$u_z = \frac{dz}{dt} = -5\sin 12t \rightarrow z = \frac{5}{12}\cos 12t + B_3$$

At t=0, (x, y, z) = (0, 1, 2) $\rightarrow B_1=0, B_2=1, B_3=\frac{19}{12}$

$$(x,y,z) = \left(\frac{5}{12}\sin 12t, 1, \frac{5}{12}\cos 12t + \frac{19}{12}\right) \qquad (4)$$

At t=10s,

$$(x,y,z) = \left(\frac{5}{12}\sin 120, 1, \frac{5}{12}\cos 120 + \frac{19}{12}\right) = \underline{(0.2419, 1, 1.923)}$$

By eliminating t from (4),

$x^2 + (z - 19/12)^2 = (5/12)^2$, $y=1$ which is a circle in the $y=1$ plane with center at (0,1,19/12). The particle gyrates.

Prob. 8.6

(a) $m\boldsymbol{a} = -e(\boldsymbol{u} \times \boldsymbol{B})$

$$-\frac{m}{e}\frac{d}{dt}(u_x, u_y, u_z) = \begin{vmatrix} u_x & u_y & u_z \\ 0 & 0 & B_o \end{vmatrix} = u_y B_o \vec{a}_x - B_o u_x \vec{a}_y$$

$$\frac{du_z}{dt} = 0 \rightarrow u_z = c = 0$$

$$\frac{du_x}{dt} = -u_y \frac{B_o e}{m} = -u_y w, \text{ where } w = \frac{B_o e}{m}$$

$$\frac{du_y}{dt} = u_x w$$

Hence,

$$\ddot{u}_x = -w\dot{u}_y = -w^2 u_x$$

or $\ddot{u}_x + w^2 u_x = 0 \rightarrow u_x = A\cos wt + B\sin wt$

$$u_y = -\frac{\dot{u}_x}{w} = A\sin wt - B\cos wt$$

At t=0, $u_x = u_o$, $u_y = 0 \rightarrow A = u_o$, B=0

Hence,

$$u_x = u_o \cos wt = \frac{dx}{dt} \rightarrow x = \frac{u_o}{w}\sin wt + c_1$$

$$u_y = u_o \sin wt = \frac{dy}{dt} \rightarrow y = -\frac{u_o}{w}\cos wt + c_2$$

At t=0, x = 0 = y $\rightarrow c_1=0$, $c_2=\frac{u_o}{w}$. Hence,

$$x = \frac{u_o}{w}\sin wt, \quad y = \frac{u_o}{w}(1 - \cos wt)$$

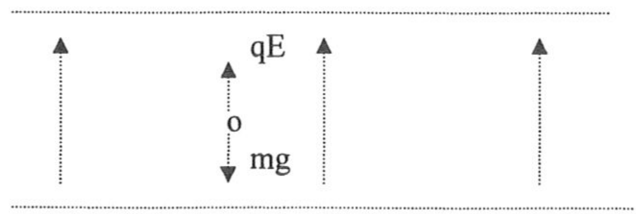
$$\frac{u_o^2}{w^2}(\cos^2 wt + \sin^2 wt) = \left(\frac{u_o}{w}\right)^2 = x^2 + (y - \frac{u_o}{w})^2$$

showing that the electron would move in a circle centered at (0, $\frac{u_o}{w}$). But since the field does not exist throughout the circular region, the electron passes through a semi-circle and leaves the field horizontally.

(b) d = twice the radius of the semi-circle

$$= \frac{2u_o}{w} = \frac{2u_o m}{B_o e}$$

Prob. 8.7



$$m = 0.4\times10^{-3}\ kg \qquad g = 9.81\, m/s^2$$
$$V_0 = 12000\ V \qquad d = 8\times10^{-2}\ m$$
$$E = \frac{V_0}{d} = 1.5\times10^5\ V/m$$
$$q = \frac{mg}{E} = \frac{0.4\times10^{-3}\times9.81}{1.5\times10^5} = 26.16\, nC$$

Prob. 8.8

$$\boldsymbol{F} = \int I d\boldsymbol{l}\times\boldsymbol{B} = \int_0^{0.2} 2dy(-\boldsymbol{a}_y)\times(4\boldsymbol{a}_x - 8\boldsymbol{a}_z)$$

$$(-\boldsymbol{a}_y)\times(4\boldsymbol{a}_x - 8\boldsymbol{a}_z) = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ 0 & -1 & 0 \\ 4 & 0 & -8 \end{vmatrix} = 8\boldsymbol{a}_x + 4\boldsymbol{a}_z$$

$$\boldsymbol{F} = 2(8\boldsymbol{a}_x + 4\boldsymbol{a}_z)(0.2) = \underline{\underline{3.2\boldsymbol{a}_x + 1.6\boldsymbol{a}_z\ \text{N}}}$$

Prob. 8.9

$$\mathfrak{F} = I\boldsymbol{L}\times\boldsymbol{B} \rightarrow \mathfrak{F} = \frac{F}{L} = I_1\boldsymbol{a}_l\times\boldsymbol{B}_2 = \frac{\mu_o I_1 I_2 \boldsymbol{a}_l\times\boldsymbol{a}_\phi}{2\pi\rho}$$

(a) $$\boldsymbol{F}_{21} = \frac{\boldsymbol{a}_z\times(-\boldsymbol{a}_y)4\pi\times10^{-7}(-100)(200)}{2\pi} = \underline{\underline{4\boldsymbol{a}_x\ \text{mN/m}}}\ \text{(repulsive)}$$

(b) $$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} = -4\boldsymbol{a}_x\ \text{mN/m (repulsive)}$$

(c) $$\boldsymbol{a}_l\times\boldsymbol{a}_\phi = \boldsymbol{a}_z\times(-\frac{4}{5}\boldsymbol{a}_x + \frac{3}{5}\boldsymbol{a}_y) = -\frac{3}{5}\boldsymbol{a}_x - \frac{4}{5}\boldsymbol{a}_y, \rho = 5$$

$$\boldsymbol{F}_{31} = \frac{4\pi\times10^{-7}(-3\times10^4)}{2\pi(5)}\left(-\frac{3}{5}\boldsymbol{a}_x - \frac{4}{5}\boldsymbol{a}_y\right)$$
$$= \underline{\underline{0.72\boldsymbol{a}_x + 0.96\boldsymbol{a}_y}}\ \text{mN/m (attractive)}$$

(d) $$\boldsymbol{F}_3 = \boldsymbol{F}_{31} + \boldsymbol{F}_{32}$$

$$\boldsymbol{F}_{32} = \frac{4\pi\times10^{-7}\times6\times10^4)}{2\pi(3)}(\boldsymbol{a}_z\times\boldsymbol{a}_y) = -4\boldsymbol{a}_x\ \text{mN/m(attractive)}$$
$$\boldsymbol{F}_3 = \underline{\underline{-3.28\boldsymbol{a}_x + 0.96\boldsymbol{a}_y}}\ \text{mN/m}$$

(attractive due to L_2 and repulsive due to L_1)

Prob. 8.10

$$F = \frac{\mu_o I_1 I_2}{2\pi\rho} = \frac{4\pi\times10^{-7}(10)10}{2\pi(20\times10^{-2})} = \underline{\underline{100\ \mu\text{N}}}$$

Prob. 8.11

$$W = -\int \boldsymbol{F}\bullet d\boldsymbol{l}, \qquad \boldsymbol{F} = \int L d\boldsymbol{l}\times\boldsymbol{B} = 3(2\boldsymbol{a}_z)\times\cos{}^{\phi}\!/_3\, \boldsymbol{a}_\phi$$
$$F = 6\cos{}^{\phi}\!/_3\, \vec{a}_\phi\ \text{N}$$
$$W = -\int_0^{2\pi} 6\cos{}^{\phi}\!/_3\, \rho_o d\phi = -6\rho_o\times3\sin{}^{\phi}\!/_3 \Big|_0^{2\pi}\ \text{J}$$
$$= -1.8\sin\frac{2\pi}{3} = \underline{-1.559\ \text{J}}$$

Prob. 8.12

(a) $$\boldsymbol{F}_1 = \int_{\rho=2}^{6} \frac{\mu_o I_1 I_2}{2\pi\rho} d\rho\boldsymbol{a}_\rho\times\boldsymbol{a}_\phi = \frac{4\pi\times10^{-7}}{2\pi}(2)(5)\ln{}^{6}\!/_2\, \boldsymbol{a}_z$$
$$= 2\ln3\boldsymbol{a}_z\ \mu\text{N} = \underline{\underline{2.197\boldsymbol{a}_z\ \mu\text{N}}}$$

(b) $$\boldsymbol{F}_2 = \int I_2 d\boldsymbol{l}_2\times\boldsymbol{B}_1$$
$$= \frac{\mu_o I_1 I_2}{2\pi}\int\frac{1}{\rho}\left[d\rho\boldsymbol{a}_\rho + dz\boldsymbol{a}_z\right]\times\boldsymbol{a}_\phi$$
$$= \frac{\mu_o I_1 I_2}{2\pi}\int\frac{1}{\rho}\left[d\rho\boldsymbol{a}_z - dz\boldsymbol{a}_\rho\right]$$

But ρ = z+2, dz=dρ

$$\boldsymbol{F}_2 = \frac{4\pi\times10^{-7}}{2\pi}(5)(2)\int_{\rho=4}^{2}\frac{1}{\rho}\left[d\rho\boldsymbol{a}_z - dz\boldsymbol{a}_\rho\right]$$
$$2\ln{}^{2}\!/_4(\boldsymbol{a}_z - \boldsymbol{a}_\rho)\mu N = 1.386\boldsymbol{a}_\rho - 1.386\boldsymbol{a}_z\boldsymbol{a}_z\mu N$$
$$\boldsymbol{F}_3 = \frac{\mu_o I_1 I_2}{2\pi}\int\frac{1}{\rho}\left[d\rho\boldsymbol{a}_z - dz\boldsymbol{a}_\rho\right]$$

But z = -ρ + 6, dz = -dρ

$$F_3 = \frac{4\pi\times10^{-7}}{2\pi}(5)(2)\int_{\rho=6}^{4}\frac{1}{\rho}\left[d\rho\,\boldsymbol{a}_z - dz\,\boldsymbol{a}_\rho\right]$$

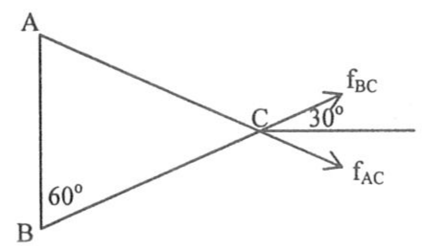
$$2\ln \tfrac{4}{6}(\boldsymbol{a}_z+\boldsymbol{a}_\rho)\mu N = -0.8109\boldsymbol{a}_\rho - 0.8109\boldsymbol{a}_z \mu N$$

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

$$= \boldsymbol{a}_\rho(\ln 4 + \ln 4 - \ln 9) + \boldsymbol{a}_z(\ln 9 - \ln 4 + \ln 4 - \ln 9)$$

$$= \underline{\underline{0.575\boldsymbol{a}_\rho\ \mu\text{N}}}$$

Prob. 8.13



From Prob. 8.7,

$$\boldsymbol{f} = \frac{\mu_o I_1 I_2}{2\pi\rho}\boldsymbol{a}_\rho$$

$$\boldsymbol{f} = \boldsymbol{f}_{AC} + \boldsymbol{f}_{BC}$$

$$|\boldsymbol{f}_{AC}| = |\boldsymbol{f}_{BC}| = \frac{4\pi\times10^{-7}\times75\times150}{2\pi\times2} = 1.125\times10^{-3}$$

$$\boldsymbol{f} = 2\times1.125\cos30^o\boldsymbol{a}_x \text{mN/m}$$

$$= \underline{\underline{1.949\boldsymbol{a}_x}}\ \text{mN/m}$$

Prob. 8.14

The field due to the current sheet is

$$\boldsymbol{B} = \frac{\mu}{2}\boldsymbol{K}\times\boldsymbol{a}_n = \frac{\mu_o}{2}10\boldsymbol{a}_x\times(-\boldsymbol{a}_z) = 5\mu_o\boldsymbol{a}_y$$

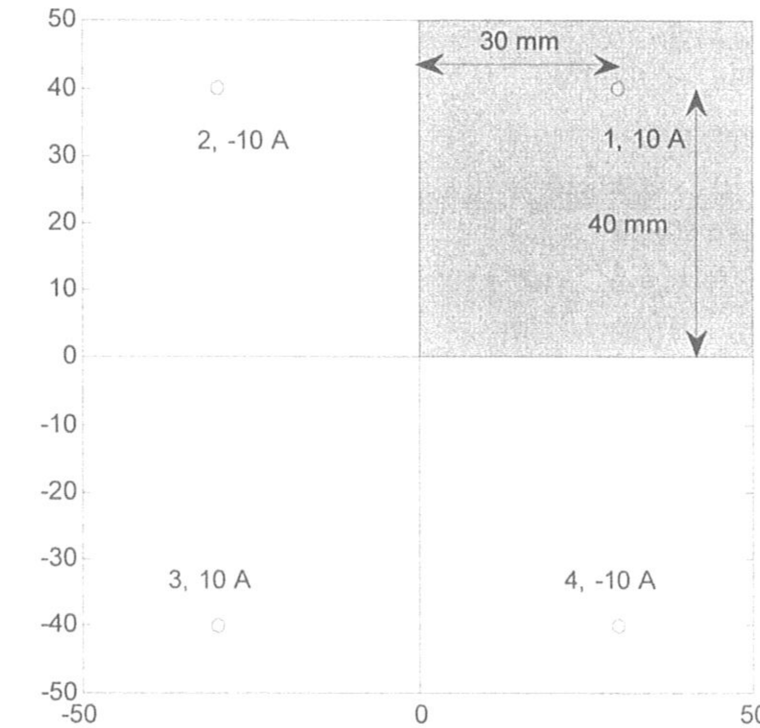
$$\boldsymbol{F} = I_2\int d\boldsymbol{l}_2\times\boldsymbol{B} = 2.5\int_0^L dx\boldsymbol{a}_x\times(5\mu_o\boldsymbol{a}_y) = 2.5L\times5\mu_o(\boldsymbol{a}_z)$$

$$\frac{\boldsymbol{F}}{L} = 12.5\times4\pi\times10^{-7}(\boldsymbol{a}_z) = \underline{\underline{15.71\boldsymbol{a}_z\ \ \mu\text{N/m}}}$$

Prob. 8.15

$$\boldsymbol{F} = \int I d\boldsymbol{l}\times\boldsymbol{B} = I\boldsymbol{L}\times\boldsymbol{B} = 5(2\boldsymbol{a}_z)\times40\boldsymbol{a}_x10^{-3} = \underline{\underline{0.4\boldsymbol{a}_y}}\ \text{N}$$

Prob. 8.16



Let $\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 + \boldsymbol{B}_3 + \boldsymbol{B}_4$

where $\boldsymbol{B}_n = \frac{\mu_o\mu_r I}{2\pi\rho}\boldsymbol{a}_\phi$

For (1), $\boldsymbol{a}_\phi = \boldsymbol{a}_\ell\times\boldsymbol{a}_\rho = \boldsymbol{a}_z\times(-\boldsymbol{a}_y) = \boldsymbol{a}_x$

$$\boldsymbol{B}_1 = \frac{4\pi\times10^{-7}\times2000\times10}{2\pi\times20\times10^{-3}}\boldsymbol{a}_x = 0.2\boldsymbol{a}_x$$

For (2), $\rho = 6\boldsymbol{a}_x - 2\boldsymbol{a}_y$,

$$\boldsymbol{a}_\phi = -\boldsymbol{a}_z\times\frac{(6\boldsymbol{a}_x - 2\boldsymbol{a}_y)}{\sqrt{40}} = \frac{(-2\boldsymbol{a}_x - 6\boldsymbol{a}_y)}{\sqrt{40}}$$

$$\boldsymbol{B}_2 = \frac{4\pi\times10^{-7}\times2000\times10}{2\pi\times400\times10^{-3}}(-2\boldsymbol{a}_x - 6\boldsymbol{a}_y)$$

$$= -0.02\boldsymbol{a}_x - 0.06\boldsymbol{a}_y$$

For (3), $\rho = 6\boldsymbol{a}_x + 6\boldsymbol{a}_y$,

$$\boldsymbol{a}_\phi = \boldsymbol{a}_z\times\frac{(6\boldsymbol{a}_x + 6\boldsymbol{a}_y)}{\sqrt{72}} = \frac{(-6\boldsymbol{a}_x + 6\boldsymbol{a}_y)}{\sqrt{72}}$$

$$B_3 = \frac{4\pi\times10^{-7}\times2000\times10}{2\pi\times720\times10^{-3}}(-6\boldsymbol{a}_x + 6\boldsymbol{a}_y)$$
$$= -0.03333\boldsymbol{a}_x + 0.03333\boldsymbol{a}_y$$

For (4), $\boldsymbol{a}_\phi = -\boldsymbol{a}_z \times \boldsymbol{a}_y = \boldsymbol{a}_x$,

$$B_4 = \frac{4\pi\times10^{-7}\times2000\times10}{2\pi\times60\times10^{-3}}\boldsymbol{a}_x = 0.06667\boldsymbol{a}_x$$
$$\boldsymbol{B} = (2 + 2/3 - 1/5 - 1/3)\times10^{-1}\boldsymbol{a}_x + (-3/5 + 1/3)\times10^{-1}\boldsymbol{a}_y$$
$$= \underline{\underline{0.21333\boldsymbol{a}_x - 0.02667\boldsymbol{a}_y \text{ Wb/m}^2}}$$

Note: We have not considered the idea of magnetic images in this problem.

Prob. 8.17

$$f(x,y,z) = x + 2y - 5z - 12 = 0 \longrightarrow \nabla f = \boldsymbol{a}_x + 2\boldsymbol{a}_y - 5\boldsymbol{a}_z$$

$$\boldsymbol{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{\boldsymbol{a}_x + 2\boldsymbol{a}_y - 5\boldsymbol{a}_z}{\sqrt{30}}$$

$$\boldsymbol{m} = NIS\boldsymbol{a}_n = 2\times60\times8\times10^{-4}\frac{(\boldsymbol{a}_x + 2\boldsymbol{a}_y - 5\boldsymbol{a}_z)}{\sqrt{30}} = \underline{\underline{17.53\boldsymbol{a}_x + 35.05\boldsymbol{a}_y - 87.64\boldsymbol{a}_z \text{ mAm}}}$$

Prob. 8.18

$$\boldsymbol{B} = \frac{k}{r^3}(2\cos\theta\boldsymbol{a}_r + \sin\theta\boldsymbol{a}_\theta)$$

At (10, 0, 0), r = 10; $\theta = \pi/2, \boldsymbol{a}_r = \boldsymbol{a}_x, \boldsymbol{a}_\theta = -\boldsymbol{a}_z$

$$-0.5\times10^{-3}\boldsymbol{a}_z = \frac{k}{10^3}(0 - \boldsymbol{a}_z) \rightarrow k = 0.5$$

Thus,

$$\boldsymbol{B} = \frac{0.5}{r^3}(2\cos\theta\boldsymbol{a}_r + \sin\theta\boldsymbol{a}_\theta)$$

(a) At (0, 3, 0), r=3, $\theta = \pi/2, \boldsymbol{a}_r = \boldsymbol{a}_y, \boldsymbol{a}_\theta = -\boldsymbol{a}_z$

$$\boldsymbol{B} = \frac{0.5}{27}(0 - \boldsymbol{a}_z) = \underline{\underline{-18.52\boldsymbol{a}_z}} \text{ mWb/m}^2$$

(b) At (3, 4, 0), r=5, $\theta = \pi/2, \vec{a}_\theta = -\vec{a}_z$

$$\boldsymbol{B} = \frac{0.5}{125}(0 - \boldsymbol{a}_z) = \underline{\underline{-4\boldsymbol{a}_z}} \text{ mWb/m}^2$$

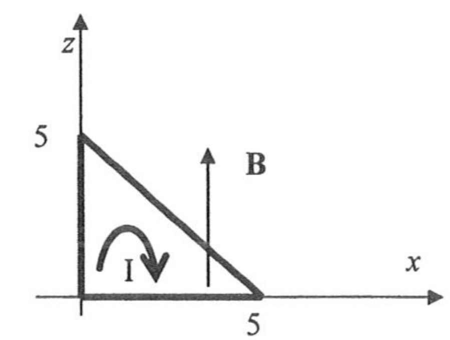
(c) At (1, 1, -1), r=$\sqrt{3}$, $\tan\theta = \rho/z = \sqrt{2}/-1$, i.e.

$$\sin\theta = \sqrt{2}/\sqrt{3}, \cos\theta = -1/\sqrt{3}$$

$$\boldsymbol{B} = \frac{0.5}{3\sqrt{3}}(-2/\sqrt{3}\,\boldsymbol{a}_r + \sqrt{2}/\sqrt{3}\,\boldsymbol{a}_\theta) = \underline{\underline{-111\boldsymbol{a}_r + 78.6\boldsymbol{a}_\theta}} \text{ mWb/m}^2$$

Prob. 8.19

Let $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$



$$\boldsymbol{F}_1 = \int I d\boldsymbol{l}\times\boldsymbol{B} = \int_5^0 2dx\boldsymbol{a}_x\times30\boldsymbol{a}_z \text{ mN}$$
$$= -60\boldsymbol{a}_y x\Big|_5^0 = 300\boldsymbol{a}_y \text{ mN}$$
$$\boldsymbol{F}_2 = \int_0^5 2dy\boldsymbol{a}_y\times30\boldsymbol{a}_z \text{ mN}$$
$$= 60\boldsymbol{a}_x y\Big|_0^5 = 300\boldsymbol{a}_x \text{ mN}$$
$$\boldsymbol{F}_3 = \int_0^5 2(dx\boldsymbol{a}_x + dz\boldsymbol{a}_z)\times30\boldsymbol{a}_z \text{ mN}$$
$$= 60(-\boldsymbol{a}_y)x\Big|_0^5 = -300\boldsymbol{a}_y \text{ mN}$$
$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 300\boldsymbol{a}_y + 300\boldsymbol{a}_x - 300\boldsymbol{a}_y \text{ mN} = \underline{\underline{300\boldsymbol{a}_x \text{ mN}}}$$
$$\boldsymbol{T} = \boldsymbol{m}\times\boldsymbol{B} = IS\boldsymbol{a}_n\times\boldsymbol{B} = 2(\tfrac{1}{2})(5)(5)\boldsymbol{a}_y\times30\boldsymbol{a}_z10^{-3} = \underline{\underline{0.75\boldsymbol{a}_x \text{ N.m}}}$$

Prob. 8.20

(a) $$\boldsymbol{M} = \chi_m\boldsymbol{H} = \chi_m\frac{\boldsymbol{B}}{\mu_o\mu}$$

$$M = \frac{4999}{5000}\times\frac{1.5}{4\pi\times10^{-7}} = \underline{\underline{1.193\times10^6 \text{ A/m}}}$$

(b) $$\boldsymbol{M} = \frac{\sum_{k=1}^{N}\boldsymbol{m}_k}{\Delta v}$$

If we assume that all $\boldsymbol{m}_k$ align with the applied **B** field,

$$M = \frac{Nm_k}{\Delta v} \rightarrow m_k = \frac{Nm_k}{N/\Delta v} = \frac{1.193\times10^6}{8.5\times10^{28}}$$

$$m_k = \underline{\underline{1.404\times10^{-23} \text{ A}\cdot\text{m}^2}}$$

Prob. 8.21

(a) $\chi_m = \mu_r - 1 = 4.6 - 1 = \underline{\underline{3.6}}$

(b) $H = \frac{B}{\mu_o \mu_r} = \frac{5x a_z}{4\pi \times 10^{-7} \times 4.6} = \underline{\underline{865 x a_z \text{ kA/m}}}$

(c) $M = \chi_m H = 3114 x a_z$ kA/m $= \underline{\underline{3.114 x a_z \text{ MA/m}}}$

Prob. 8.22

(a) $\chi_m = \mu_r - 1 = \underline{\underline{3.5}}$

(b) $H = \frac{B}{\mu} = \frac{4y\, a_z \times 10^{-3}}{4\pi \times 10^{-7} \times 4.5} = \underline{\underline{707.3y\, a_z \text{ A/m}}}$

(c) $M = \chi_m H = \underline{\underline{2.476y\, a_z \text{ kA/m}}}$

(d) $$J_b = \nabla \times M = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & M_z(y) \end{vmatrix} = \frac{dM_z}{dy} a_x$$

$= \underline{\underline{2.476 a_x \text{ kA/m}^2}}$

Prob. 8.23

For case 1,

$$\mu = \frac{B_1}{H_1} = \frac{2}{1200}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1}{600} \times \frac{1}{4\pi \times 10^{-7}} = 1326.3$$

$$\chi_m = \mu_r - 1 = 1325.3$$

$$M_1 = \chi_m H_1 = 1{,}590{,}366$$

For case 2,

$$\mu = \frac{B_2}{H_2} = \frac{1.4}{400}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.4}{400} \times \frac{1}{4\pi \times 10^{-7}} = 2785.2$$

$$\chi_m = \mu_r - 1 = 2784.2$$

$$M = \chi_m H = 1{,}113{,}630$$

$$\Delta M = M_2 - M_1 = -476{,}680$$

$= \underline{\underline{-476.7 \text{ kA/m}}}$

Prob. 8.24

$$\oint H \cdot dl = I_{enc}$$

$$H_\phi \cdot 2\pi\rho = \frac{\pi\rho^2}{\pi a^2} \cdot I \rightarrow H_\phi = \frac{I\rho}{2\pi a^2}$$

$$M = \chi_m H = \underline{\underline{(\mu_r - 1)\frac{I\rho}{2\pi a^2} a_\phi}}$$

$$J_b = \nabla \times M = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho M_\phi)\, a_z = \underline{\underline{(\mu_r - 1)\frac{I}{\pi a^2} a_z}}$$

Prob. 8.25

(a) From $H_{1t} - H_{2t} = K$ and $M = \chi_m H$, we obtain:

$$\underline{\underline{\frac{M_{1t}}{\chi_{m1}} - \frac{M_{2t}}{\chi_{m2}} = K}}$$

Also from $B_{1n} - B_{2n} = 0$ and $B = \mu H = (\mu/\chi_m)M$, we get:

$$\underline{\underline{\frac{\mu_1 M_{1n}}{\chi_{m1}} = \frac{\mu_2 M_{2n}}{\chi_{m2}}}}$$

(b) From $B_1\cos\theta_1 = B_{1n} = B_{2n} = B_2\cos\theta_2$ (1)

and $\frac{B_1 \sin\theta_1}{\mu_1} = H_{1t} = K + H_{2t} = K + \frac{B_2 \sin\theta_2}{\mu_2}$ (2)

Dividing (2) by (1) gives

$$\frac{\tan\theta_1}{\mu_1} = \frac{k}{B_2\cos\theta_2} + \frac{\tan\theta_2}{\mu_2} = \frac{\tan\theta_2}{\mu_2}\left(1 + \frac{k\mu_2}{B_2 \sin\theta_2}\right)$$

i.e. $\underline{\underline{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}\left(1 + \frac{k\mu_2}{B_2 \sin\theta_2}\right)}}$

Prob. 8.26

$B_{2n} = B_{1n} = 30 a_z$

$H_{2t} = H_{1t} \longrightarrow \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}$

$$B_{2t} = \frac{\mu_2}{\mu_1} B_{1t} = \frac{20\mu_o}{\mu_o}(20a_x - 15a_y) = 400a_x - 300a_y$$

$$B_2 = B_{2n} + B_{2t} = 400a_x - 300a_y + 30a_z \text{ mWb/m}^2$$

$$H_2 = \frac{B_2}{\mu_2} = \frac{(400, -300, 30)}{4\pi \times 10^{-7}(20)} = \underline{\underline{15.915a_x - 11.94a_y + 1.194a_z \text{ kA/m}}}$$

Prob. 8.27

$$H_{2t} = H_{1t} = \alpha a_x + \delta a_z$$

$$B_{2n} = B_{1n} \longrightarrow \mu_2 H_{2n} = \mu_1 H_{1n}$$

$$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = \frac{\mu_{r1}}{\mu_{r2}} \beta a_y$$

$$H = \alpha a_x + \frac{\mu_{r1}}{\mu_{r2}} \beta a_y + \delta a_z$$

Prob. 8.28

(a) $$B_{1n} = B_{2n} = 15a_\phi$$

$$H_{1t} = H_{2t} \rightarrow \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$$

$$B_{1t} = \frac{\mu_1}{\mu_2} B_{2t} = \frac{2}{5}\left(10a_\rho - 20a_z\right) = 4a_\rho - 8a_z$$

Hence,

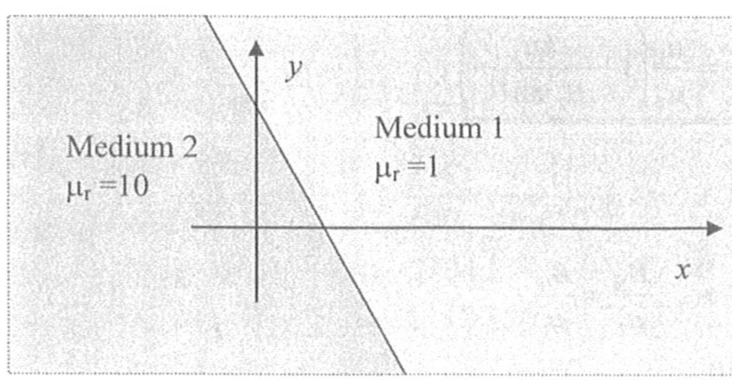
$$B_1 = 4a_\rho + 15a_\phi - 8a_z \text{ mWb/m}^2$$

(b) $$w_{m1} = \frac{1}{2} B_1 \cdot H_1 = \frac{B_1^2}{2\mu_1} = \frac{\left(4^2 + 15^2 + 8^2\right)\times 10^{-6}}{2\times 2\times 4\pi \times 10^{-7}}$$

$$w_{m1} = 60.68 \text{ J}/m^3$$

$$w_{m2} = \frac{B_2^2}{2\mu_2} = \frac{\left(10^2 + 15^2 + 20^2\right)\times 10^{-6}}{2\times 5\times 4\pi\times 10^{-7}} = 57.7 \text{ J}/m^3$$

Prob. 8.29



(a) $$w_{m1} = \frac{1}{2} B_1 \cdot H_1 = \frac{1}{2}\mu_0 \mu_{r1} H_1 \cdot H_1, \quad \mu_r = 1$$

$$w_{m1} = \frac{1}{2}\times 4\pi \times 10^{-7} \times 1 \left(16+9+1\right)$$

$$= 16.34 \ \mu\text{J}/m^3$$

(b)

$$f(x,y) = 2x + y - 8 = 0$$

$$\nabla f = 2a_x + a_y, \quad a_n = \frac{\nabla f}{|\nabla f|} = \frac{2a_x + a_y}{\sqrt{5}}$$

$$H_{1n} = (H_1 \cdot a_n) a_n = \left(\frac{-8+3}{5}\right)\left(2a_x + a_y\right) = -2a_x - a_y$$

$$H_{1t} = H_1 - H_{1n} = -2a_x + 4a_y - a_z = H_{2t}$$

$$B_{2n} = B_{1n} \rightarrow \mu_2 H_{2n} = \mu_1 H_{1n}$$

$$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = \frac{1}{10}\left(-2a_x - a_y\right)$$

$$= -0.2a_x - 0.1a_y$$

$$H_2 = H_{2t} + H_{2n} = -2.2a_x + 3.9a_y - a_z$$

$$M_2 = \chi_{m2} H_2 = 9H_2 = -19.8a_x + 35.1a_y - 9a_z \text{ A/m}$$

$$B_2 = \mu_2 H_2 = 10\mu_o H_2 = 4\pi \times (-2.2, 3.9, -1) \ \mu\text{Wb/m}^2$$

$$= -27.65a_x + 49a_y - 12.57a_z \ \mu\text{Wb/m}^2$$

(c) $$H_1 \bullet a_n = H_1 \cos\theta_1$$

$$\cos\theta_1 = \frac{H_1 \bullet a_n}{H_1} = \frac{(-8+3)/\sqrt{5}}{\sqrt{16+9+1}} = -0.4385 \longrightarrow \theta_1 = 116^\circ$$

$$\cos\theta_2 = \frac{H_2 \bullet a_n}{H_2} = \frac{(-4.4+3.9)/\sqrt{5}}{4.588} = -0.0487 \longrightarrow \theta_1 = 92.8^o$$

If the unit normal is defined as $\mathbf{a}_n = \left(\frac{-2\mathbf{a}_x - \mathbf{a}_y}{\sqrt{5}}\right)$, the above angles would be acute.

Prob. 8.30

$$a_n = a_\rho$$

$$B_{2n} = B_{1n} = 22\mu_o a_\rho$$

$$H_{2t} = H_{1t} \longrightarrow \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}$$

$$\mathbf{B}_{2t} = \frac{\mu_2}{\mu_1}\mathbf{B}_{1t} = \frac{\mu_o}{800\mu_o}(45\mu_o\mathbf{a}_\phi) = 0.05625\mu_o\mathbf{a}_\phi$$

$$\mathbf{B}_2 = \mu_o(22\mathbf{a}_\rho + 0.05625\mathbf{a}_\phi)\ \text{Wb/m}^2$$

Prob. 8.31

$$\mathbf{H}_{1n} = -3\mathbf{a}_z, \qquad \mathbf{H}_{1t} = 10\mathbf{a}_x + 15\mathbf{a}_y$$

$$\mathbf{H}_{2t} = \mathbf{H}_{1t} = 10\mathbf{a}_x + 15\mathbf{a}_y$$

$$\mathbf{H}_{2n} = \frac{\mu_1}{\mu_2}\mathbf{H}_{1n} = \frac{1}{200}(-3\mathbf{a}_z) = -0.015\mathbf{a}_z$$

$$\mathbf{H}_2 = 10\mathbf{a}_x + 15\mathbf{a}_y - 0.015\mathbf{a}_z$$

$$\mathbf{B}_2 = \mu_2\mathbf{H}_2 = 200\times 4\pi\times 10^{-7}(10, 15, -0.015)$$

$$\mathbf{B}_2 = 2.51\mathbf{a}_x + 3.77\mathbf{a}_y - 0.0037\mathbf{a}_z\ \text{mWb/m}^2$$

$$\tan\alpha = \frac{B_{2n}}{B_{2t}}$$

or $\alpha = \tan^{-1}\dfrac{0.0037}{\sqrt{2.51^2 + 3.77^2}} = 0.047^\circ$

Prob. 8.32

(a) $\mathbf{H} = \frac{1}{2}\mathbf{K}\times\mathbf{a}_n = \frac{1}{2}(30-40)\mathbf{a}_x\times(-\mathbf{a}_z) = -5\mathbf{a}_y$ A/m

$$\mathbf{B} = \mu_o\mathbf{H} = 4\pi\times 10^{-7}(-5\mathbf{a}_y) = -6.28\mathbf{a}_y\mu\ \text{Wb/m}^2$$

(b) $\mathbf{H} = \frac{1}{2}(-30-40)\mathbf{a}_y = -35\mathbf{a}_y$ A/m

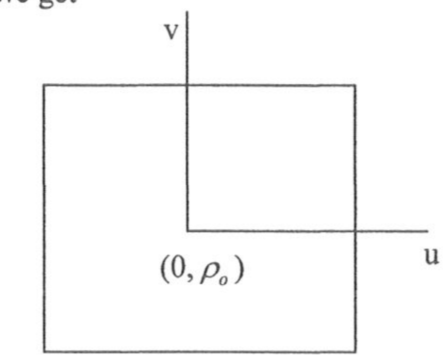
$$\mathbf{B} = \mu_o\mu_r\mathbf{H} = 4\pi\times 10^{-7}(2.5)(-35\mathbf{a}_y) = -110\mathbf{a}_y\ \mu\text{Wb/m}^2$$

(c) $\mathbf{H} = \frac{1}{2}(-30+40)\mathbf{a}_y = 5\mathbf{a}_y$

$$\mathbf{B} = \mu_o\mathbf{H} = 6.283\mathbf{a}_y\mu\ \text{Wb/m}^2$$

Prob. 8.33

(a) The square cross-section of the toroid is shown below. Let (u,v) be the local coordinates and ρ_o=mean radius. Using Ampere's law around a circle passing through P, we get



$$H(2\pi)(\rho_o + v) = NI \longrightarrow H = \frac{NI}{2\pi(\rho_o + v)}$$

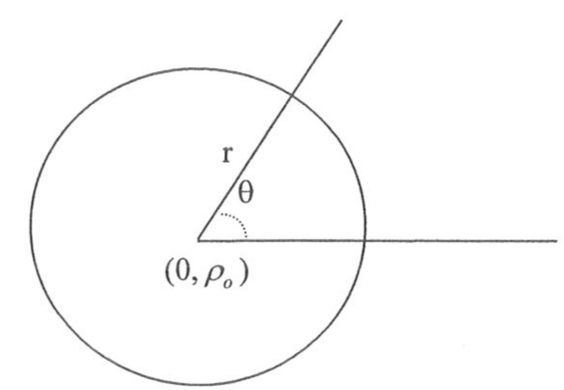
The flux per turn is

$$\Psi = \int_{u=-a/2}^{a/2}\int_{v=-a/2}^{a/2} B\,du\,dv = \frac{\mu_o NIa}{2\pi}\ln\left(\frac{\rho_o + a/2}{\rho_o - a/2}\right)$$

$$L = \frac{N\Psi}{I} = \frac{\mu_o N^2 a}{2\pi}\ln\left(\frac{2\rho_o + a}{2\rho_o - a}\right)$$

(b) The circular cross-section of the toroid is shown below. Let (r,θ) be the local coordinates. Consider a point P($r\cos\theta, \rho_o + r\sin\theta$) and apply Ampere's law around a circle that passes through P.

$$H(2\pi)(\rho_o + r\sin\theta) = NI \longrightarrow H = \frac{NI}{2\pi(\rho_o + r\sin\theta)} \approx \frac{NI}{2\pi\rho_o}\left(1 - \frac{r\sin\theta}{\rho_o}\right)$$



Flux per turn $\Psi = \int_{r=0}^{a}\int_{\theta}^{2\pi} \frac{\mu NI}{2\pi\rho_o}\left(1-\frac{r\sin\theta}{\rho_o}\right) r dr d\theta = \frac{\mu NI}{2\pi\rho_o}\frac{a^2}{2}(2\pi)$

$$L = \frac{N\Psi}{I} = \frac{\mu N^2 a^2}{2\rho_o}$$

Or from Example 8.10,

$$L = L'l = \frac{\mu_o N^2 lS}{l^2} = \frac{\mu_o N^2 \pi a^2}{2\pi\rho_o} = \frac{\mu_o N^2 a^2}{2\rho_o}$$

Prob. 8.34

From Problem 8.33,

$$L = \frac{\mu_o N^2 S}{\ell} = \frac{4\pi \text{x} 10^{-7} \text{x} (450)^2 \text{x} \pi (10^{-2})^2}{0.1} = \underline{\underline{800\mu\text{H}}}$$

Prob. 8.35
Recall the solution of problem 8.33.

$\rho_o = \frac{1}{2}(3+5) = 4\text{cm}$

$a = 2$ cm

$L = \frac{\mu_o N^2 a}{2\pi}\ln\left[\frac{2\rho_o + a}{2\rho_o - a}\right]$

$N^2 = \frac{2\pi L}{\mu_o a \ln\left[\frac{2\rho_o + a}{2\rho_o - a}\right]} = \frac{2\pi(45\times10^{-6})}{4\pi\times10^{-7}(2\times10^{-2})\ln\left(\frac{8+2}{8-2}\right)} = 22,023.17$

$\underline{\underline{N = 148.4 \text{ or } 148}}$

Prob. 8.36

$L_{in} = \frac{\mu_o \ell}{8\pi}, \quad L_{ext} = \frac{\mu_o \ell}{2\pi}\ln\frac{b}{a}$

If $L_{in} = L_{ext} \longrightarrow 4\ln\frac{b}{a} = 1 \longrightarrow \frac{b}{a} = 1.284$

$b = 1.284\times 8$ mm $= \underline{\underline{10.272 \text{ mm}}}$

Prob. 8.37
From Table 8.3,

$L = \frac{\mu_o \ell}{\pi}\ln\frac{d}{a}$

$\ln\frac{d}{a} = \frac{\pi L/\ell}{\mu_o} = \frac{\pi(1.37\times10^{-6})}{4\pi\times10^{-7}} = 3.425$

$\frac{d}{a} = e^{3.425} \longrightarrow a = \frac{d}{e^{3.425}} = \frac{1.2}{30.72} = 0.0391$ m

$\underline{\underline{a = 3.91 \text{ cm}}}$

Prob. 8.38

$$L' = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\frac{b}{a}\right] = \frac{4\pi\times10^{-7}}{2\pi}[0.25 + \ln(6/2.5)] = \underline{\underline{225 \text{ nH}}}$$

Prob. 8.39

$$\psi_{12} = \int \mathbf{B}_1 \bullet d\mathbf{S} = \int_{\rho=\rho_o}^{\rho_o+a}\int_{z=0}^{b} \frac{\mu_o I}{2\pi\rho} dz d\rho = \frac{\mu_o I b}{2\pi}\ln\frac{a+\rho_o}{\rho_o}$$

$$M_{12} = \frac{N\psi_{12}}{I} = \frac{N\mu_o b}{2\pi}\ln\frac{a+\rho_o}{\rho_o}$$

For N = 1,

$$M_{12} = \frac{\psi_{12}}{I_1} = \frac{\mu_o b}{2\pi}\ln\frac{a+\rho_o}{\rho_o}$$

$$= \frac{4\pi\times10^{-7}}{2\pi}(1)\ln 2 = \underline{\underline{0.1386\mu\text{ H}}}$$

Prob. 8.40

We may approximate the longer solenoid as infinite so that $B_1 = \frac{\mu_o N_1 I_1}{l_1}$. The flux linking the second solenoid is:

$$\psi_2 = N_2 B_1 S_1 = \frac{\mu_o N_1 I_1}{l_1} \bullet \pi r_1^2 \square N_2$$

$$M = \frac{\psi_2}{I_1} = \frac{\mu_o N_1 N_2}{l_1} \bullet \pi r_1^2$$

Here we assume air-core solenoids.

Prob. 8.41

$$H = \frac{I}{2\pi\rho} a_\rho$$

$$w_m = \frac{1}{2}\mu |H|^2 = \frac{1}{2}\mu \frac{I^2}{4\pi^2\rho^2}$$

$$W = \int w_m dv = \iiint \frac{1}{2}\mu \frac{I^2}{4\pi^2\rho^2}\rho d\phi d\rho dz = \frac{1}{4\pi}\mu I^2 L \ln(b/a)$$

$$= \frac{1}{4\pi}\times 4\times 4\pi\times 10^{-7}(625\times 10^{-6})3\ln(18/12) = \underline{\underline{304.1 \text{ pJ}}}$$

Alternatively,

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu L}{2\pi}\ln\frac{b}{a}\times I^2 = \frac{\mu I^2 L}{4\pi}\ln\frac{b}{a}$$

Prob. 8.42

$$\mu_r = \chi_m + 1 = 20$$

$$w_m = \frac{1}{2}B_1 \cdot H_1 = \frac{1}{2}\mu \mathbf{H}\cdot\mathbf{H}$$

$$= \frac{1}{2}\mu\left(25x^4y^2z^2 + 100x^2y^4z^2 + 225x^2y^2z^4\right)$$

$$W_m = \int w_m dv$$

$$= \frac{1}{2}\mu\left[25\int_0^1 x^4dx\int_0^2 y^2dy\int_{-1}^2 z^2dz + 100\int_0^1 x^2dx\int_0^2 y^4dy\int_{-1}^2 z^2dz + 225\int_0^1 x^2dx\int_0^2 y^2dy\int_{-1}^2 zdz\right]$$

$$= \frac{25\mu}{2}\left[\frac{x^5}{5}\Big|_0^1 \frac{y^3}{3}\Big|_0^2 \frac{z^3}{3}\Big|_{-1}^2 + 4\frac{x^3}{3}\Big|_0^1 \frac{y^5}{5}\Big|_0^2 \frac{z^3}{3}\Big|_{-1}^2 + 9\frac{x^3}{3}\Big|_0^1 \frac{y^3}{3}\Big|_0^2 \frac{z^5}{5}\Big|_{-1}^2\right]$$

$$= \frac{25\mu}{2}\left(\frac{1}{5}\cdot\frac{8}{3}\cdot\frac{9}{3} + \frac{4}{3}\cdot\frac{32}{3}\cdot\frac{9}{3} + \frac{9}{3}\cdot\frac{8}{3}\cdot\frac{33}{5}\right)$$

$$= \frac{25}{2}\times 4\pi\times 10^{-7}\times 20\times\frac{3600}{45}$$

$$W_m = \underline{\underline{25.13 \text{ mJ}}}$$

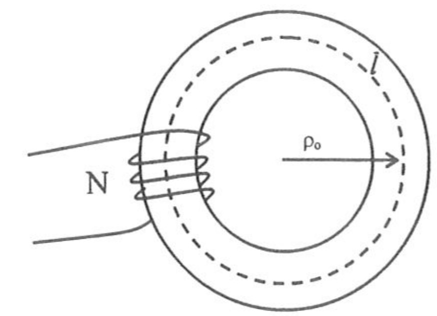
Prob. 8.43

$$W = \frac{1}{2}\int_v \mu H^2 dv = \frac{1}{2}\iiint 4.5\times 4\pi\times 10^{-7}[200^2 + 500^2]10^{-6}dxdydz$$

$$= 2\pi(4.5)10^{-7}(29\times 10^4)10^{-6}(2)(2)(2)10^{-6} = \underline{\underline{6.56 \text{ pJ}}}$$

Prob. 8.44

$$NI = Hl = \frac{Bl}{\mu}$$



$$N = \frac{Bl}{\mu_o\mu_r I} = \frac{1.5\times 0.6\pi}{4\pi\times 10^{-7}\times 600\times 12}$$

$$= \underline{\underline{313 \text{ turns}}}$$

Prob. 8.45

F = NI = 400 x 0.5 = 200 A.t

$$R_a = \frac{100}{4\pi}\text{ MAt/Wb},\quad R_1 = R_2 = \frac{6}{4\pi}\text{ MAt/Wb},\quad R_3 = \frac{1.8}{4\pi}\text{ MAt/Wb}$$

$$F_a = \frac{R_a F}{R_a + R_3 + R_1 // R_2} = \underline{\underline{190.8}}\text{ A.t}$$

$$H_a = \frac{F_a}{l_a} = \frac{190.8}{1\times 10^{-2}} = \underline{\underline{19080}}\text{ A/m}$$

Prob. 8.46

Total F = NI = 2000 x 10 = 20,000 A.t

$$R_c = \frac{l_c}{\mu_o\mu_r S} = \frac{(24+20-0.6)\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 2\times 10^{-4}} = \underline{0.115\text{ x }10^7\text{ A.t/m}}$$

$$R_a = \frac{l_a}{\mu_o\mu_r S} = \frac{0.6\times 10^{-2}}{4\pi\times 10^{-7}(1)\times 2\times 10^{-4}} = \underline{2.387\text{ x }10^7\text{ A.t/m}}$$

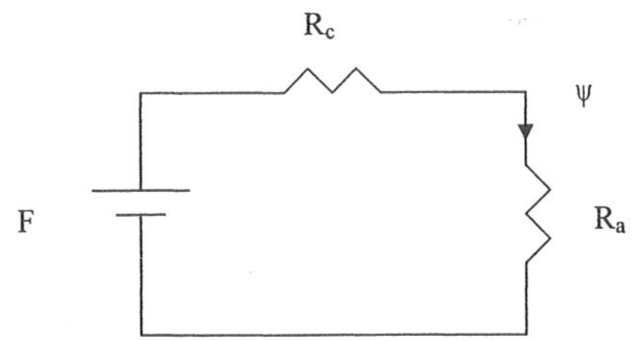
$R = R_a + R_c = 2.502 \text{ x } 10^7$ A.t/m

$$\psi = \frac{\Im}{R} = \psi_a = \psi_c = \frac{20,000}{2.502\times 10^7} = \underline{8\text{ x }10^{-4}\text{ Wb/m}^2}$$

$$\Im_a = \frac{R_a}{R_a + R_c}\Im = \frac{2.387\times 20,000}{2.502} = \underline{\underline{19,081}}\text{ A.t}$$

$$\Im_c = \frac{R_c}{R_a + R_c}\Im = \frac{0.115\times 20,000}{2.502} = \underline{\underline{919}}\text{ A.t}$$

Prob. 8.47



F = NI = 500 x 0.2 = 100 A.t

$$R_c = \frac{l_c}{\mu S} = \frac{42\times10^{-2}}{4\pi\times10^{-7}\times10^{3}\times4\times10^{-4}} = \frac{42\times10^{6}}{16\pi}$$

$$R_a = \frac{l_a}{\mu_o S} = \frac{10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}} = \frac{10^{8}}{16\pi}$$

$$R_a + R_c = \frac{1.42\times10^{8}}{16\pi}$$

$$\psi = \frac{F}{R_a + R_c} = \frac{16\pi\times100}{1.42\times10^{8}} = \frac{16\pi}{1.42}\ \mu\text{Wb}$$

$$B_a = \frac{\psi}{S} = \frac{16\pi\times10^{-6}}{1.42\times4\times10^{-4}} = \underline{\underline{88.5\ \text{mWb/m}^2}}$$

Prob. 8.48

$$NI = \Psi R = \Psi\frac{l}{\mu S} = \frac{2.56\times10^{-3}\times2.5\times10^{-3}}{4\pi\times10^{-7}\times40\times60\times10^{-6}} = \underline{\underline{2122.1\ \text{A.t}}}$$

Prob. 8.49

Area of cross section A = 0.00015m^2

Mmf = NI = 500(2mA) = 1 At. Mean radius = $\overline{R} = 5.5\times10^{-2}\,m$

Reluctance = $R = \frac{NI}{\psi} = \frac{l}{\mu A} = \frac{2\pi\overline{R}}{\mu A}$ =83.3333 At/Weber

Solving, $\mu = 27.646$ H/m

Magnetic flux density $B = \frac{\psi}{A} = 80\ Tesla$

Prob. 8.50

$$F = \frac{B^2 S}{2\mu_o} = \frac{\psi^2}{2\mu_o S} = \frac{4\times10^{-6}}{2\times4\pi\times10^{-7}\times0.3\times10^{-4}} = \underline{\underline{53.05}}\ \text{kN}$$

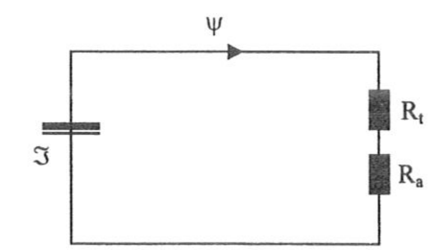
Prob. 8.51

(a) F = NI = 200 x 10^{-3} x 750 = 150 A.t.

$$R_a = \frac{l_a}{\mu_o S} = \frac{10^{-3}}{25\times10^{-6}\mu_o} = 3.183\times10^{7}$$

$$R_t = \frac{l_t}{\mu_o\mu_r S} = \frac{2\pi\times0.1}{\mu_o\times300\times25\times10^{-6}} = 6.7\text{ x }10^{7}$$

$$\psi = \frac{\Im}{R_a + R_t} = \frac{150}{10^{7}(3.183+20/3)} = 15.23\times10^{-7}$$



$$F = \frac{B^2 S}{2\mu_o} = \frac{\psi^2}{2\mu_o S} = \frac{2.32\times10^{-12}}{2\times4\pi\times10^{-7}\times25\times10^{-6}}$$

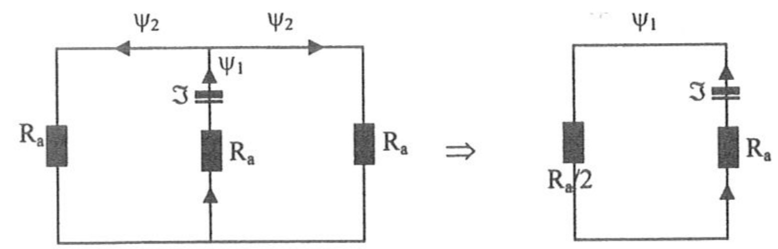
= $\underline{37\ \text{mN}}$

(b)If $\mu_t \to \infty$, $R_t = 0$, $\psi = \frac{\Im}{R_a} = \frac{150}{3.183\times10^{7}}$

$$F_2 = I_2 dl_2 \bullet B_1 = I_2 dl_2 \frac{\psi_1}{S} = \frac{2\times10^{-3}\times5\times10^{-3}\times150}{3.183\times10^{7}\times25\times10^{-6}}$$

F_2 = $\underline{1.885\ \mu\text{N}}$

Prob. 8.52



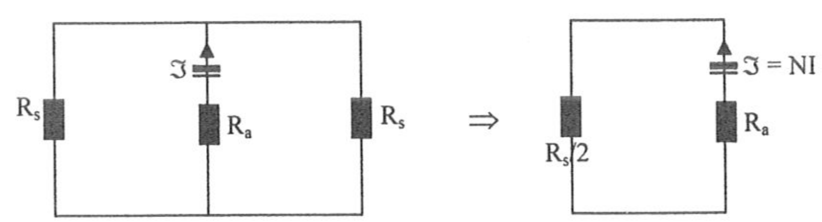
$$\psi_1 = 2\psi_2, \psi_1 = \frac{\Im}{\frac{3}{2}R_a} = \frac{2\Im}{3R_a} \to \psi_2 = \frac{\Im}{3R_a}$$

$$\Im = 2\left(\frac{\psi_2^{\,2}}{2\mu_o S}\right) + \frac{\psi_1}{2\mu_o S} = \frac{3\psi_1^{\,2}}{4\mu_o S} = \frac{\Im^2}{3R_a^{\,2}\mu_o S}$$

$$= \frac{\mu_o S\Im^2}{3l_a^{\,2}} = \frac{4\pi\times10^{-7}\times200\times10^{-4}\times9\times10^{6}}{3\times10^{-6}}$$

$$= 24\pi\times10^{3} = mg \to m = \frac{24\pi\times10^{3}}{9.8} = \underline{\underline{7694}} \text{ kg}$$

Prob. 8.53



Since $\mu \to \infty$ for the core (see Figure), $R_c = 0$.

$$\Im = NI = \psi\left(R_a + \frac{R_s}{2}\right) = \frac{\psi(a/2 + x)}{\mu_o S}$$

$$= \frac{\psi(2x+a)}{2\mu_o S}$$

$$\Im = \frac{B^2 S}{2\mu_o} = \psi^2\frac{1}{2\mu_o S} = \frac{1}{2\mu_o S}\bullet\frac{N^2I^2 4\mu_o^2 S^2}{(a+2x)^2}$$

$$= \frac{2N^2I^2\mu_o S}{(a+2x)^2}$$

$F = -F\boldsymbol{a}_x$ since the force is attractive, i.e.

$$F = \frac{-2N^2I^2\mu_o S\boldsymbol{a}_x}{(a+2x)^2}$$

CHAPTER 9

P.E. 9.1

(a) $V_{emf} = \int(\boldsymbol{u}\times\boldsymbol{B})\cdot d\boldsymbol{l} = uBl = 8(0.5)(0.1) = \underline{\underline{0.4}}$ V

(b) $I = \frac{V_{emf}}{R} = \frac{0.4}{20} = \underline{\underline{20}}$ mA

(c) $F_m = I\boldsymbol{l}\times\boldsymbol{B} = 0.02(-0.1\boldsymbol{a}_y\times0.5\boldsymbol{a}_z) = \underline{\underline{-\boldsymbol{a}_x}}$ mN

(d) $P = FU = I^2R = 8$ mW

or $\quad P = \frac{V_{emf}}{R} = \frac{(0.4)^2}{20} = \underline{\underline{8}}$ mW

P.E. 9.2

(a) $V_{emf} = \int(\boldsymbol{u}\times\boldsymbol{B})\cdot d\boldsymbol{l}$

where $\boldsymbol{B} = B_o\boldsymbol{a}_y = B_o(\sin\phi\boldsymbol{a}_\rho + \cos\phi\boldsymbol{a}_\phi)$, $B_o = 0.05$ Wb/m^2

$$(\boldsymbol{u}\times\boldsymbol{B})\cdot d\boldsymbol{l} = -\rho\omega B_o\sin\phi dz = -0.2\pi\sin\left(\omega t + \pi/2\right)dz$$

$$V_{emf} = \int_0^{0.03}(\boldsymbol{u}\times\boldsymbol{B})\cdot d\boldsymbol{l} = -6\pi\cos(100\pi t) \text{ mV}$$

At t = 1ms,

$$V_{emf} = -6\pi\cos 0.1\pi = \underline{\underline{-17.93}} \text{ mV}$$

$$i = \frac{V_{emf}}{R} = -60\pi\cos(100\pi t) \text{ mA}$$

At t = 3ms, $i = -60\pi\cos 0.3\pi = \underline{\underline{-110.8 \text{ mA}}}$

(b) Method 1:

$$\Psi = \int\boldsymbol{B}\cdot d\boldsymbol{S} = \int B_o t(\cos\phi\boldsymbol{a}_\rho - \sin\phi\boldsymbol{a}_\phi)\cdot d\rho dz\boldsymbol{a}_\phi = -\int_0^{\rho_o}\int_0^{z_o} B_o t\sin\phi d\rho dz = -B_o\rho_o z_o t\sin\phi$$

where $B_o = 0.02$, $\rho_o = 0.04$, $z_o = 0.03$

$$\phi = \omega t + \pi/2$$

$$\Psi = -B_o\rho_o z_o t\cos\omega t$$

$$V_{emf} = -\frac{\partial\Psi}{\partial t} = B_o\rho_o z_o\cos\omega t - B_o\rho_o z_o t\omega\sin\omega t$$