5.5 Both ε_o and χ_e are dimensionless.

(a) True (b) False

5.6 If $\nabla \cdot \mathbf{D} = \varepsilon\nabla \cdot \mathbf{E}$ and $\nabla \cdot \mathbf{J} = \sigma\nabla \cdot \mathbf{E}$ in a given material, the material is said to be

(a) Linear
(b) Homogeneous
(c) Isotropic
(d) Linear and homogeneous
(e) Linear and isotropic
(f) Isotropic and homogeneous

5.7 The relaxation time of mica ($\sigma = 10^{-15}$ S/m, $\varepsilon_r = 6$) is

(a) 5×10^{-10} s
(b) 10^{-6} s
(c) 5 hrs
(d) 10 hrs
(e) 15 hrs

5.8 The uniform fields shown in Figure 5.17 are near a dielectric–dielectric boundary but on opposite sides of it. Which configurations are correct? Assume that the boundary is charge free and that $\varepsilon_2 > \varepsilon_1$.

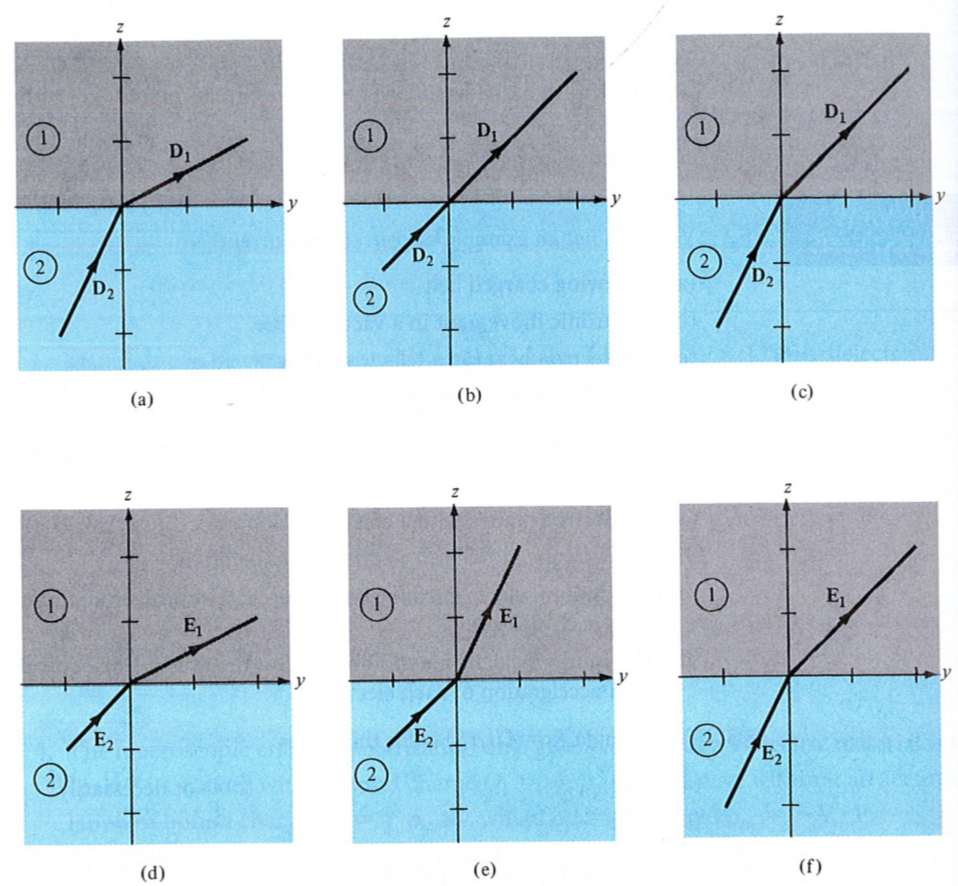


FIGURE 5.17 For Review Question 5.8.

5.9 Which of the following statements are incorrect?

(a) The conductivities of conductors and insulators vary with temperature and frequency.
(b) A conductor is an equipotential body in steady state, and **E** is always tangential to the conductor.
(c) Nonpolar molecules have no permanent dipoles.
(d) In a linear dielectric, P varies linearly with E.

5.10 The electric conditions (charge and potential) inside and outside an electric screening are completely independent of one another.

(a) True (b) False

Answers: 5.1d, 5.2c, 5.3c, 5.4d, 5.5b, 5.6d, 5.7e, 5.8e, 5.9b, 5.10a.

PROBLEMS

Section 5.3—Convection and Conduction Currents

5.1 Find the current crossing $x = 0, 0 \leq y \leq \pi/3, -1 \leq z \leq 4$ if $\mathbf{J} = 50 \sin 4y\mathbf{a}_x$ A/m^2.

5.2 In a certain region, $\mathbf{J} = 3r^2 \cos\theta\mathbf{a}_r - r^2 \sin\theta\mathbf{a}_\theta$ A/m^2.

(a) Find the current crossing the surface defined by $\theta = 30°, 0 < \phi < 2\pi, 0 < r < 2$m.
(b) Calculate the current through surface $r = 2, 0 < \theta < 30°, 0 < \phi < 2\pi$.

5.3 Given that $\mathbf{J} = \dfrac{10}{\rho} \sin\phi\,\mathbf{a}_\rho$ A/m^2, determine the current flowing through the surface $\rho = 2, 0 < \phi < \pi, 0 < z < 5$m.

5.4 The current density in a cylindrical conductor of radius a placed along the z-axis is

$$\mathbf{J} = 10e^{-(1-\rho/a)}\mathbf{a}_z \text{ A/m}^2$$

Find the current through the cross section of the conductor.

5.5 The charge $10^{-4}e^{-3t}$ C is removed from a sphere through a wire. Find the current in the wire at $t = 0$ and $t = 2.5$ s.

Section 5.4—Conductors

5.6 A 1 MΩ resistor is formed by a cylinder of graphite–clay mixture having a length of 2 cm and a radius of 4 mm. Determine the conductivity of the resistor.

5.7 If the ends of a cylindrical bar of carbon ($\sigma = 3 \times 10^4$ S/m) of radius 5 mm and length 8 cm are maintained at a potential difference of 9 V, find (a) the resistance of the bar, (b) the current through the bar, (c) the power dissipated in the bar.

5.8 A conducting wire is 2 mm in radius and 100 m in length. When a dc voltage of 9 V is applied to the wire, it results in a current of 0.3 A. Find: (a) the **E**-field in the wire, (b) the conductivity of the wire.

5.9 Two wires have the same diameter and same resistance. If one is made of copper, and the other is of silver, which wire is longer?

5.10 A 50-m long brass wire dissipates an average power of 2 kW when 120 V rms at 60 Hz is applied. If $\sigma = 1.5 \times 10^7$ S/m for brass, find the radius of the wire.

5.11 A coil is made of 150 turns of copper wire wound on a cylindrical core. If the mean radius of the turns is 6.5 mm and the diameter of the wire is 0.4 mm, calculate the resistance of the coil.

5.12 A composite conductor 10 m long consists of an inner core of steel of radius 1.5 cm and an outer sheath of copper whose thickness is 0.5 cm. Take the resistivities of copper and steel as 1.77×10^{-8} and 11.8×10^{-8} $\Omega \cdot$ m, respectively.

(a) Determine the resistance of the conductor.

(b) If the total current in the conductor is 60 A, what current flows in each metal?

(c) Find the resistance of a solid copper conductor of the same length and cross-sectional areas as the sheath.

5.13 A hollow cylinder of length 2 m has its cross section as shown in Figure 5.18. If the cylinder is made of carbon ($\sigma = 3 \times 10^4$ S/m), determine the resistance between the ends of the cylinder. Take $a = 3$ cm, $b = 5$ cm.

Sections 5.5–5.7—Polarization and Dielectric Constant

5.14 At a particular temperature and pressure, a helium gas contains 5×10^{25} atoms/m³. If a 10 kV/m field applied to the gas causes an average electron cloud shift of 10^{-18} m, find the dielectric constant of helium.

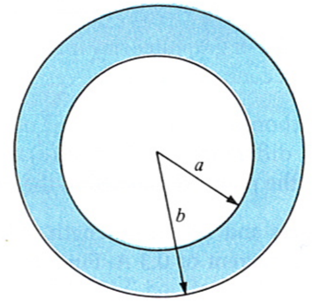
5.15 A dielectric material contains 2×10^{19} polar molecules/m³, each of dipole moment 1.8×10^{-27} C · m. Assuming that all the dipoles are aligned in the direction of the electric field $\mathbf{E} = 10^5\mathbf{a}_x$ V/m, find **P** and ε_r.

5.16 A 10-mC point charge is embedded in wood, which has $\varepsilon = 4.0$. Assuming that the charge is located at the origin, find **P** at $r = 1$ m.

5.17 In a certain dielectric for which $\varepsilon_r = 3.5$, given that $\mathbf{P} = \dfrac{100}{\rho}\mathbf{a}_\rho$ nC/m², find **E** and **D** at $\rho = 2$m.

5.18 In a slab of Teflon ($\varepsilon = 2.1\,\varepsilon_o$), $\mathbf{E} = 6\mathbf{a}_x + 12\mathbf{a}_y - 20\mathbf{a}_z$ V/m, find **D** and **P**.

FIGURE 5.18 For Problems 5.13 and 5.23.



5.19 In a dielectric material ($\varepsilon = 5\varepsilon_o$), the potential field $V = 10x^2yz - 5z^2$ V, determine (a) **E**, (b) **D**, (c) **P**, (d) ρ_v.

5.20 A point charge Q is located at the center of a spherical dielectric ($\varepsilon = \varepsilon_r\varepsilon_o$) shell of inner radius a and outer radius b. Determine **E**, **D**, **P**, and V.

5.21 Determine the polarization **P** in a dielectric material with $\varepsilon_r = 2.4$ and $\mathbf{D} = 450\,\mathbf{a}_x$ nC/m².

5.22 If $\mathbf{D} = 350$ nC/m² and $\varepsilon_r = 6.4$, find the magnitudes of χ_e, **E**, and **P**.

5.23 Consider Figure 5.18 as a spherical dielectric shell so that $\varepsilon = \varepsilon_o\varepsilon_r$ for $a < r < b$ and $\varepsilon = \varepsilon_o$ for $0 < r < a$. If a charge Q is placed at the center of the shell, find

(a) **P** for $a < r < b$

(b) ρ_{pv} for $a < r < b$

(c) ρ_{ps} at $r = a$ and $r = b$

5.24 Two point charges in free space are separated by distance d and exert a force 2.6 nN on each other. The force becomes 1.5 nN when the free space is replaced by a homogeneous dielectric material. Calculate the dielectric constant of the material.

5.25 The electric flux density is five times the polarization in a certain material. Determine the dielectric constant of the material.

5.26 A solid sphere of radius a and dielectric constant ε_r has a uniform volume charge density of ρ_o.

(a) At the center of the sphere, show that

$$V = \frac{\rho_o a^2}{6\varepsilon_o\varepsilon_r}(2\varepsilon_r + 1)$$

(b) Find the potential at the surface of the sphere.

5.27 In an anisotropic medium, **D** is related to **E** as

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_o \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Find **D** due to $\mathbf{E} = E_o(\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z)$ V/m.

Section 5.8—Continuity Equation and Relaxation Time

5.28 For static (time-independent) fields, which of the following current densities are possible?

(a) $\mathbf{J} = 2x^3y\mathbf{a}_x + 4x^2z^2\mathbf{a}_y - 6x^2yz\mathbf{a}_z$ (b) $\mathbf{J} = xy\mathbf{a}_x + y(z+1)\mathbf{a}_y + 2y\mathbf{a}_z$

(c) $\mathbf{J} = \dfrac{z^2}{\rho}\mathbf{a}_\rho + z\cos\phi\,\mathbf{a}_z$ (d) $\mathbf{J} = \dfrac{\sin\theta}{r^2}\mathbf{a}_r$

5.29 If $\mathbf{J} = \frac{100}{\rho^2}\mathbf{a}_\rho$ A/m², find (a) the time rate of increase in the volume charge density, (b) the total current passing through surface defined by $\rho = 2, 0 < z < 1, 0 < \phi < 2\pi$.

5.30 Given that $\mathbf{J} = \frac{5e^{-10^4 t}}{r}\mathbf{a}_r$ A/m², at $t = 0.1$ ms, find (a) the current passing surface $r = 2$ m, (b) the charge density ρ_v on that surface.

5.31 Determine the relaxation time for each of the following media:

(a) Hard rubber ($\sigma = 10^{-15}$ S/m, $\varepsilon = 3.1\varepsilon_o$)
(b) Mica ($\sigma = 10^{-15}$ S/m, $\varepsilon = 6\varepsilon_o$)
(c) Distilled water ($\sigma = 10^{-4}$ S/m, $\varepsilon = 80\varepsilon_o$)

5.32 In a certain medium, the volume current density is $\mathbf{J} = e^{-z}\cos wz\,\mathbf{a}_z$ A/m². Determine the rate of change of the volume charge density in the medium.

5.33 Lightning strikes a dielectric sphere of radius 20 mm for which $\varepsilon_r = 2.5$, $\sigma = 5 \times 10^{-6}$ S/m and deposits uniformly a charge of 1 C. Determine the initial volume charge density and the volume charge density 2 μs later.

Section 5.9—Boundary Conditions

5.34 The plane $z = 0$ separates region 1 ($z > 0$), which is a dielectric material with $\varepsilon_r = 4$, from region 2 ($z < 0$), which is also a dielectric material with $\varepsilon_r = 6.5$. If $\mathbf{D}_1 = 16\mathbf{a}_x + 30\mathbf{a}_y - 20\mathbf{a}_z$ nC/m², find $\mathbf{P}_1$ and $\mathbf{D}_2$.

5.35 If $\mathbf{D} = 4\mathbf{a}_x - 6\mathbf{a}_y + 8\mathbf{a}_z$ nC/m² in a region $y > 0$ where $\varepsilon = 2.5\varepsilon_o$, find **D** and **E** in region $y < 0$ where $\varepsilon = 8.1\varepsilon_o$.

5.36 A dielectric interface is defined by $4x + 3y = 10$ m. The region including the origin is free space, where $\mathbf{D}_1 = 2\mathbf{a}_x - 4\mathbf{a}_y + 6.5\mathbf{a}_z$ nC/m². In the other region, $\varepsilon_{r2} = 2.5$. Find $\mathbf{D}_2$ and the angle θ_2 that $\mathbf{D}_2$ makes with the normal.

5.37 Given that $\mathbf{E}_1 = 10\mathbf{a}_x - 6\mathbf{a}_y + 12\mathbf{a}_z$ V/m in Figure 5.19, find (a) $\mathbf{P}_1$, (b) $\mathbf{E}_2$ and the angle $\mathbf{E}_2$ makes with the y-axis, (c) the energy density in each region.

5.38 Two homogeneous dielectric regions 1 ($\rho \leq 4$ cm) and 2 ($\rho \geq 4$ cm) have dielectric constants 3.5 and 1.5, respectively. If $\mathbf{D}_2 = 12\mathbf{a}_\rho - 6\mathbf{a}_\phi + 9\mathbf{a}_z$ nC/m², calculate (a) $\mathbf{E}_1$ and $\mathbf{D}_1$, (b) $\mathbf{P}_2$ and ρ_{pv2}, (c) the energy density for each region.

5.39 A conducting sphere of radius a is half-embedded in a liquid dielectric medium of permittivity ε_1 as in Figure 5.20. The region above the liquid is a gas of permittivity ε_2. If the total free charge on the sphere is Q, determine the electric field intensity everywhere.

***5.40** Two parallel sheets of glass ($\varepsilon_r = 8.5$) mounted vertically are separated by a uniform air gap between their inner surface. The sheets, properly sealed, are immersed in oil ($\varepsilon_r = 3.0$) as shown in Figure 5.21. A uniform electric field of strength 2 kV/m in the horizontal direction exists in the oil. Calculate the magnitude and direction of the electric field in the glass and in the enclosed air gap when (a) the field is normal to the glass surfaces and (b) the field in the oil makes an angle of 75° with a normal to the glass surfaces. Ignore edge effects.

5.41 (a) Given that $\mathbf{E} = 15\mathbf{a}_x - 8\mathbf{a}_z$ V/m at a point on a conductor surface, what is the surface charge density at that point? Assume $\varepsilon = \varepsilon_o$.

(b) Region $y \geq 2$ is occupied by a conductor. If the surface charge on the conductor is -20 nC/m², find **D** just outside the conductor.

5.42 Two planar slabs of equal thickness but with different dielectric constants are shown in Figure 5.22. $\mathbf{E}_o$ in air makes an angle of 30° with the z-axis. Calculate the angle E that makes with the z-axis in each of the two dielectric layers.

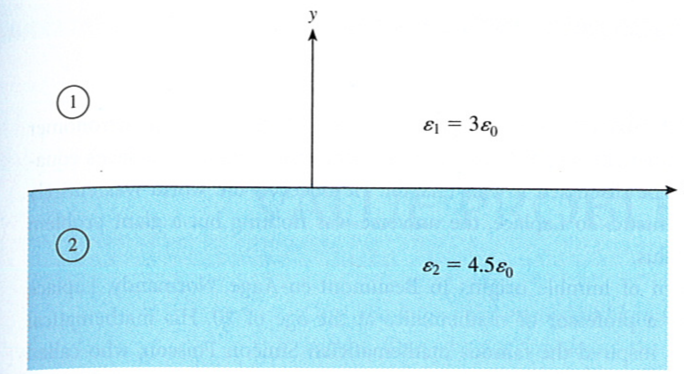


FIGURE 5.19 For Problem 5.37.

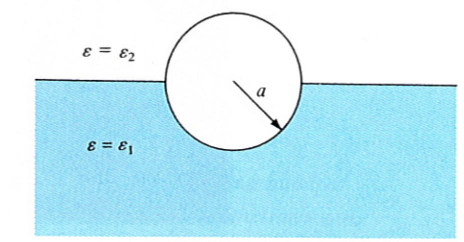


FIGURE 5.20 For Problem 5.39.

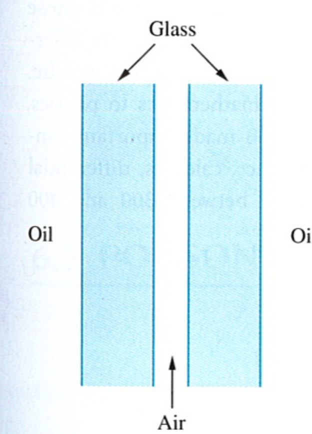


FIGURE 5.21 For Problem 5.40.

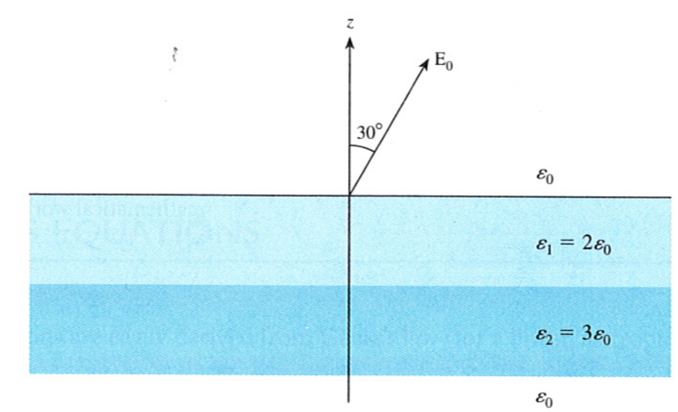


FIGURE 5.22 For Problem 5.42.