(d) Atmospheric pressure in a given region

(e) Humidity of a city

1.3 Of the rectangular coordinate systems shown in Figure 1.13, which are not are right-handed?

1.4 Which of these is correct?

(a) $\mathbf{A} \times \mathbf{A} = |\mathbf{A}|^2$

(b) $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} = 0$

(c) $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \cdot \mathbf{A}$

(d) $\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_z$

(e) $\mathbf{a}_k = \mathbf{a}_x - \mathbf{a}_y$, where $\mathbf{a}_k$ is a unit vector

1.5 Which of the following identities is not valid?

(a) $\mathbf{a}(\mathbf{b} + \mathbf{c}) = \mathbf{ab} + \mathbf{bc}$

(b) $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

(c) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

(d) $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = -\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})$

(e) $\mathbf{a}_A \cdot \mathbf{a}_B = \cos \theta_{AB}$

1.6 Which of the following statements are meaningless?

(a) $\mathbf{A} \cdot \mathbf{B} + 2\mathbf{A} = 0$

(b) $\mathbf{A} \cdot \mathbf{B} + 5 = 2\mathbf{A}$

(c) $\mathbf{A}(\mathbf{A} + \mathbf{B}) + 2 = 0$

(d) $\mathbf{A} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B} = 0$

1.7 Let $\mathbf{F} = 2\mathbf{a}_x - 6\mathbf{a}_y + 10\mathbf{a}_z$ and $\mathbf{G} = \mathbf{a}_x + G_y\mathbf{a}_y + 5\mathbf{a}_z$. If **F** and **G** have the same unit vector, G_y is

(a) 6

(b) −3

(c) 0

(d) 6

1.8 Given that $\mathbf{A} = \mathbf{a}_x + \alpha\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = \alpha\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, if **A and B** are normal to each other, α is

(a) −2

(b) −1/2

(c) 0

(d) 1

(e) 2

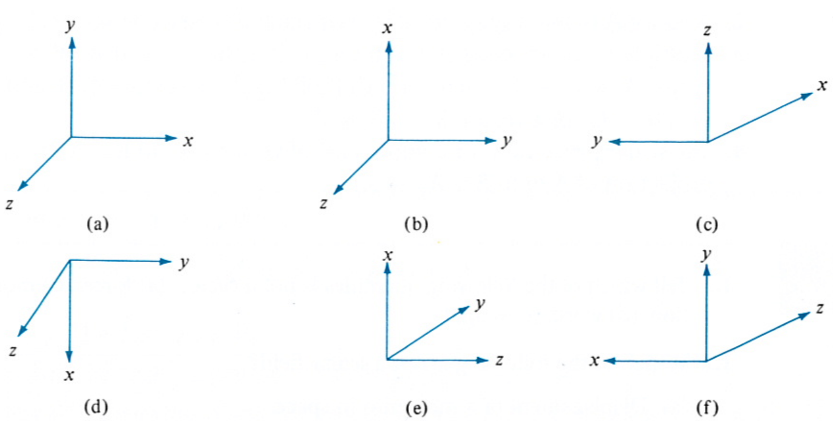

FIGURE 1.13 For Review Question 1.3.

1.9 The component of $6\mathbf{a}_x + 2\mathbf{a}_y - 3\mathbf{a}_z$ along $3\mathbf{a}_x - 4\mathbf{a}_y$ is

(a) $-12\mathbf{a}_x - 9\mathbf{a}_y - 3\mathbf{a}_z$

(b) $30\mathbf{a}_x - 40\mathbf{a}_y$

(c) 10/7

(d) 2

(e) 10

1.10 Given $\mathbf{A} = -6\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$, the projection of **A along** $\mathbf{a}_y$ is

(a) −12

(b) −4

(c) 3

(d) 7

(e) 12

Answers: 1.1d, 1.2a, 1.3b,e, 1.4b, 1.5a, 1.6a,b,c, 1.7b, 1.8b, 1.9d, 1.10c.

PROBLEMS

Section 1.4—Unit Vector

1.1 Find the unit vector along the line joining point (2, 4, 4) to point (−3, 2, 2).

1.2 Determine the unit vector along the direction *OP*, where *O* is the origin and *P* is point (4, −5, 1).

Sections 1.5–1.7—Vector Addition, Subtraction, and Multiplication

1.3 Given vectors $\mathbf{A} = 2\mathbf{a}_x + 5\mathbf{a}_z$ and $\mathbf{B} = \mathbf{a}_x - 3\mathbf{a}_y + 4\mathbf{a}_z$, find $|\mathbf{A} \times \mathbf{B}| + \mathbf{A} \cdot \mathbf{B}$

1.4 The position vectors of points *M* and *N* are $\mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$ and $3\mathbf{a}_x + 5\mathbf{a}_y - \mathbf{a}_z$, respectively. Find the distance vector from *M* to *N*.

1.5 If $\mathbf{A} = 4\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 12\mathbf{a}_x + 18\mathbf{a}_y - 8\mathbf{a}_z$, determine:

(a) $\mathrm{A} - 3\mathrm{B}$

(b) $(2\mathrm{A} + 5\mathbf{B})/|\mathbf{B}|$

(c) $\mathbf{a}_x \times \mathbf{A}$

(d) $(B \times \mathbf{a}_x) \cdot \mathbf{a}_y$

1.6 Let $\mathbf{A} = \mathbf{a}_x - \mathbf{a}_z$, $\mathbf{B} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, $\mathbf{C} = \mathbf{a}_y + 2\mathbf{a}_z$, find:

(a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

(b) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$

(c) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$

(d) $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$

1.7 If the position vectors of points *T* and *S* are $3\mathbf{a}_x - 2\mathbf{a}_y + a_z$ and $4\mathbf{a}_x + 6\mathbf{a}_y + 2\mathbf{a}_x$, respectively, find (a) coordinates of *T* and *S*, (b) the distance vector from *T* to *S*, (c) the distance between *T* and *S*.

1.8 Let $\mathbf{A} = \alpha\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z$ and $\mathbf{B} = 4\mathbf{a}_x + \beta\mathbf{a}_y + 8\mathbf{a}_z$.

(a) Find α and β if **A** and **B** are parallel.

(b) Determine the relationship between α and β if **B** is perpendicular to **A**.

1.9 (a) Show that

$$(\mathbf{A} \cdot \mathbf{B})^2 + |\mathbf{A} \times \mathbf{B}|^2 = (AB)^2$$

(b) Show that

$$\mathbf{a}_x = \frac{\mathbf{a}_y \times \mathbf{a}_z}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \quad \mathbf{a}_y = \frac{\mathbf{a}_z \times \mathbf{a}_x}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \quad \mathbf{a}_z = \frac{\mathbf{a}_x \times \mathbf{a}_y}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}$$

1.10 Given that

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z$$
$$\mathbf{Q} = 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{R} = -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

find (a) $|\mathbf{P} + \mathbf{Q} - \mathbf{R}|$, (b) $\mathbf{P} \cdot \mathbf{Q} \times \mathbf{R}$, (c) $\mathbf{Q} \times \mathbf{P} \cdot \mathbf{R}$, (d) $(\mathbf{P} \times \mathbf{Q}) \cdot (\mathbf{Q} \times \mathbf{R})$, (e) $(\mathbf{P} \times \mathbf{Q}) \times (\mathbf{Q} \times \mathbf{R})$, (f) $\cos \theta_{PR}$, (g) $\sin \theta_{PQ}$.

1.11 If $\mathbf{A} = 4\mathbf{a}_x - 6\mathbf{a}_y + \mathbf{a}_z$ and $B = 2\mathbf{a}_x + 5\mathbf{a}_z$, find:

(a) A * B + 2|B|^2

(b) a unit vector perpendicular to both A and B

1.12 Determine the dot product, cross product, and angle between

$$\mathbf{P} = 2\mathbf{a}_x - 6\mathbf{a}_y + 5\mathbf{a}_z \quad \text{and} \quad \mathbf{Q} = 3\mathbf{a}_y + \mathbf{a}_z$$

1.13 Show that vectors $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -2\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z$ are parallel.

1.14 Simplify the following expressions:

(a) $\mathbf{A} \times (\mathbf{A} \times \mathbf{B})$

(b) $\mathbf{A} \times [\mathbf{A} \times (\mathbf{A} \times \mathbf{B})]$

1.15 Show that the dot and cross in the triple scalar product may be interchanged, that is, $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.

1.16 Points $P_1(1, 2, 3)$, $P_2(-5, 2, 0)$, and $P_3(2, 7, -3)$ form a triangle in space. (a) Calculate the area of the triangle. (b) Find the three angles of the triangle.

1.17 Points P, Q, and R are located at $(-1, 4, 8)$, $(2, -1, 3)$, and $(-1, 2, 3)$, respectively. Determine (a) the distance between P and Q, (b) the distance vector from P to R, (c) the angle between QP and QR, (d) the area of triangle PQR, (e) the perimeter of triangle PQR.

1.18 Two points $P(2, 4, -1)$ and $Q(12, 16, 9)$ form a straight line. Calculate the time taken for a sonar signal traveling at 300 m/s to get from the origin to the midpoint of PQ.

***1.19** (a) Prove that $\mathbf{P} = \cos\theta_1 \mathbf{a}_x + \sin\theta_1 \mathbf{a}_y$ and $\mathbf{Q} = \cos\theta_2 \mathbf{a}_x + \sin\theta_2 \mathbf{a}_y$ are unit vectors in the xy-plane, respectively, making angles θ_1 and θ_2 with the x-axis.

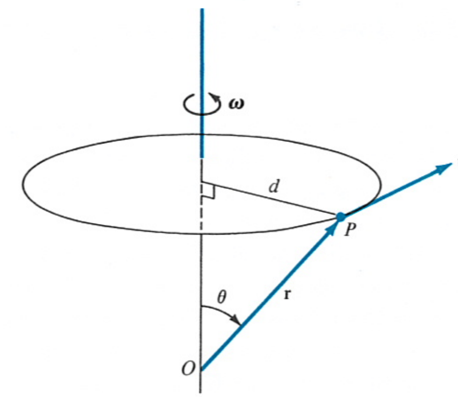

FIGURE 1.14 For Problem 1.20.

(b) By means of dot product, obtain the formula for $\cos(\theta_2 - \theta_1)$. By similarly formulating **P** and **Q**, obtain the formula for $\cos(\theta_2 + \theta_1)$.

(c) If θ is the angle between P and Q, find $\frac{1}{2}|\mathbf{P} - \mathbf{Q}|$ in terms of θ.

1.20 Consider a rigid body rotating with a constant angular velocity ω radians per second about a fixed axis through O as in Figure 1.14. Let $\mathbf{r}$ be the distance vector from O to P, the position of a particle in the body. The magnitude of the velocity $\mathbf{u}$ of the body at P is $|\mathbf{u}| = d\omega = |\mathbf{r}| \sin\theta\, |\boldsymbol{\omega}|$ or $\mathbf{u} = \boldsymbol{\omega} \times \mathbf{r}$. If the rigid body is rotating at 3 rad/s about an axis parallel to $\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$ and passing through point $(2, -3, 1)$, determine the velocity of the body at $(1, 3, 4)$.

1.21 Given vectors $\mathbf{T} = 2\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{S} = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, find (a) the scalar projection of **T** on **S**, (b) the vector projection of **S** on **T**, (c) the smaller angle between **T** and **S**.

Section 1.8—Components of a Vector

1.22 If $\mathbf{H} = 2xy\mathbf{a}_x - (x + z)\mathbf{a}_y + z^2\mathbf{a}_z$, find:

(a) A unit vector parallel to **H** at $P(1, 3, -2)$

(b) The equation of the surface on which $|\mathbf{H}| = 10$

1.23 Let $\mathbf{A} = -3\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z$, $\mathbf{B} = 2\mathbf{a}_x - 5\mathbf{a}_y + \mathbf{a}_z$, $\mathbf{C} = \mathbf{a}_y + 4\mathbf{a}_z$. Determine:

(a) the minimum angle between **A** and **B**

(b) the component of **A** along **C**

(c) $\mathbf{D} = \mathbf{A} + 2\mathbf{B} - 3\mathbf{C}$

(d) $(\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})$

1.24 Let $\mathbf{A} = 2x\mathbf{a}_x + y\mathbf{a}_y - z^2\mathbf{a}_z$ and $\mathbf{B} = 3x^2\mathbf{a}_x + 6\mathbf{a}_y + \mathbf{a}_z$. At point $(1, 2, -4)$, (a) calculate $\mathbf{A} \cdot \mathbf{B}$, (b) determine the angle between **A** and **B**, (c) find the vector component of **A** on **B**.

1.25 Determine the scalar component of vector $\mathbf{H} = y\mathbf{a}_x - x\mathbf{a}_z$ at point $P(1, 0, 3)$ that is directed toward point $Q(-2, 1, 4)$.

1.26 **E** and **F** are vector fields given by $\mathbf{E} = 2x\mathbf{a}_x + \mathbf{a}_y + yz\mathbf{a}_z$ and $\mathbf{F} = xy\mathbf{a}_x - y^2\mathbf{a}_y + xyz\mathbf{a}_z$. Determine:

(a) $|\mathbf{E}|$ at (1, 2, 3)

(b) The component of **E** along **F** at (1, 2, 3)