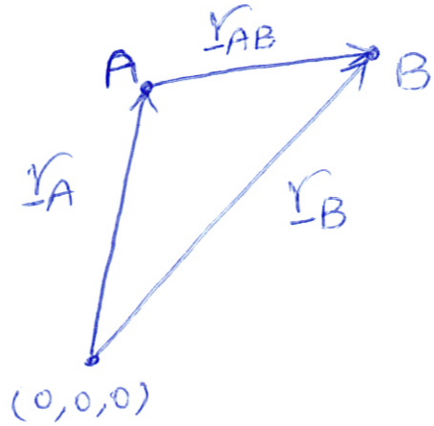
1.1 unit vector along the line joining point (2,4,4) to point (−3,2,2) = ?

A = (2,4,4), B = (−3,2,2)

$$\underline{a}_{AB} = \frac{\underline{r}_{AB}}{|\underline{r}_{AB}|}$$



$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A$$

$$\begin{cases} \underline{r}_A = 2\,\underline{a}_x + 4\,\underline{a}_y + 4\,\underline{a}_z \\ \underline{r}_B = -3\,\underline{a}_x + 2\,\underline{a}_y + 2\,\underline{a}_z \end{cases}$$

$$\underline{r}_{AB} = (-3-2)\,\underline{a}_x + (2-4)\,\underline{a}_y + (2-4)\,\underline{a}_z$$

$$= -5\,\underline{a}_x - 2\,\underline{a}_y - 2\,\underline{a}_z$$

$$|\underline{r}_{AB}| = \sqrt{(-5)^2 + (-2)^2 + (-2)^2} = \sqrt{25+4+4} = \sqrt{33}$$

$$= 5.7446$$

$$\underline{a}_{AB} = \frac{-5\,\underline{a}_x - 2\,\underline{a}_y - 2\,\underline{a}_z}{\sqrt{33}}$$

$$= -0.8704\,\underline{a}_x - 0.3482\,\underline{a}_y - 0.3482\,\underline{a}_z$$

1.3 $\underline{A} = 2\underline{a}_x + 5\underline{a}_z$

$\underline{B} = \underline{a}_x - 3\underline{a}_y + 4\underline{a}_z$

$|\underline{A} \times \underline{B}| + \underline{A} \cdot \underline{B} = ?$

$$\underline{A} \times \underline{B} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 2 & 0 & 5 \\ 1 & -3 & 4 \end{vmatrix} = (0-(-15))\underline{a}_x - (8-5)\underline{a}_y + (-6-0)\underline{a}_z$$

$$= 15\underline{a}_x - 3\underline{a}_y - 6\underline{a}_z$$

$$|\underline{A} \times \underline{B}| = \sqrt{15^2 + (-3)^2 + (-6)^2} = 16.4317$$

$$\underline{A} \cdot \underline{B} = 2 \times 1 + 0 \times (-3) + 5 \times 4 = 22$$

$$|\underline{A} \times \underline{B}| + \underline{A} \cdot \underline{B} = 16.4317 + 22 = 38.4317$$

1.6 $\underline{A} = \underline{a}_x - \underline{a}_z \qquad = \underline{a}_x + 0\underline{a}_y - \underline{a}_z$

$\underline{B} = \underline{a}_x + \underline{a}_y + \underline{a}_z = \underline{a}_x + \underline{a}_y + \underline{a}_z$

$\underline{C} = \underline{a}_y + 2\underline{a}_z \qquad = 0\underline{a}_x + \underline{a}_y + 2\underline{a}_z$

a) $\underline{A} \cdot (\underline{B} \times \underline{C}) = ?$

$$\underline{B} \times \underline{C} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (2-1)\underline{a}_x - (2-0)\underline{a}_y + (1-0)\underline{a}_z$$

$$= \underline{a}_x - 2\underline{a}_y + \underline{a}_z$$

$$\underline{A} \cdot (\underline{B} \times \underline{C}) = 1 \times 1 + 0 \times (-2) + (-1) \times 1 = 0$$

b) $(\underline{A} \times \underline{B}) \cdot \underline{C} = ?$

$$\underline{A} \times \underline{B} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix} = (0-(-1))\underline{a}_x - (1-(-1))\underline{a}_y + (1-0)\underline{a}_z$$

$$= \underline{a}_x - 2\underline{a}_y + \underline{a}_z$$

$$(\underline{A} \times \underline{B}) \cdot \underline{C} = 1 \times 0 + (-2) \times 1 + 1 \times 2 = 0$$

c) $\underline{A} \times (\underline{B} \times \underline{C}) = ?$

from part (a): $\underline{B} \times \underline{C} = \underline{a}_x - 2\underline{a}_y + \underline{a}_z$

$$\underline{A} \times (\underline{B} \times \underline{C}) = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{vmatrix} = (0-2)\underline{a}_x - (1-(-1))\underline{a}_y + (-2-0)\underline{a}_z$$

$$= -2\underline{a}_x - 2\underline{a}_y - 2\underline{a}_z$$

1.6

d) $(\underline{A} \times \underline{B}) \times \underline{C} = ?$

from part (b): $\underline{A} \times \underline{B} = \underline{a}_x - 2\underline{a}_y + \underline{a}_z$

$$(\underline{A} \times \underline{B}) \times \underline{C} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (-4-1)\underline{a}_x - (2-0)\underline{a}_y + (1-0)\underline{a}_z$$

$$= -5\underline{a}_x - 2\underline{a}_y + \underline{a}_z$$

1.9

a) $(\underline{A}\cdot\underline{B})^2 + |\underline{A}\times\underline{B}|^2 \overset{?}{=} (AB)^2 = |\underline{A}|^2|\underline{B}|^2$

$$\underline{A}\cdot\underline{B} = |\underline{A}||\underline{B}|\cos\theta_{AB}$$

$$|\underline{A}\times\underline{B}| = |\underline{A}||\underline{B}|\sin\theta_{AB}$$

$$(\underline{A}\cdot\underline{B})^2 + |\underline{A}\times\underline{B}|^2 = |\underline{A}|^2|\underline{B}|^2\cos^2\theta_{AB} + |\underline{A}|^2|\underline{B}|^2\sin^2\theta_{AB}$$

$$= |\underline{A}|^2|\underline{B}|^2\underbrace{(\cos^2\theta_{AB} + \sin^2\theta_{AB})}_{=1}$$

$$= |\underline{A}|^2|\underline{B}|^2$$

b) $\underline{a}_x \overset{?}{=} \dfrac{\underline{a}_y\times\underline{a}_z}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)}$ $\quad\underline{a}_y \overset{?}{=} \dfrac{\underline{a}_z\times\underline{a}_x}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)}$ $\quad\underline{a}_z \overset{?}{=} \dfrac{\underline{a}_x\times\underline{a}_y}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)}$

we know that: $\begin{cases}\underline{a}_y\times\underline{a}_z = \underline{a}_x \\ \underline{a}_z\times\underline{a}_x = \underline{a}_y \\ \underline{a}_x\times\underline{a}_y = \underline{a}_z\end{cases}$ & $\begin{cases}\underline{a}_x\cdot\underline{a}_x = 1 \\ \underline{a}_y\cdot\underline{a}_y = 1 \\ \underline{a}_z\cdot\underline{a}_z = 1\end{cases}$

Therefore, $\underline{a}_x\cdot\underbrace{(\underline{a}_y\times\underline{a}_z)}_{\underline{a}_x} = \underline{a}_x\cdot\underline{a}_x = 1$

Now, we can show that,

$$\begin{cases}\dfrac{\underline{a}_y\times\underline{a}_z}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)} = \dfrac{\underline{a}_x}{1} = \underline{a}_x \\[2ex] \dfrac{\underline{a}_z\times\underline{a}_x}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)} = \dfrac{\underline{a}_y}{1} = \underline{a}_y \\[2ex] \dfrac{\underline{a}_x\times\underline{a}_y}{\underline{a}_x\cdot(\underline{a}_y\times\underline{a}_z)} = \dfrac{\underline{a}_z}{1} = \underline{a}_z\end{cases}$$

1.11 $\underline{A} = 4\underline{a}_x - 6\underline{a}_y + \underline{a}_z = (4,-6,1)$

$\underline{B} = 2\underline{a}_x + 5\underline{a}_z = (2,0,5)$

a) $\underline{A} \cdot \underline{B} + 2|\underline{B}|^2 = ?$

$$\underline{A} \cdot \underline{B} = 4 \times 2 + (-6) \times 0 + 1 \times 5 = 13$$

$$|\underline{B}|^2 = (2)^2 + (0)^2 + (5)^2 = 29$$

$$\underline{A} \cdot \underline{B} + 2|\underline{B}|^2 = 13 + 2 \times 29 = 71$$

b) a unit vector perpendicular to both $\underline{A}$ and $\underline{B}$ = ?

We know that $\underline{A} \times \underline{B}$ is perpendicular to both $\underline{A}$ and $\underline{B}$. So, by dividing it by its magnitude, we can find this unit vector.

$$\underline{a}^{\perp} = \frac{\underline{A} \times \underline{B}}{|\underline{A} \times \underline{B}|}$$

$$\underline{A} \times \underline{B} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 4 & -6 & 1 \\ 2 & 0 & 5 \end{vmatrix} = (-30-0)\underline{a}_x - (20-2)\underline{a}_y + (0-(-12))\underline{a}_z$$

$$= -30\underline{a}_x - 18\underline{a}_y + 12\underline{a}_z$$

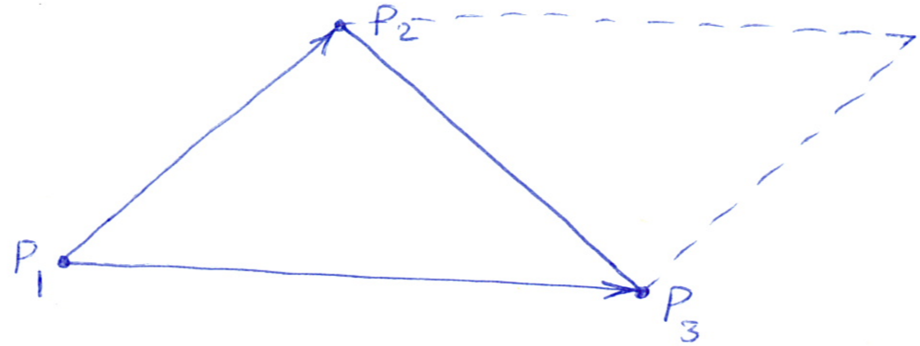
$$|\underline{A} \times \underline{B}| = \sqrt{(-30)^2 + (-18)^2 + (12)^2} = 36.9865$$

$$\underline{a}^{\perp} = \frac{\underline{A} \times \underline{B}}{|\underline{A} \times \underline{B}|} = \frac{-30\underline{a}_x - 18\underline{a}_y + 12\underline{a}_z}{36.9865}$$

$$= -0.8111\underline{a}_x - 0.4867\underline{a}_y + 0.3244\underline{a}_z$$

Also, we know if $\underline{a}^{\perp}$ is a unit vector perpendicular to both $\underline{A}$ and $\underline{B}$, then $-\underline{a}^{\perp}$ is a unit vector perpendicular to both $\underline{A}$ and $\underline{B}$ too.

1.16 $P_1 = (1,2,3)$ $P_2 = (-5,2,0)$ $P_3 = (2,7,-3)$



a) The area of the triangle = ?

$$= \frac{1}{2} \times (\text{Area of the parallelogram})$$

$$= \frac{1}{2} \times |\underline{r}_{P_1P_2} \times \underline{r}_{P_1P_3}|$$

$$\underline{r}_{P_1P_2} = \underline{r}_{P_2} - \underline{r}_{P_1} = (-5-1)\underline{a}_x + (2-2)\underline{a}_y + (0-3)\underline{a}_z$$

$$= -6\underline{a}_x + 0\,\underline{a}_y - 3\,\underline{a}_z$$

$$\underline{r}_{P_1P_3} = \underline{r}_{P_3} - \underline{r}_{P_1} = (2-1)\underline{a}_x + (7-2)\underline{a}_y + (-3-3)\underline{a}_z$$

$$= \underline{a}_x + 5\underline{a}_y - 6\underline{a}_z$$

$$\underline{r}_{P_1P_2} \times \underline{r}_{P_1P_3} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ -6 & 0 & -3 \\ 1 & 5 & -6 \end{vmatrix} = (0-(-15))\underline{a}_x - (36-(-3))\underline{a}_y + (-30-0)\underline{a}_z$$

$$= 15\underline{a}_x - 39\underline{a}_y - 30\,\underline{a}_z$$

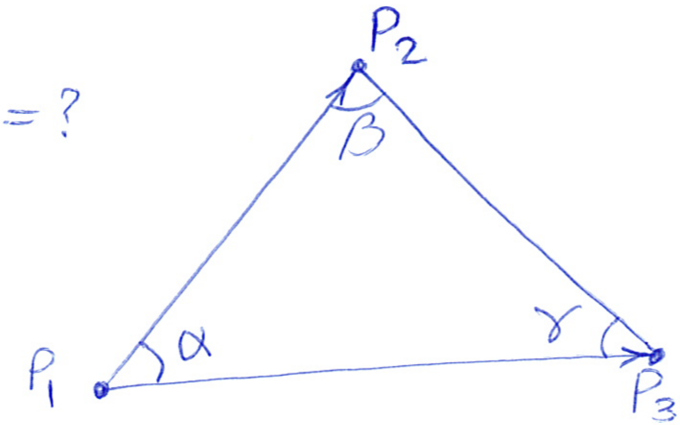
The area of the triangle $= \frac{1}{2}|\underline{r}_{P_1P_2} \times \underline{r}_{P_1P_3}|$

$$= \frac{1}{2} \times \sqrt{15^2 + (-39)^2 + (-30)^2}$$

$$= 25.7196$$

1.16

b) The three angles of the triangle = ?



We know that $\underline{A} \cdot \underline{B} = |\underline{A}||\underline{B}| \cos \theta$

so, $\theta = \cos^{-1}\left(\dfrac{\underline{A} \cdot \underline{B}}{|\underline{A}||\underline{B}|}\right)$

In this problem, we have,

$$\underline{r}_{P_1P_2} \cdot \underline{r}_{P_1P_3} = |\underline{r}_{P_1P_2}||\underline{r}_{P_1P_3}| \cos \alpha$$

$$\begin{cases} \underline{r}_{P_1P_2} = (-6, 0, -3) \quad \& \quad |\underline{r}_{P_1P_2}| = \sqrt{36+0+9} = 6.7082 \\ \underline{r}_{P_1P_3} = (1, 5, -6) \quad \& \quad |\underline{r}_{P_1P_3}| = \sqrt{1+25+36} = 7.8740 \end{cases}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{\underline{r}_{P_1P_2} \cdot \underline{r}_{P_1P_3}}{|\underline{r}_{P_1P_2}||\underline{r}_{P_1P_3}|}\right) = \cos^{-1}\left(\frac{(-6)\times 1 + 0\times 5 + (-3)\times(-6)}{6.7082 \times 7.8740}\right)$$

$$\alpha = \cos^{-1}(0.2272) = 76.8686^\circ$$
$$= 1.3416 \text{ rad.}$$

With the same argument, we can calculate β and γ.

$$\beta = \cos^{-1}\left(\frac{\underline{r}_{P_2P_1} \cdot \underline{r}_{P_2P_3}}{|\underline{r}_{P_2P_1}||\underline{r}_{P_2P_3}|}\right)$$

$$\begin{cases} \underline{r}_{P_2P_1} = \underline{r}_{P_1} - \underline{r}_{P_2} = (6, 0, 3) \quad \& \quad |\underline{r}_{P_2P_1}| = |\underline{r}_{P_1P_2}| = 6.7082 \\ \underline{r}_{P_2P_3} = \underline{r}_{P_3} - \underline{r}_{P_2} = (7, 5, -3) \quad \& \quad |\underline{r}_{P_2P_3}| = \sqrt{49+25+9} = 9.1104 \end{cases}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{42+0-9}{6.7082 \times 9.1104}\right) = 57.3185^\circ$$
$$= 1.0004 \text{ rad.}$$

1.16

b)

$$\gamma = \cos^{-1}\left(\frac{\underline{r}_{P_3P_1} \cdot \underline{r}_{P_3P_2}}{|\underline{r}_{P_3P_1}||\underline{r}_{P_3P_2}|}\right)$$

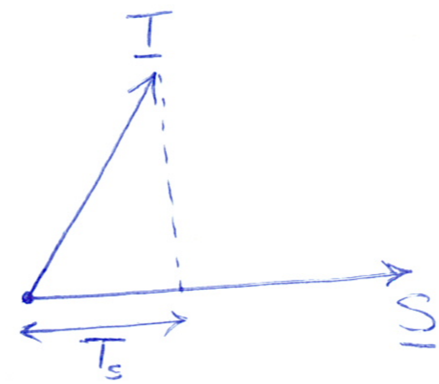
$$\begin{cases} \underline{r}_{P_3P_1} = \underline{r}_{P_1} - \underline{r}_{P_3} = (-1,-5,6) \ \& \ |\underline{r}_{P_3P_1}| = |\underline{r}_{P_1P_3}| = 7.8740 \\ \underline{r}_{P_3P_2} = \underline{r}_{P_2} - \underline{r}_{P_3} = (-7,-5,3) \ \& \ |\underline{r}_{P_3P_2}| = |\underline{r}_{P_2P_3}| = 9.1104 \end{cases}$$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{7+25+18}{7.8740 \times 9.1104}\right) = 45.8129^\circ$$

$$= 0.7996 \text{ rad.}$$

1.21 $\underline{T} = 2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z$

$\underline{S} = \underline{a}_x + 2\underline{a}_y + \underline{a}_z$



a) scalar projection of $\underline{T}$ on $\underline{S}$ = ?

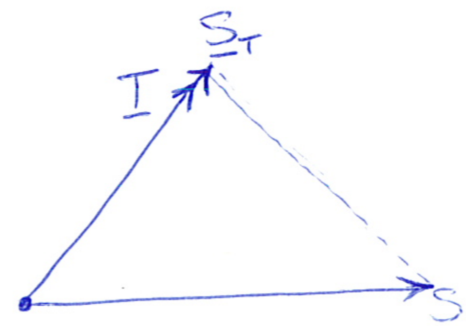
$T_S = \underline{T} \cdot \underline{a}_S$

$$\underline{a}_S = \frac{\underline{S}}{|\underline{S}|} = \frac{\underline{a}_x + 2\underline{a}_y + \underline{a}_z}{\sqrt{1+4+1}}$$

$$T_S = \underline{T} \cdot \underline{a}_S = \frac{1}{\sqrt{6}}(2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z)\cdot(\underline{a}_x + 2\underline{a}_y + \underline{a}_z)$$

$$= \frac{2\times1 + (-6)\times2 + 3\times1}{\sqrt{6}} = \frac{-7}{\sqrt{6}}$$

$$= -2.8577$$



b) vector projection of $\underline{S}$ on $\underline{T}$ = ?

$\underline{S}_T = (\underline{S} \cdot \underline{a}_T)\underline{a}_T$

$$\underline{a}_T = \frac{\underline{T}}{|\underline{T}|} = \frac{2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z}{\sqrt{4+36+9}} = \frac{1}{7}(2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z)$$

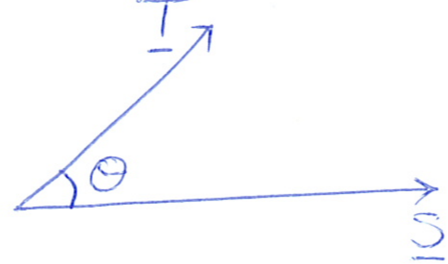
$$\underline{S}_T = \frac{(\underline{a}_x + 2\underline{a}_y + \underline{a}_z)\cdot(2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z)}{7} \times \frac{2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z}{7}$$

$$= \frac{(\overbrace{2-12+3}^{=-7})}{7} \times \frac{2\underline{a}_x - 6\underline{a}_y + 3\underline{a}_z}{7}$$

$$= -0.2857\,\underline{a}_x + 0.8571\,\underline{a}_y - 0.4286\,\underline{a}_z$$

1.21

c) smaller angle between $\underline{S}$ and $\underline{I}$ = ?



We know that

$$\underline{S}.\underline{I} = |\underline{S}||\underline{I}|\cos\theta \quad (1)$$

also, we know that

$$|\underline{S}\times\underline{I}| = |\underline{S}||\underline{I}|\sin\theta \quad (2)$$

Using either of these equations, we can calculate θ.

From (1) we have,

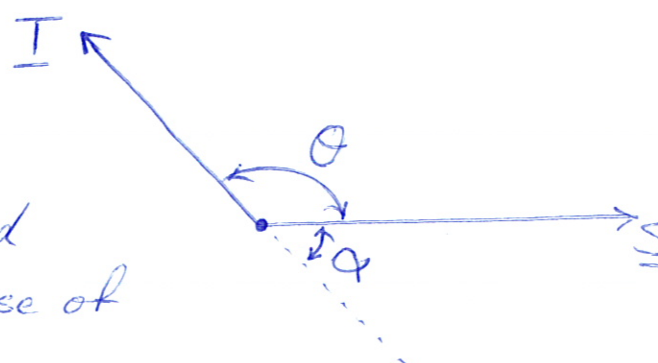
$$\underline{S}.\underline{I} = 2\times1+(-6)\times2+3\times1 = -7$$

$$|\underline{S}| = \sqrt{6}$$

$$|\underline{I}| = 7$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\underline{S}.\underline{I}}{|\underline{S}||\underline{I}|}\right) = \cos^{-1}\left(\frac{-7}{\sqrt{6}\times7}\right) = 114.0948^\circ$$

It means that the angle between $\underline{S}$ and $\underline{I}$ is bigger than 90°.



But if you use equation (2), you get α which is $(180-\theta)$ and smaller angle. And it is because of this equality:

$$|\underline{S}\times\underline{I}| = |\underline{S}\times(-\underline{I})|$$

1.21

c)

$$\underline{S} \times \underline{T} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 1 & 2 & 1 \\ 2 & -6 & 3 \end{vmatrix} = 12\underline{a}_x - \underline{a}_y - 10\underline{a}_z$$

$$|\underline{S} \times \underline{T}| = \sqrt{144+1+100} = \sqrt{245} \qquad \begin{cases} |\underline{S}| = \sqrt{6} \\ |\underline{T}| = 7 \end{cases}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{|\underline{S} \times \underline{T}|}{|\underline{S}||\underline{T}|}\right) = \sin^{-1}\left(\frac{\sqrt{245}}{\sqrt{6} \times 7}\right)$$

$$= 65.9052^\circ$$

1.24 $\underline{A} = 2x\underline{a}_x + y\underline{a}_y - z^2\underline{a}_z$ at point $(1,2,-4)$,

$\underline{B} = 3x^2\underline{a}_x + 6\underline{a}_y + \underline{a}_z$

a) $\underline{A}\cdot\underline{B} = ?$

$$\underline{A} @ (1,2,-4) = 2\times(1)\underline{a}_x + 2\underline{a}_y - (-4)^2\underline{a}_z$$

$$= 2\underline{a}_x + 2\underline{a}_y - 16\underline{a}_z$$

$$\underline{B} @ (1,2,-4) = 3\times(1)^2\underline{a}_x + 6\underline{a}_y + \underline{a}_z$$

$$= 3\underline{a}_x + 6\underline{a}_y + \underline{a}_z$$

$$\underline{A}\cdot\underline{B} = (2\times3) + (2\times6) + (-16\times1) = 6+12-16 = 2$$

b) The angle between $\underline{A}$ and $\underline{B} = ?$

$$\theta = \cos^{-1}\left(\frac{\underline{A}\cdot\underline{B}}{|\underline{A}||\underline{B}|}\right)$$

$$|\underline{A}| = \sqrt{4+4+256} = \sqrt{264}$$

$$|\underline{B}| = \sqrt{9+36+1} = \sqrt{46}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{264}\sqrt{46}}\right) = 88.9601^\circ$$

c) vector component of $\underline{A}$ on $\underline{B} = ?$

$$\underline{A}_B = (\underline{A}\cdot\underline{a}_B)\underline{a}_B = \left(\underline{A}\cdot\frac{\underline{B}}{|\underline{B}|}\right)\frac{\underline{B}}{|\underline{B}|} = \left(\frac{\underline{A}\cdot\underline{B}}{|\underline{B}|^2}\right)\underline{B}$$

1.24

c)

$$\underline{A} \cdot \underline{B} = 2$$

$$|\underline{B}|^2 = 46$$

$$\Rightarrow \underline{A}_B = \frac{2}{46}\left(3\underline{a}_x + 6\underline{a}_y + \underline{a}_z\right)$$

$$= 0.1304\,\underline{a}_x + 0.2609\,\underline{a}_y + 0.0435\,\underline{a}_z$$

1.26 $\quad \underline{E} = 2x\,\underline{a}_x + \underline{a}_y + yz\,\underline{a}_z$

$$\underline{F} = xy\,\underline{a}_x - y^2\underline{a}_y + xyz\,\underline{a}_z$$

a) $|\underline{E}|$ at $(1,2,3) = ?$

$$\underline{E}\ @\ (1,2,3) = 2\times(1)\underline{a}_x + \underline{a}_y + (2)\times(3)\underline{a}_z$$
$$= 2\underline{a}_x + \underline{a}_y + 6\underline{a}_z$$

$$|\underline{E}| = \sqrt{4+1+36} = \sqrt{41} = 6.4031$$

b) component of $\underline{E}$ along $\underline{F}$ at $(1,2,3) = ?$

$$\underline{E}_F = (\underline{E}\cdot\underline{a}_F)\underline{a}_F = \left(\underline{E}\cdot\frac{\underline{F}}{|\underline{F}|}\right)\frac{\underline{F}}{|\underline{F}|}$$

$$= \left(\frac{\underline{E}\cdot\underline{F}}{|\underline{F}|^2}\right)\underline{F}$$

$$\underline{E}\cdot\underline{F} = ?$$

$$\underline{F}\ @\ (1,2,3) = (1)\times(2)\underline{a}_x - (2)^2\underline{a}_y + (1)\times(2)\times(3)\underline{a}_z$$
$$= 2\underline{a}_x - 4\underline{a}_y + 6\underline{a}_z$$

$$|\underline{F}|^2 = 4+16+36 = 56$$

$$\underline{E}.\underline{F} = 2\times2 + 1\times(-4) + 6\times6 = 36$$

$$\underline{E}_F = \frac{36}{56}(2\underline{a}_x - 4\underline{a}_y + 6\underline{a}_z)$$
$$= 1.2857\,\underline{a}_x - 2.5714\,\underline{a}_y + 3.8571\,\underline{a}_z$$

1.26

c) unit vector perpendicular to both $\underline{\underline{E}}$ and $\underline{E}$ at $(0,+1,-3) = ?$

$$\underline{\underline{E}} \ @ (0,1,-3) = 2x(0)\underline{a}_x + \underline{a}_y + (1)(-3)\underline{a}_z$$
$$= \underline{a}_y - 3\underline{a}_z$$

$$\underline{E} \ @ (0,1,-3) = (0)(1)\underline{a}_x - (1)^2\underline{a}_y + (0)(1)(-3)\underline{a}_z$$
$$= -\underline{a}_y$$

$$\underline{a}^{\perp} = \frac{\underline{\underline{E}} \times \underline{E}}{|\underline{\underline{E}} \times \underline{E}|}$$

$$\underline{\underline{E}} \times \underline{E} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = (0-3)\underline{a}_x - (0-0)\underline{a}_y + (0-0)\underline{a}_z$$
$$= -3\underline{a}_x$$

$$|\underline{\underline{E}} \times \underline{E}| = \sqrt{(-3)^2} = 3$$

$$\underline{a}^{\perp} = \frac{-3\underline{a}_x}{3} = -\underline{a}_x$$

also, $\underline{a}_x$ is a unit vector perpendicular to both $\underline{\underline{E}}$ and $\underline{E}$ at $(0,1,-3)$.

$$\underline{a}^{\perp} = \pm \underline{a}_x$$