4.7 Suppose a uniform electric field exists in the room in which you are working, such that the lines of force are horizontal and at right angles to one wall. As you walk toward the wall from which the lines of force emerge into the room, are you walking toward

(a) Points of higher potential?

(b) Points of lower potential?

(c) Points of the same potential (equipotential line)?

4.8 A charge Q is uniformly distributed throughout a sphere of radius a. Taking the potential at infinity as zero, the potential at $r = b < a$ is

(a) $-\int_{\infty}^{b} \frac{Qr}{4\pi\varepsilon_o a^3}\, dr$

(b) $-\int_{\infty}^{b} \frac{Q}{4\pi\varepsilon_o r^2}\, dr$

(c) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_o r^2}\, dr - \int_{a}^{b} \frac{Qr}{4\pi\varepsilon_o a^3}\, dr$

(d) $-\int_{\infty}^{a} \frac{Q}{4\pi\varepsilon_o r^3}\, dr$

4.9 A potential field is given by $V = 3x^2y - yz$. Which of the following is not true?

(a) At point $(1, 0, -1)$, V and $\mathbf{E}$ vanish.

(b) $x^2y = 1$ is an equipotential line on the xy-plane.

(c) The equipotential surface $V = -8$ passes through point $P(2, -1, 4)$.

(d) The electric field at P is $12\mathbf{a}_x - 8\mathbf{a}_y - \mathbf{a}_z$ V/m.

(e) A unit normal to the equipotential surface $V = -8$ at P is $-0.83\mathbf{a}_x + 0.55\mathbf{a}_y + 0.07\mathbf{a}_z$.

4.10 An electric potential field is produced by point charges 1 μC and 4 μC located at $(-2, 1, 5)$ and $(1, 3, -1)$, respectively. The energy stored in the field is

(a) 2.57 mJ

(b) 5.14 mJ

(c) 10.28 mJ

(d) None of the above

Answers: 4.1c,e, 4.2d, 4.3b, 4.4a, 4.5d, 4.6e, 4.7a, 4.8c, 4.9a, 4.10b.

PROBLEMS

Section 4.2—Coulomb's Law and Field Intensity

4.1 Point charges $Q_1 = 5\,\mu$C and $Q_2 = -4\,\mu$C are placed at $(3, 2, 1)$ and $(-4, 0, 6)$, respectively. Determine the force on Q_1.

4.2 Point charges Q_1 and Q_2 are, respectively, located at $(4, 0, -3)$ and $(2, 0, 1)$. If $Q_2 = 4$ nC, find Q_1 such that

(a) The $\mathbf{E}$ at $(5, 0, 6)$ has no z-component

(b) The force on a test charge at $(5, 0, 6)$ has no x-component.

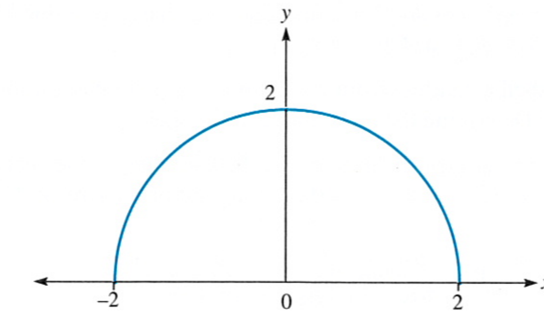

FIGURE 4.25 For Problem 4.6.

4.3 Charges $+Q$ and $+3Q$ are separated by a distance 2 m. A third charge is located such that the electrostatic system is in equilibrium. Find the location and the value of the third charge in terms of Q.

4.4 A point Q is located at (a, 0, 0), while another charge −Q is at (−a, 0, 0). Find E at: (a) (0, 0, 0) (b) (0, a, 0), (c) (a, 0, a).

Section 4.3—Electric Fields Due to Continuous Charge Distributions

4.5 Determine the total charge

(a) On line $0 < x < 5$ m if $\rho_L = 12x^2$ mC/m

(b) On the cylinder $\rho = 3, 0 < z < 4$ m if $\rho_S = \rho z^2$ nC/m^2

(c) Within the sphere $r = 4$ m if $\rho_v = \dfrac{10}{r \sin\theta}$ C/m^3

4.6 A point charge Q is located at point $P(0, -4, 0)$, while a 10 nC charge is uniformly distributed along a semicircular ring as shown in Figure 4.25. Find the value of Q such that $E(0, 0, 0) = 0$.

4.7 Given that $\rho_s = 6xy$ C/m^2, calculate the total charge on the triangular region in Figure 4.26.

4.8 A charge distribution is given by $\rho_v = 6x^2y^2$ nC/m^3. Determine the total charge enclosed by a cube of side 2 m centered at the origin and whose edges are parallel to the axes.

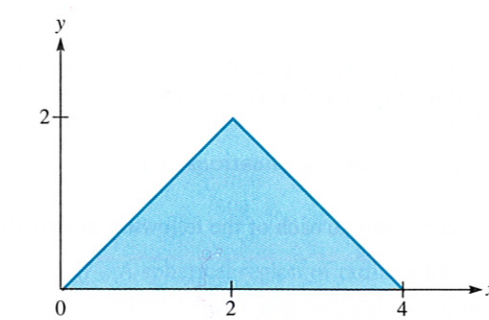

FIGURE 4.26 For Problem 4.7.

4.9 Given that $\rho_v = 4\rho^2 z \cos\phi$ nC/m^3, find the total charge contained in a wedge defined by $0 < \rho < 2, 0 < \phi < \pi/4, 0 < z < 1$.

4.10 The spherical shell extending from $r = 2$ cm to $r = 4$ cm has a uniform charge density $\rho_v = 5$ mC/m^3. Determine the total charge in the shell.

***4.11** (a) Show that the electric field at point $(0, 0, h)$ due to the rectangle described by $-a \leq x \leq a, -b \leq y \leq b, z = 0$ carrying uniform charge ρ_S C/m^2 is

$$\mathbf{E} = \frac{\rho_S}{\pi\varepsilon_o}\tan^{-1}\left[\frac{ab}{h(a^2 + b^2 + h^2)^{1/2}}\right]\mathbf{a}_z$$

(b) If $a = 2, b = 5, \rho_S = 10^{-5}$, find the total charge on the plate and the electric field intensity at $(0, 0, 10)$.

4.12 Plane $x + 2y = 5$ carries charge $\rho_S = 6$ nC/m^2. Determine $\mathbf{E}$ at $(-1, 0, 1)$.

4.13 Plane $x = 0$ has a uniform charge density ρ_s, while plane $x = a$ has $-\rho_s$. Determine the electric field intensity in regions: (a) $x < 0$, (b) $0 < x < a$, (c) $x > a$.

4.14 Three surface charge distributions are located in free space as follows: 10 μC/m^2 at $x = 2$, -20 μC/m^2 at $y = -3$, and 30 μC/m^2 at $z = 5$. Determine $\mathbf{E}$ at (a) $P(5, -1, 4)$, (b) $R(0, -2, 1)$, (c) $Q(3, -4, 10)$.

4.15 The gravitation force between two bodies of masses m_1 and m_2 is

$$\mathbf{F}_g = \frac{Gm_1m_2}{r^2}\mathbf{a}_r$$

where $G = 6.67 \times 10^{-11}$ m^3/kg $-$ s^2. Find the ratio of the electrostatic and gravitational forces between two electrons.

Section 4.4—Electric Flux Density

***4.16** State Gauss's law. Deduce Coulomb's law from Gauss's law, thereby affirming that Gauss's law is an alternative statement of Coulomb's law and that Coulomb's law is implicit in Maxwell's equation $\nabla \cdot \mathbf{D} = \rho_v$.

4.17 A ring placed along $y^2 + z^2 = 4, x = 0$ carries a uniform charge of 5 μC/m.

(a) Find $\mathbf{D}$ at $P(3, 0, 0)$.

(b) If two identical point charges Q are placed at $(0, -3, 0)$ and $(0, 3, 0)$ in addition to the ring, find the value of Q such that $\mathbf{D} = 0$ at P.

Sections 4.5 and 4.6—Gauss's Law and Applications

4.18 Determine the charge density due to each of the following electric flux densities:

(a) $\mathbf{D} = 8xy\mathbf{a}_x + 4x^2\mathbf{a}_y$ C/m^2

(b) $\mathbf{D} = 4\rho\sin\phi\,\mathbf{a}_\rho + 2\rho\cos\phi\,\mathbf{a}_\phi + 2z^2\mathbf{a}_z$ C/m^2

(c) $\mathbf{D} = \dfrac{2\cos\theta}{r^3}\mathbf{a}_r + \dfrac{\sin\theta}{r^3}\mathbf{a}_\theta$ C/m^2

4.19 If spherical surfaces $r = 1$ m and $r = 2$ m, respectively, carry uniform surface charge densities 8 nC/m^2 and -6 mC/m^2, find $\mathbf{D}$ at $r = 3$ m.

4.20 A solid sphere of radius 20 mm is charged with 50 nC. Calculate the flux density $\mathbf{D}$ on the surface of the sphere.

4.21 Let $\mathbf{D} = 2xy\mathbf{a}_x + x^2\mathbf{a}_y$ C/m^2 and find

(a) The volume charge density ρ_v.

(b) The flux through surface $0 < x < 1, 0 < z < 1, y = 1$.

(c) The total charge contained in the region $0 < x < 1, 0 < y < 1, 0 < z < 1$.

4.22 In a certain region, the electric field is given by

$$\mathbf{D} = 2\rho(z + 1)\cos\phi\,\mathbf{a}_\rho - \rho(z + 1)\sin\phi\,\mathbf{a}_\phi + \rho^2\cos\phi\,\mathbf{a}_z\ \mu\text{C/m}^2$$

(a) Find the charge density.

(b) Calculate the total charge enclosed by the volume $0 < \rho < 2$, $0 < \phi < \pi/2$, $0 < z < 4$.

(c) Confirm Gauss's law by finding the net flux through the surface of the volume in (b).

***4.23** The Thomson model of a hydrogen atom is a sphere of positive charge with an electron (a point charge) at its center. The total positive charge equals the electronic charge e. Prove that when the electron is at a distance r from the center of the sphere of positive charge, it is attracted with a force

$$F = \frac{e^2 r}{4\pi\varepsilon_o R^3}$$

where R is the radius of the sphere.

4.24 Let $\rho_v = \rho_o/r$ nC/m^3, $0 < r < a$, where ρ_o is a constant. (a) Find $\mathbf{E}$ inside and outside $r = a$. (b) Calculate the total charge.

4.25 Let

$$\rho_v = \begin{cases} \dfrac{10}{r^2}\ \text{mC/m}^3, & 1 < r < 4 \\ 0, & r > 4 \end{cases}$$

(a) Find the net flux crossing surface $r = 2$ m and $r = 6$ m.

(b) Determine $\mathbf{D}$ at $r = 1$ m and $r = 5$ m.

4.26 Let $\rho_v = \dfrac{50e^{-r}}{r}$ nC/m^3. Use Gauss's law to find $\mathbf{E}$.

4.27 A spherical region of radius a has total charge Q. If the charge is uniformly distributed, apply Gauss's law to find $\mathbf{D}$ both inside and outside the sphere.

Sections 4.7 and 4.8—Electric Potential and Relationship with E

4.28 Two point charges $Q = 2$ nC and $Q = -4$ nC are located at $(1, 0, 3)$ and $(-2, 1, 5)$, respectively. Determine the potential at $P(1, -2, 3)$.

4.29 (a) A total charge $Q = 60\ \mu$C is split into two equal charges located at 180° intervals around a circular loop of radius 4 m. Find the potential at the center of the loop.

(b) If Q is split into three equal charges spaced at 120° intervals around the loop, find the potential at the center.

(c) If in the limit $\rho_L = \dfrac{Q}{2\pi}$, find the potential at the center.

4.30 Three point charges $Q_1 = 1$ mC, $Q_2 = -2$ mC, and $Q_3 = 3$ mC are, respectively, located at $(0, 0, 4)$, $(-2, 5, 1)$, and $(3, -4, 6)$.

(a) Find the potential V_P at $P(-1, 1, 2)$.

(b) Calculate the potential difference V_{PQ} if Q is $(1, 2, 3)$.

4.31 A circular disk of radius a carries charge $\rho_S = \dfrac{1}{\rho}$ C/m². Calculate the potential at $(0, 0, h)$.

4.32 $V = x^2y(z + 3)$ V. Find

(a) $\mathbf{E}$ at $(3, 4, -6)$

(b) the charge within the cube $0 < x < 1, 0 < y < 1, 0 < z < 1$.

4.33 Given that $\rho_v = 1 - \left(\dfrac{r}{a}\right)^2$ within a sphere of radius a, determine $\mathbf{E}$ at $r = 5a$.

4.34 A spherically symmetric charge distribution is given by

$$\rho_v = \begin{cases} \rho_o\left(1 - \dfrac{r}{a}\right)^2, & r \leq a \\ 0, & r \geq a \end{cases}$$

(a) Find $\mathbf{E}$ and V for $r \geq a$.

(b) Find $\mathbf{E}$ and V for $r \leq a$.

(c) Find the total charge.

4.35 Let charge Q be uniformly distributed on a circular ring defined by $a < \rho < b$ and shown in Figure 4.27. Find $\mathbf{D}$ at $(0, 0, h)$.

4.36 If $\mathbf{D} = 4x\mathbf{a}_x - 10y^2\mathbf{a}_y + z^2\mathbf{a}_z$ C/m², find the charge density at $P(1, 2, 3)$.

4.37 A 10-nC charge is uniformly distributed over a spherical shell $r = 3$ cm and a -5 nC charge is uniformly distributed over another spherical shell $r = 5$ cm. Find $\mathbf{D}$ for regions $r < 3$ cm, $3 \text{ cm} < r < 5$ cm, $r > 5$ cm.

4.38 In free space, an electric field is given by

$$\mathbf{E} = \begin{cases} E_0(\rho/a)\mathbf{a}_\rho, & 0 < \rho < a \\ 0, & \text{otherwise} \end{cases}$$

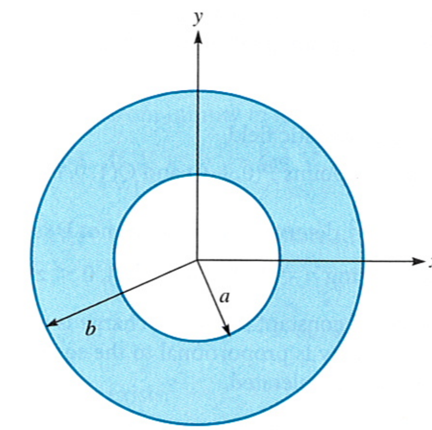

FIGURE 4.27 For Problem 4.35.

Calculate the volume charge density.

4.39 The electric field intensity in free space is given by

$$\mathbf{E} = 2\,xyz\mathbf{a}_x + x^2z\mathbf{a}_y + x^2y\mathbf{a}_z \text{ V/m}$$

Calculate the amount of work necessary to move a $2\ \mu$C charge from $(2, 1, -1)$ to $(5, 1, 2)$.

4.40 Determine the energy expended in moving a 20-μC point charge in

$$\mathbf{E} = 2\rho \sin\phi\, \mathbf{a}_\rho + \rho\cos\phi\, \mathbf{a}_\phi \text{ V/m}$$

4.41 In an electric field $\mathbf{E} = 20r\sin\theta\ \mathbf{a}_r + 10r\cos\theta\ \mathbf{a}_\theta$ V/m, calculate the energy expended in transferring a 10 nC charge

(a) From $A(5, 30°, 0°)$ to $B(5, 90°, 0°)$

(b) From A to $C(10, 30°, 0°)$

(c) From A to $D(5, 30°, 60°)$

(d) From A to $E(10, 90°, 60°)$

4.42 Let $\mathbf{E} = \dfrac{10}{r^2}\mathbf{a}_2$ V/m. Find V_{AB}, where A is $(1, \pi/4, \pi/2)$ and B is $(5, \pi, 0)$.

4.43 Let $\mathbf{E} = 2xy^2\mathbf{a}_x + 2y(x^2 + 1)\mathbf{a}_y$ V/m. Find the work done in transforming a 2-nC charge from point $A(2, 3, -1)$ to $B(8, 0, -1)$.

4.44 For each of the following potential distributions, find the electric field intensity and the volume charge distribution:

(a) $V = 2x^2 + 4y^2$

(b) $V = 10\rho^2\sin\phi + 6\rho z$

(c) $V = 5r^2\cos\theta\sin\phi$

4.45 A uniform surface charge with density ρ_s exists on a hemispherical surface with $r = a$ and $\theta \leq \pi/2$. Calculate the electric potential at the center.

4.46 Let $\mathbf{E} = 2x\mathbf{a}_x + 6y\mathbf{a}_y$ V/m.

(a) Show that **E** is a genuine electrostatic field.

(b) Calculate the voltage between points $P(0, 1, 0)$ and $Q(1, 0, 0)$.

4.47 If $\mathbf{D} = 2\rho \sin \phi \mathbf{a}_\rho - \dfrac{\cos \phi}{2\rho}\mathbf{a}_\phi$ C/m^2, determine whether or not **D** is a genuine electric flux density. Determine the flux crossing $\rho = 1, 0 \leq \phi \leq \pi/4, 0 < z < 1$.

***4.48** (a) Prove that when a particle of constant mass and charge is accelerated from rest in an electric field, its final velocity is proportional to the square root of the potential difference through which it is accelerated.

(b) Find the magnitude of the proportionality constant if the particle is an electron.

(c) Through what voltage must an electron be accelerated, assuming no change in its mass, to require a velocity one-tenth that of light? (At such velocities, the mass of a body becomes appreciably larger than its "rest mass" and cannot be considered constant.)

***4.49** An electron is projected with an initial velocity $u_o = 10^7$ m/s into the uniform field between the parallel plates of Figure 4.28. It enters the field at the midway between the plates. If the electron just misses the upper plate as it emerges from the field.

(a) Find the electric field intensity.

(b) Calculate its velocity as it emerges from the field. Neglect edge effects.

Section 4.9—Electric Dipole and Flux Lines

4.50 An electric dipole with $\mathbf{p} = p\mathbf{a}_z$ C · m is placed at $(x, z) = (0, 0)$. If the potential at $(0, 1$ nm$)$ is 9 V, find the potential at $(1$ nm$, 1$ nm$)$.

4.51 Point charges Q and $-Q$ are located at $(0, d/2, 0)$ and $(0, -d/2, 0)$. Show that at point (r, θ, ϕ), where $r \gg d$,

$$V = \frac{Qd \sin \theta \sin \phi}{4\pi\varepsilon_o r^2}$$

Find the corresponding **E** field.

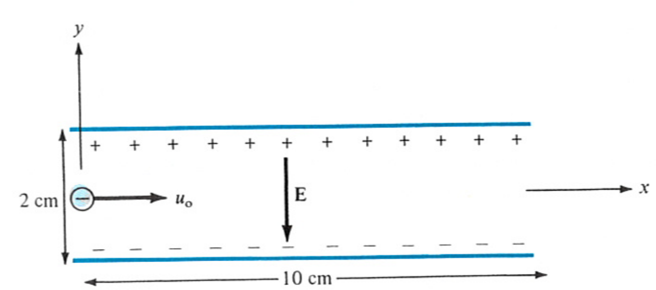

FIGURE 4.28 For Problem 4.49.

Section 4.10—Energy Density

4.52 Determine the amount of work needed to transfer two charges of 40 nC and −50 nC from infinity to locations (0, 0, 1) and (2, 0, 0), respectively.

4.53 If $V = 2x^2 + 6y^2$ V in free space, find the energy stored in a volume defined by $-1 \leq x \leq 1, -1 \leq y \leq 1$, and $-1 \leq z \leq 1$.

4.54 Find the energy stored in the hemispherical region $r \leq 2$ m, $0 < \theta < \pi, 0 < \phi < \pi$ where

$$\mathbf{E} = 2r \sin \theta \cos \phi \,\mathbf{a}_r + r \cos \theta \cos \phi \,\mathbf{a}_\theta - r \sin \phi \,\mathbf{a}_\phi \text{ V/m}$$

exists.

4.55 If $V = \rho^2 z \sin \phi$, calculate the energy within the region defined by $1 < \rho < 4$, $-2 < z < 2$, and $0 < \phi < \pi/3$.