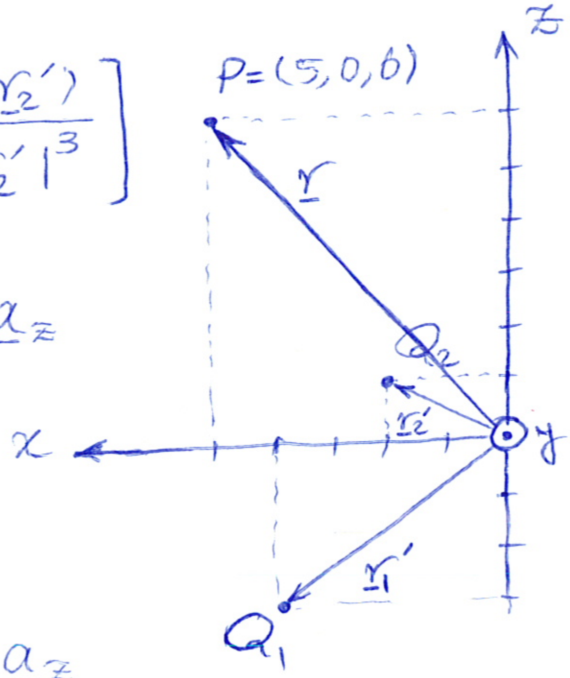
4.2 Q_1 is at $(4,0,-3)$

Q_2 is at $(2,0,1)$ $\qquad Q_2 = 4\ nC$

a) E_z at $(5,0,6) = 0 \Rightarrow Q_1 = ?$



$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1(\underline{r}-\underline{r}_1')}{|\underline{r}-\underline{r}_1'|^3} + \frac{Q_2(\underline{r}-\underline{r}_2')}{|\underline{r}-\underline{r}_2'|^3}\right]$$

$$\begin{cases} \underline{r}-\underline{r}_1' = (5-4)\underline{a}_x + 0\,\underline{a}_y + (6-(-3))\underline{a}_z \\ \qquad = \underline{a}_x + 9\,\underline{a}_z \\ |\underline{r}-\underline{r}_1'| = \sqrt{1+9^2} = \sqrt{82} \end{cases}$$

$$\begin{cases} \underline{r}-\underline{r}_2' = (5-2)\underline{a}_x + 0\,\underline{a}_y + (6-1)\underline{a}_z \\ \qquad = 3\,\underline{a}_x + 5\,\underline{a}_z \\ |\underline{r}-\underline{r}_2'| = \sqrt{3^2+5^2} = \sqrt{34} \end{cases}$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 \times 9}{(\sqrt{82})^3} + \frac{Q_2 \times 5}{(\sqrt{34})^3}\right] = 0$$

$$\Rightarrow \frac{9Q_1}{82^{3/2}} = -\frac{5Q_2}{34^{3/2}} = -\frac{5\times 4\times 10^{-9}}{34^{3/2}}$$

$$\Rightarrow Q_1 = -\frac{5\times 4\times 10^{-9}}{34^{3/2}} \times \frac{82^{3/2}}{9} = -8.3232\ nC$$

b) $\underline{F} = q\underline{E}$, F_x at $(5,0,6) = 0 \Rightarrow Q_1 = ?$

$F_x = 0 \Rightarrow E_x = 0$

4.2

b)

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 \times 1}{(82)^{3/2}} + \frac{Q_2 \times 3}{(34)^{3/2}}\right] = 0$$

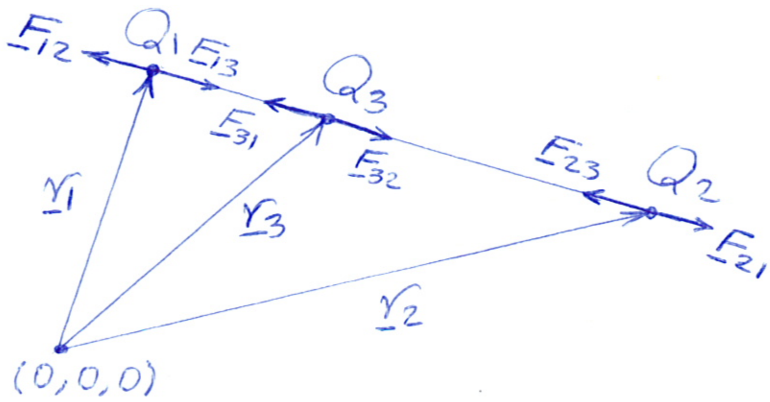
$$\Rightarrow \frac{Q_1}{82^{3/2}} = -\frac{3Q_2}{34^{3/2}} = -\frac{3\times 4\times 10^{-9}}{34^{3/2}}$$

$$\Rightarrow Q_1 = -\frac{3\times 4 \times 10^{-9}}{34^{3/2}} \times 82^{3/2} = -44.9452 \text{ nC}$$

4.3 $\quad Q_1 = +Q$ at $\underline{r}_1$

$Q_2 = +3Q$ at $\underline{r}_2$ $\quad$ & $|\underline{r}_2 - \underline{r}_1| = 2$ m

$Q_3 = ?$ at $\underline{r}_3$ $\quad$ & $\underline{F}_{31} + \underline{F}_{32} = \underline{0}$



To have a equilibrium,

1) Q_3 should be negative;

2) Q_3 should be colinear with Q_1 & Q_2;

3) Q_3 should be in the middle.

In fact,

$$\begin{cases} \underline{F}_{31} + \underline{F}_{32} = \underline{0} & \Leftrightarrow \dfrac{Q_3 Q_1 (\underline{r}_1 - \underline{r}_3)}{4\pi\varepsilon_0 |\underline{r}_1 - \underline{r}_3|^3} + \dfrac{Q_3 Q_2 (\underline{r}_2 - \underline{r}_3)}{4\pi\varepsilon_0 |\underline{r}_2 - \underline{r}_3|^3} = \underline{0} \\ \underline{F}_{12} + \underline{F}_{13} = \underline{0} & \Leftrightarrow \dfrac{Q_1 Q_2 (\underline{r}_2 - \underline{r}_1)}{4\pi\varepsilon_0 |\underline{r}_2 - \underline{r}_1|^3} + \dfrac{Q_1 Q_3 (\underline{r}_3 - \underline{r}_1)}{4\pi\varepsilon_0 |\underline{r}_3 - \underline{r}_1|^3} = \underline{0} \\ \underline{F}_{21} + \underline{F}_{23} = \underline{0} & \end{cases}$$

The 3rd Eq. is the summation of the 1st & the 2nd Eqs.

From the first eq., we have:

$$|\underline{F}_{31}| = |\underline{F}_{32}| \Rightarrow \frac{|Q_3||Q_1||\underline{r}_1 - \underline{r}_3|}{4\pi\varepsilon_0 |\underline{r}_1 - \underline{r}_3|^3} = \frac{|Q_3||Q_2||\underline{r}_2 - \underline{r}_3|}{4\pi\varepsilon_0 |\underline{r}_2 - \underline{r}_3|^3}$$

$$\Rightarrow \frac{|Q_1|}{|Q_2|} = \frac{|\underline{r}_1 - \underline{r}_3|^2}{|\underline{r}_2 - \underline{r}_3|^2}$$

4.3

$$\Rightarrow \frac{|Q_1|}{|Q_2|} = \frac{1}{3} = \frac{|\underline{r}_1 - \underline{r}_3|^2}{|\underline{r}_2 - \underline{r}_3|^2}$$

$$\Rightarrow |\underline{r}_2 - \underline{r}_3| = \sqrt{3}\,|\underline{r}_1 - \underline{r}_3|$$

Also, we have $|\underline{r}_1 - \underline{r}_3| + |\underline{r}_2 - \underline{r}_3| = |\underline{r}_1 - \underline{r}_2| = |\underline{r}_2 - \underline{r}_1| = 2$

$$\Rightarrow |\underline{r}_2 - \underline{r}_3| = 2 - |\underline{r}_1 - \underline{r}_3| = \sqrt{3}\,|\underline{r}_1 - \underline{r}_3|$$

$$\Rightarrow |\underline{r}_1 - \underline{r}_3| = |\underline{r}_3 - \underline{r}_1| = \frac{2}{1+\sqrt{3}}$$

On the other hand,

$$\underline{r}_3 = \underline{r}_1 + (\underline{r}_3 - \underline{r}_1)$$

$$= \underline{r}_1 + |\underline{r}_3 - \underline{r}_1|\,\underline{a}_{r_{13}}$$

$$= \underline{r}_1 + |\underline{r}_3 - \underline{r}_1|\,\underline{a}_{r_{12}}$$

$$= \underline{r}_1 + |\underline{r}_3 - \underline{r}_1| \frac{(\underline{r}_2 - \underline{r}_1)}{|\underline{r}_2 - \underline{r}_1|} = \underline{r}_1 + \frac{\cancel{2}}{1+\sqrt{3}} \times \frac{\underline{r}_2 - \underline{r}_1}{\cancel{2}}$$

$$= \frac{(1+\sqrt{3})\underline{r}_1 + (\underline{r}_2 - \underline{r}_1)}{1+\sqrt{3}}$$

$$\Rightarrow \underline{r}_3 = \frac{\sqrt{3}\,\underline{r}_1 + \underline{r}_2}{1+\sqrt{3}}$$

From the second eq., we have :

$$|\underline{F}_{12}| = |\underline{F}_{13}| \Rightarrow \frac{|Q_1||Q_2|\,|\underline{r}_2 - \underline{r}_1|}{4\pi\varepsilon_0 |\underline{r}_2 - \underline{r}_1|^3} = \frac{|Q_1||Q_3||\underline{r}_3 - \underline{r}_1|}{4\pi\varepsilon_0 |\underline{r}_3 - \underline{r}_1|^3}$$

4.3

$$\Rightarrow \frac{|Q_3|}{|Q_2|} = \frac{|\underline{r}_3 - \underline{r}_1|^2}{|\underline{r}_2 - \underline{r}_1|^2} = \frac{\left(\frac{2}{1+\sqrt{3}}\right)^2}{2^2} = \frac{1}{(1+\sqrt{3})^2}$$

$$\Rightarrow Q_3 = -|Q_3|$$

$$= -\frac{|Q_2|}{(1+\sqrt{3})^2}$$

$$\Rightarrow Q_3 = \frac{-3Q}{(1+\sqrt{3})^2} \quad at \quad \underline{r}_3 = \frac{\sqrt{3}\,\underline{r}_1 + \underline{r}_2}{\sqrt{3}+1}$$

4.5 Total charge = ?

a) Line : $0 < x < 5$

$$\rho_L = 12x^2 \ mC/m = 12x^2 \times 10^{-3} \ C/m$$

$$Q = \int \rho_L \, dl = \int_{x=0}^{5} 12x^2 \times 10^{-3} \, dx$$

$dl = dx$

$x = 0 \rightarrow 5$

$$= 4x^3 \times 10^{-3} \Big|_{x=0}^{5}$$

$$= 500 \times 10^{-3} \ C$$

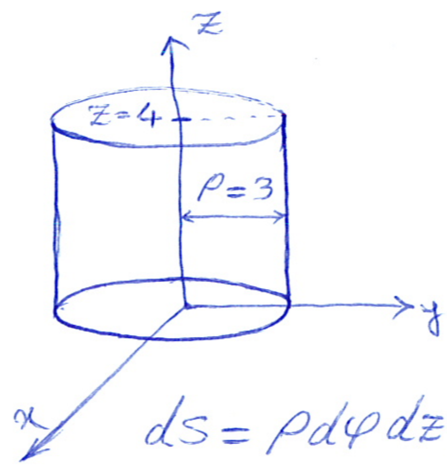
$$= 0.5 \ C$$

b) Cylinder : $\rho = 3$

$0 < z < 4$

$$\rho_s = \rho z^2 \ nC/m^2 = \rho z^2 \times 10^{-9} \ C/m^2$$

$$Q = \iint \rho_s \, ds = \int_{z=0}^{4} \int_{\varphi=0}^{2\pi} \rho z^2 \rho \, d\varphi \, dz, \ nC$$



$ds = \rho \, d\varphi \, dz$

$\rho = 3$ const.

$\varphi = 0 \rightarrow 2\pi$

$z = 0 \rightarrow 4$

$$= \rho^2 \int_{z=0}^{4} z^2 \int_{\varphi=0}^{2\pi} d\varphi \times 10^{-9} \, dz$$

$$= 3^2 \times 10^{-9} \times \frac{z^3}{3} \Big|_{z=0}^{4} \times \varphi \Big|_{\varphi=0}^{2\pi}$$

$$= 9 \times \frac{64}{3} \times 2\pi \times 10^{-9}$$

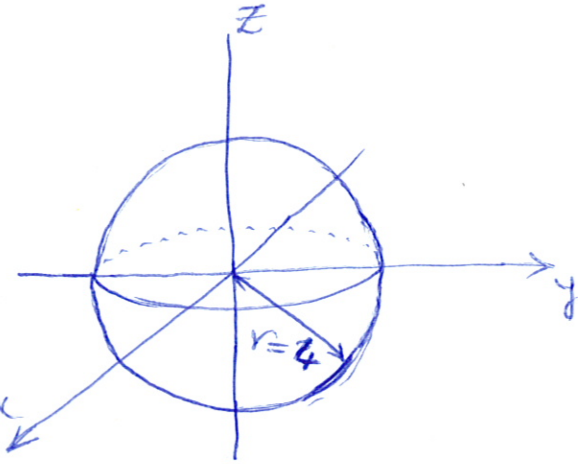
$$= 384\pi \times 10^{-9} \ C \simeq 1.2064 \ \mu C$$

4.5

C) Sphere :

$$r = 4 \text{ m}$$

$$\rho_v = \frac{10}{r\sin\theta} \ C/m^3$$



$$Q = \iiint \rho_v \, dv$$

$$= \int_{r=0}^{4}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} \frac{10}{r\sin\theta} r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$dv = r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$r = 0 \rightarrow 4$$

$$\theta = 0 \rightarrow \pi$$

$$\varphi = 0 \rightarrow 2\pi$$

$$= 10 \int_{r=0}^{4} r\,dr \int_{\theta=0}^{\pi} d\theta \int_{\varphi=0}^{2\pi} d\varphi$$

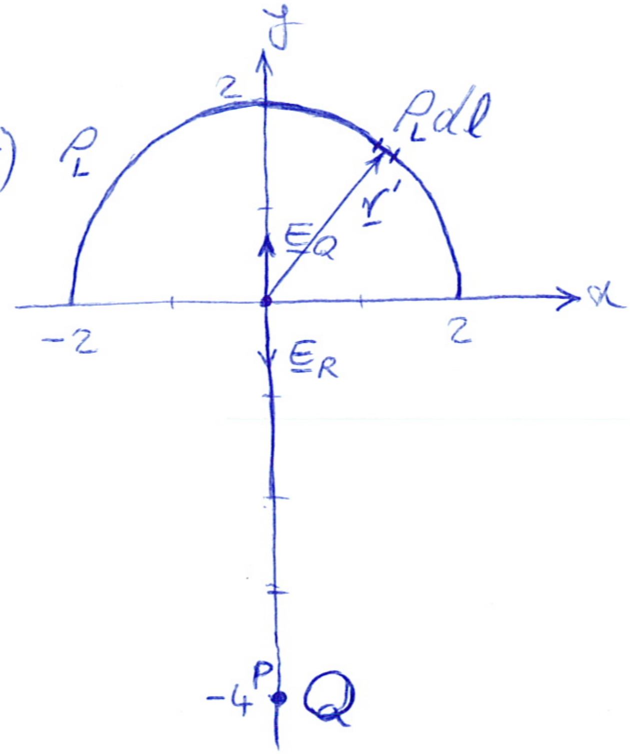
$$= 10 \times \frac{r^2}{2}\Big|_{r=0}^{4} \times \theta\Big|_{\theta=0}^{\pi} \times \varphi\Big|_{\varphi=0}^{2\pi}$$

$$= 10 \times \frac{16}{2} \times \pi \times 2\pi = 160\pi^2 \ C$$

$$= 1579.1 \ C$$

4.6 Q is at $P(0,-4,0)$

$\rho_L = \frac{10}{2\pi}\ nC/m$ (length of Ring $= 2\pi$)



$\underline{E}(0,0,0) = \underline{0} \Rightarrow Q = ?$

$\underline{E} = \underline{E}_R + \underline{E}_Q$

$$\begin{cases} \underline{r} = (0,0,0) \\ \underline{r}'_Q = (0,-4,0) \end{cases}$$

$$\underline{E}_Q = \frac{Q\,(\underline{r}-\underline{r}'_Q)}{4\pi\varepsilon_0 |\underline{r}-\underline{r}'_Q|^3} = \frac{Q \times 4\underline{a}_y}{4\pi\varepsilon_0 \times 4^3} = \frac{Q}{64\pi\varepsilon_0}\underline{a}_y$$

$$\begin{cases} \underline{r} = (0,0,0) \\ \underline{r}' = 2\,\underline{a}_\rho = 2(\cos\varphi\,\underline{a}_x + \sin\varphi\,\underline{a}_y) \end{cases}$$

$$\underline{E}_R = \int \frac{\rho_L dl\,(\underline{r}-\underline{r}')}{4\pi\varepsilon_0 |\underline{r}-\underline{r}'|^3} \qquad dl = \rho d\varphi = 2d\varphi$$

$$|\underline{r}-\underline{r}'| = 2$$

$E_{Rx} = 0 \Leftarrow$ Problem is symmetric

$$\underline{E}_R = E_{Ry}\underline{a}_y = \int_{\varphi=0}^{\pi} \frac{\rho_L \rho d\varphi\,(-2\sin\varphi\,\underline{a}_y)}{4\pi\varepsilon_0 \underbrace{|\underline{r}-\underline{r}'|^3}_{=2}}$$

$$= \frac{\frac{10}{2\pi}\times 10^{-9} \times 2\,\underline{a}_y}{4\pi \times \frac{1}{36\pi}\times 10^{-9} \times 2^3} \int_{\varphi=0}^{\pi} (-2\sin\varphi)\,d\varphi$$

$$= \frac{90}{2\pi} \times \frac{2^2 \underline{a}_y}{2^3} \cos\varphi \Big|_{\varphi=0}^{\pi} = -\frac{90}{2\pi}\underline{a}_y$$

4.6

$$\underline{E} = \underline{E}_Q + \underline{E}_R$$

$$= \frac{Q}{64\pi\varepsilon_0}\underline{a_y} - \frac{90}{2\pi}\underline{a_y} = \underline{0}$$

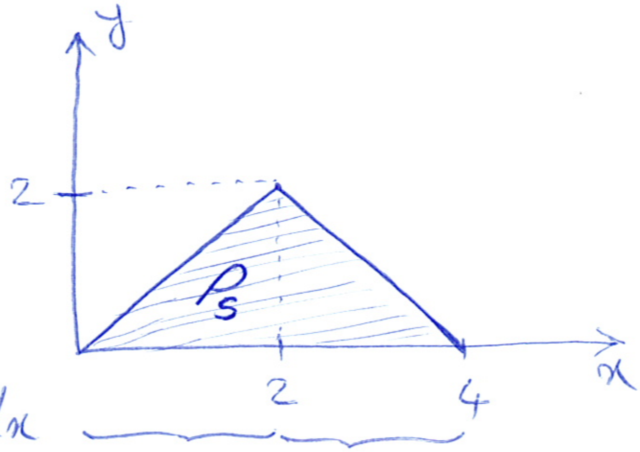
$$\Rightarrow \frac{Q}{64\pi \times \frac{1}{36\pi} \times 10^{-9}} = \frac{90}{2\pi}$$

$$\Rightarrow Q = \frac{90}{2\pi} \times 64\pi \times \frac{1}{36\pi} \times 10^{-9} = \frac{80}{\pi} \times 10^{-9}\ C$$

$$\Rightarrow Q = 25.4648\ nC$$

4.7 $\rho_s = 6xy \ C/m^2$

Total Charge = ?



$x=0 \to 2$ $\quad x=2 \to 4$

$y=0 \to x$ $\quad y=0 \to (-x+4)$

$$Q = \iint_S \rho_s \, ds$$

$$= \int_{x=0}^{2} \int_{y=0}^{x} 6xy \, dy \, dx + \int_{x=2}^{4} \int_{y=0}^{-x+4} 6xy \, dy \, dx$$

$$= \int_{x=0}^{2} 6x \left.\frac{y^2}{2}\right|_{y=0}^{x} dx + \int_{x=2}^{4} 6x \left.\frac{y^2}{2}\right|_{y=0}^{-x+4} dx$$

$ds = dx\,dy$

$$= \int_{x=0}^{2} 3x^3 \, dx + \int_{x=2}^{4} 3x(-x+4)^2 \, dx$$

$$= \int_{x=0}^{2} 3x^3 \, dx + \int_{x=2}^{4} (3x^3 - 24x^2 + 48x) \, dx$$

$$= \int_{x=0}^{4} 3x^3 \, dx + \int_{x=2}^{4} (-24x^2 + 48x) \, dx$$

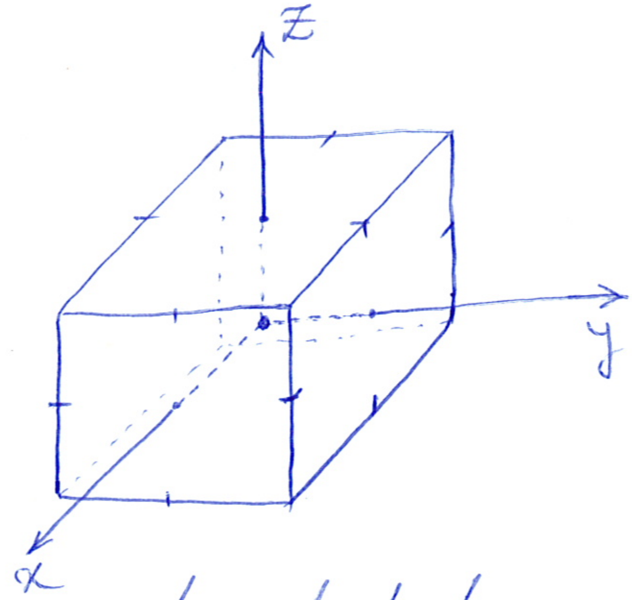
$$= \left.\frac{3}{4} x^4 \right|_{x=0}^{4} + \left.(-8x^3 + 24x^2)\right|_{x=2}^{4}$$

$$= 192 - 448 + 288 = 32 \, C$$

$Q = 32 \, C$

4.8 $\rho_v = 6x^2y^2 \; nC/m^3$

Total Charge = ?



$dv = dx\,dy\,dz$

$$Q = \iiint_v \rho_v \, dv$$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} 6x^2y^2 \times 10^{-9}\, dx\,dy\,dz$$

$$= 6\times10^{-9} \times \left.\frac{x^3}{3}\right|_{x=-1}^{1} \times \left.\frac{y^3}{3}\right|_{y=-1}^{1} \times \left. z\right|_{z=-1}^{1}$$

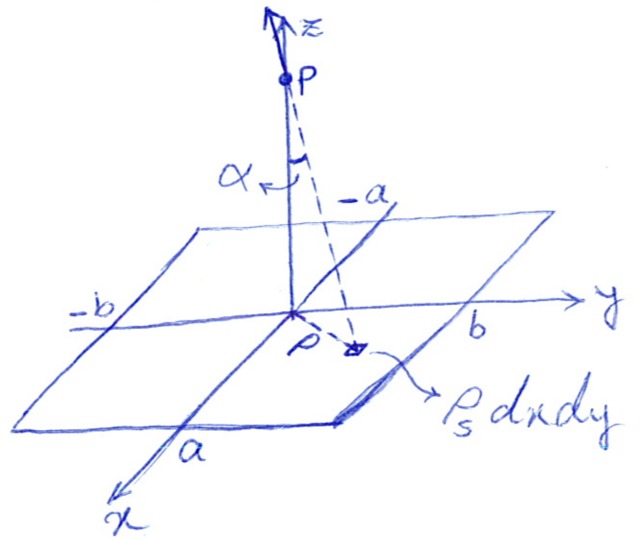
$$= 6\times10^{-9} \times \frac{2}{3} \times \frac{2}{3} \times 2 = \frac{16}{3}\times10^{-9}\ C$$

$Q = 5.3333 \; nC$

4.11

a) Rectangle $\begin{matrix} -a \leqslant x \leqslant a \\ -b \leqslant y \leqslant b \\ z = 0 \end{matrix}$ with ρ_s C/m^2 (uniform)

at $(0,0,h)$: $\underline{E} \overset{?}{=} \frac{\rho_s}{\pi \varepsilon_0} \tan^{-1}\left[\frac{ab}{h(a^2+b^2+h^2)^{1/2}}\right] \underline{a}_z$



Due to symmetry :

$$\begin{cases} \underline{E} = E_z \underline{a}_z \\ E_x = E_y = 0 \end{cases}$$

$$\underline{E} = \iint_S \frac{\rho_s ds\,(\underline{r} - \underline{r}')}{4\pi\varepsilon_0 |\underline{r} - \underline{r}'|^3}$$

$$\begin{cases} \underline{r} - \underline{r}' = -x\underline{a}_x - y\underline{a}_y + h\underline{a}_z \\ |\underline{r} - \underline{r}'| = \sqrt{x^2+y^2+h^2} \end{cases}$$

$$E_z = \int_{x=-a}^{a} \int_{y=-b}^{b} \frac{\rho_s\,dx\,dy \times h}{4\pi\varepsilon_0 (x^2+y^2+h^2)^{3/2}}$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0} \int_{x=-a}^{a} \int_{y=-b}^{b} \frac{dy}{(x^2+y^2+h^2)^{3/2}}\, dx$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0} \int_{x=-a}^{a} \left. \frac{y}{(x^2+h^2)(x^2+y^2+h^2)^{1/2}} \right|_{y=-b}^{b} dx$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0} \int_{x=-a}^{a} \frac{2b\, dx}{(x^2+h^2)(x^2+b^2+h^2)^{1/2}}$$

4.11

$$\Rightarrow E_z = \frac{\rho_s h \times 2b}{4\pi\varepsilon_0} \times \frac{1}{hb} \tan^{-1}\left(\frac{bx}{h(x^2+b^2+h^2)^{1/2}}\right)\Bigg|_{x=-a}^{a}$$

$$= \frac{\rho_s h \times 2b}{4\pi\varepsilon_0} \times \frac{1}{hb} \times 2 \tan^{-1}\left(\frac{ab}{h(a^2+b^2+h^2)^{1/2}}\right)$$

$$= \frac{\rho_s}{\pi\varepsilon_0} \tan^{-1}\left[\frac{ab}{h(a^2+b^2+h^2)^{1/2}}\right]$$

$$\Rightarrow \underline{E} = \frac{\rho_s}{\pi\varepsilon_0} \tan^{-1}\left[\frac{ab}{h(a^2+b^2+h^2)^{1/2}}\right] \underline{a}_z$$

b) $a = 2$, $b = 5$, $\rho_s = 10^{-5}$ $\Rightarrow \begin{cases} Q = ? \\ \underline{E}(0,0,10) = ? \end{cases}$

$$Q = \iint_S \rho_s \, ds = \int_{x=-a}^{a} \int_{y=-b}^{b} \rho_s \, dx\,dy = \rho_s \times x\Big|_{-a}^{a} \times y\Big|_{-b}^{b} = 4ab\rho_s$$

$$\Rightarrow Q = 4 \times 2 \times 5 \times 10^{-5} = 400\,\mu C = 0.4\ mC$$

$$\underline{E} = \frac{10^{-5}}{\pi \times \frac{1}{36\pi} \times 10^{-9}} \tan^{-1}\left[\frac{2 \times 5}{10(2^2+5^2+10^2)^{1/2}}\right] \underline{a}_z$$

$$= 36 \times 10^4 \tan^{-1}\left[\frac{1}{\sqrt{129}}\right] \underline{a}_z = 36 \times 10^4 \times (0.0878^{rad}) \underline{a}_z$$

$$\Rightarrow \underline{E} = 31.6147\ KV/m$$

4.14 $\rho_{s1} = 10\ \mu C/m^2$ @ $x=2$

$\rho_{s2} = -20\ \mu C/m^2$ @ $y=-3$ $\quad \Rightarrow \underline{E} = ?$

$\rho_{s3} = 30\ \mu C/m^2$ @ $z=5$

a) $P(5,-1,4)$

$$\underline{E} = \sum_{k=1}^{3} \frac{\rho_{sk}}{2\varepsilon_0} \underline{a}_{nk} \qquad \begin{cases} \underline{a}_{n1} = \underline{a}_x & (5>2) \\ \underline{a}_{n2} = \underline{a}_y & (-1>-3) \\ \underline{a}_{n3} = -\underline{a}_z & (4<5) \end{cases}$$

$$\Rightarrow \underline{E} = \frac{10\times10^{-6}}{2\times\frac{1}{36\pi}\times10^{-9}}\underline{a}_x + \frac{-20\times10^{-6}}{2\times\frac{1}{36\pi}\times10^{-9}}\underline{a}_y - \frac{30\times10^{-6}}{2\times\frac{1}{36\pi}\times10^{-9}}\underline{a}_z$$

$$= 180\pi\times10^3\underline{a}_x - 360\pi\times10^3\underline{a}_y - 540\pi\times10^3\underline{a}_z \ \text{V/m}$$

$$= 565.49\,\underline{a}_x - 1130.98\,\underline{a}_y - 1696.46\,\underline{a}_z \ \text{KV/m}$$

b) $R(0,-2,1)$

$$\underline{E} = \frac{10^{-6}}{2\times\frac{1}{36\pi}\times10^{-9}}\left[10(-\underline{a}_x) - 20(\underline{a}_y) + 30(-\underline{a}_z)\right] \quad \begin{cases} \underline{a}_{n1} = -\underline{a}_x & (0<2) \\ \underline{a}_{n2} = \underline{a}_y & (-2>-3) \\ \underline{a}_{n3} = -\underline{a}_z & (1<5) \end{cases}$$

$$= -565.49\,\underline{a}_x - 1130.98\,\underline{a}_y - 1696.46\,\underline{a}_z \ \text{KV/m}$$

c) $Q(3,-4,10)$

$$\underline{E} = \frac{10^{-6}}{2\times\frac{10^{-9}}{36\pi}}\left[10\,\underline{a}_x - 20(-\underline{a}_y) + 30\,\underline{a}_z\right] \quad \begin{cases} \underline{a}_{n1} = \underline{a}_x & (3>2) \\ \underline{a}_{n2} = -\underline{a}_y & (-4<-3) \\ \underline{a}_{n3} = \underline{a}_z & (10>5) \end{cases}$$

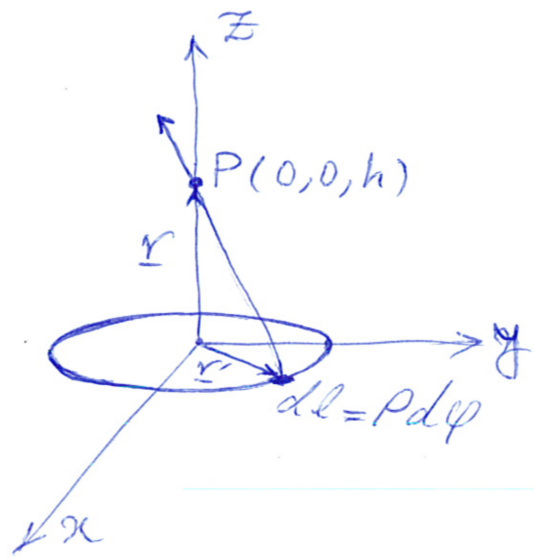
$$= 565.49\,\underline{a}_x + 1130.98\,\underline{a}_y + 1696.46\,\underline{a}_z \ \text{KV/m}$$

4.17 Ring : $\begin{cases} y^2+z^2=4 \\ x=0 \end{cases}$ uniform charge $\rho_L = 5\ \mu C/m$

a) $\underline{D}$ at $P(3,0,0)=?$

It's similar to a ring located at $z=0$, $x^2+y^2=4$.
Now, we calculate $\underline{D}$ at $P(0,0,3)$.

$$\begin{cases} \underline{r} = h\underline{a}_z \\ \underline{r}' = \rho \underline{a}_\rho \end{cases}$$



$$\underline{D} = \int \frac{\rho_L dl}{4\pi} \frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

Due to symmetry :

$$D_\rho = 0 \Rightarrow \underline{D} = D_z \underline{a}_z$$

$$D_z = \frac{\rho_L}{4\pi} \int_{\varphi=0}^{2\pi} \frac{\rho d\varphi \times h}{(\rho^2+h^2)^{3/2}} \qquad \rho = 2 \quad (x^2+y^2=4)$$

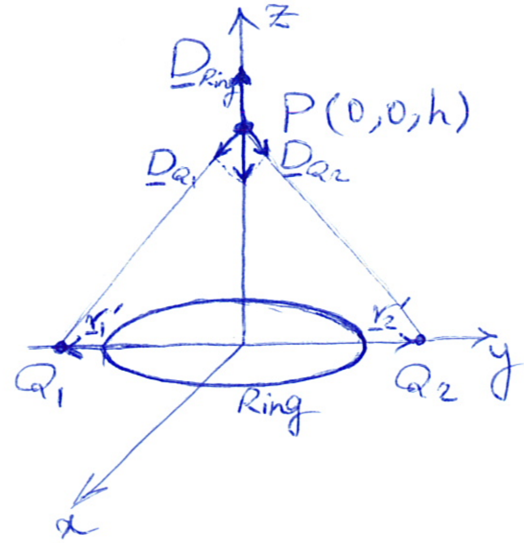
$$\Rightarrow \underline{D} = \frac{\rho_L \rho h \times 2\pi}{4\pi(\rho^2+h^2)^{3/2}} \underline{a}_z = \frac{\rho_L \rho h}{2(\rho^2+h^2)^{3/2}} \underline{a}_z$$

at $P(0,0,3)$: $\underline{D} = \dfrac{5\times10^{-6} \times 2 \times 3}{2(2^2+3^2)^{3/2}} \underline{a}_z = 0.32 \times 10^{-6}\ \underline{a}_z\ C/m^2$

b) Q at $(0,-3,0)$ & Q at $(0,3,0)$
at P:
$$\underline{D}_{Ring} + \underline{D}_{Q_1} + \underline{D}_{Q_2} = 0 \Rightarrow Q = ?$$

4.17

b) $Q_1 = Q_2 = Q$



$$\begin{cases} \underline{r} = h\,\underline{a}_z = 3\,\underline{a}_z \\ \underline{r}_1' = -3\underline{a}_y \\ \underline{r}_2' = 3\underline{a}_y \end{cases}$$

$$\underline{D}_{Q_1} = \frac{Q}{4\pi}\frac{(\underline{r}-\underline{r}_1')}{|\underline{r}-\underline{r}_1'|^3} = \frac{Q}{4\pi}\frac{(3\underline{a}_z+3\underline{a}_y)}{(3^2+3^2)^{3/2}}$$

$$\underline{D}_{Q_2} = \frac{Q}{4\pi}\frac{(\underline{r}-\underline{r}_2')}{|\underline{r}-\underline{r}_2'|^3} = \frac{Q}{4\pi}\frac{(3\underline{a}_z-3\underline{a}_y)}{(3^2+3^2)^{3/2}}$$

$$\underline{D}_{Q_1} + \underline{D}_{Q_2} = \frac{Q \times 6\underline{a}_z}{4\pi \times (18)^{3/2}}$$

$$\underline{D}_{Ring} + \underline{D}_{Q_1} + \underline{D}_{Q_2} = 0.32\times10^{-6}\,\underline{a}_z + \frac{6Q}{4\pi\times18^{3/2}}\underline{a}_z = \underline{0}$$

$$\Rightarrow Q = -0.32\times10^{-6}\times\frac{4\pi}{6}\times(18)^{3/2} = -51.182\ \mu C$$

4.18 Charge Density = ? $\rho_v = \nabla \cdot \underline{D}$

a) $\underline{D} = 8xy\,\underline{a}_x + 4x^2\,\underline{a}_y \ C/m^2$

$$\rho_v = \nabla \cdot \underline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 8y + \underset{\nwarrow D_z = 0}{0} = 8y \ C/m^2$$

b) $\underline{D} = 4\rho \sin\varphi\,\underline{a}_\rho + 2\rho\cos\varphi\,\underline{a}_\varphi + 2z^2\,\underline{a}_z \ C/m^2$

$$\rho_v = \nabla \cdot \underline{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial \varphi} + \frac{\partial D_z}{\partial z}$$

$$= 8\sin\varphi + 2(-\sin\varphi) + 4z$$

$$= 6\sin\varphi + 4z \ C/m^2$$

c) $\underline{D} = \frac{2\cos\theta}{r^3}\underline{a}_r + \frac{\sin\theta}{r^3}\underline{a}_\theta \ C/m^2$

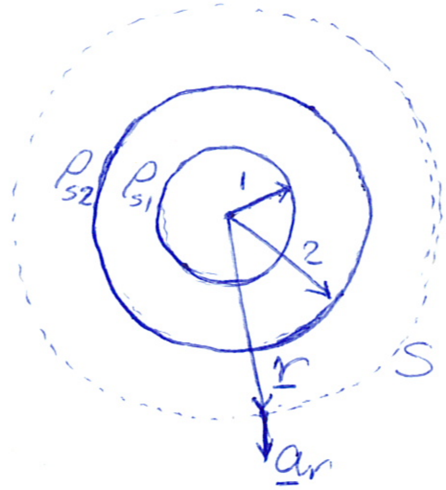
$$\rho_v = \nabla \cdot \underline{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\varphi}{\partial \varphi}$$

$$= \frac{-2\cos\theta}{r^4} + \frac{2\cos\theta}{r^4} + 0$$

$$= 0 \ C/m^2$$

4.19 $\begin{cases} \rho_{s1} = 8\ nC/m^2 & \text{at } r = 1\ m \\ \rho_{s2} = -6\ mC/m^2 & \text{at } r = 2\ m \end{cases}$

$\underline{D}$ at $(r = 3\ m) = ?$



Gauss's Law:

$$\oint_S \underline{D} \cdot d\underline{s} = Q$$

Due to symmetry: $\underline{D} = D_r \underline{a_r}$

$$Q = \oint_S \underline{D} \cdot d\underline{S} = \oint_S D_r\, ds = D_r \oint_S ds = D_r \times 4\pi r^2$$

$$Q = Q_1 + Q_2 = \rho_{s1} \times 4\pi r_1^2 + \rho_{s2} \times 4\pi r_2^2$$

$$= 8 \times 10^{-9} \times 4\pi \times 1^2 + (-6 \times 10^{-3}) \times 4\pi \times 2^2$$

$$= -0.30159\ C$$

$$\Rightarrow \underline{D} = \frac{Q}{4\pi r^2}\underline{a_r} = \frac{-0.30159}{4\pi \times 3^2}\underline{a_r} = -2.6667\,\underline{a_r}\ mC/m^2$$

$$\Rightarrow \underline{D} = -2.6667\,\underline{a_r}\ mC/m^2$$

4.21 $\underline{D} = 2xy\,\underline{a}_x + x^2\underline{a}_y \quad C/m^2$

a) $\rho_v = ?$

$$\rho_v = \nabla\cdot\underline{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= 2y + 0 + 0 = 2y \quad C/m^3$$

b) $\Psi = ?$ $\quad \begin{cases} 0 < x < 1 \\ 0 < z < 1 \\ y = 1 \end{cases} \Rightarrow \underline{a}_n = \underline{a}_y$

$$\Psi = \iint \underline{D}\cdot d\underline{S} \qquad d\underline{S} = dx\,dz\,\underline{a}_y$$

$$= \int_{x=0}^{1}\int_{z=0}^{1} (2xy\,\underline{a}_x + x^2\underline{a}_y)\cdot(dx\,dz\,\underline{a}_y)$$

$$= \int_{x=0}^{1}\int_{z=0}^{1} x^2\,dx\,dz = \frac{x^3}{3}\Big|_0^1 \times z\Big|_0^1 = 1/3 \ C$$

c) Total charge $= ?$ $\quad \begin{cases} 0 < x < 1 \\ 0 < y < 1 \\ 0 < z < 1 \end{cases}$

$$Q = \iiint \rho_v\,dv = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} 2y\,dz\,dy\,dx$$

$$= x\Big|_0^1 \times y^2\Big|_0^1 \times z\Big|_0^1$$

$$= 1 \ C$$

4.22 $\underline{D} = 2\rho(z+1)\cos\varphi\,\underline{a}_\rho - \rho(z+1)\sin\varphi\,\underline{a}_\varphi + \rho^2\cos\varphi\,\underline{a}_z \quad \mu C/m^2$

a) $\rho_v = ?$

$$\rho_v = \nabla\cdot\underline{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial\varphi} + \frac{\partial D_z}{\partial z}$$

$$= 4(z+1)\cos\varphi - (z+1)\cos\varphi + 0$$

$$= 3(z+1)\cos\varphi \quad \mu C/m^3$$

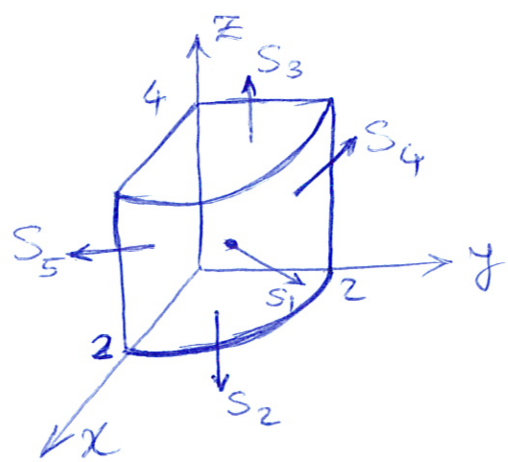
b) Total charge = ? $\begin{cases} 0<\rho<2 \\ 0<\varphi<\pi/2 \\ 0<z<4 \end{cases}$

$$Q = \iiint \rho_v\,dv = \int_{\rho=0}^{2}\int_{\varphi=0}^{\pi/2}\int_{z=0}^{4} 3(z+1)\cos\varphi\;\rho\,d\rho\,d\varphi\,dz \quad \mu C$$

$$= 3\times\frac{\rho^2}{2}\Big|_0^2 \times \sin\varphi\Big|_0^{\pi/2} \times \left(\frac{z^2}{2}+z\right)\Big|_0^4 \quad \mu C$$

$$= 3\times\frac{4}{2}\times 1\times\left(\frac{16}{2}+4\right) \quad \mu C$$

$$= 72 \quad \mu C$$



c) Confirm Gauss's Law $\Psi = ?$

$$\Psi = \oint_S \underline{D}\cdot d\underline{S}$$

$$= \int_{S_1}\underline{D}\cdot d\underline{S} + \int_{S_2}\underline{D}\cdot d\underline{S} + \int_{S_3}\underline{D}\cdot d\underline{S} + \int_{S_4}\underline{D}\cdot d\underline{S} + \int_{S_5}\underline{D}\cdot d\underline{S}$$

$$= \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \Psi_5$$

4.22

c) $\Psi_1 = \int_{S_1} \underline{D}\cdot d\underline{S} = \int_{z=0}^{4}\int_{\varphi=0}^{\pi/2} \underline{D}\cdot(\rho\, d\varphi\, dz\, \underline{a}_\rho)$, $\rho = 2$

$$= \int_{z=0}^{4}\int_{\varphi=0}^{\pi/2} 2\rho^2 (z+1)\cos\varphi\, d\varphi\, dz$$

$$= 2\rho^2\Big|_{\rho=2} \times \left(\frac{z^2}{2}+z\right)\Big|_0^4 \times \sin\varphi\Big|_0^{\pi/2}$$

$$= 2\times 4\times 12\times 1 = 96 \quad \mu C$$

$\Psi_2 = \int_{S_2} \underline{D}\cdot d\underline{S} = \int_{S_2} \underline{D}\cdot(\rho\, d\rho\, d\varphi(-\underline{a}_z))$, $z = 0$

$$= \int_{\rho=0}^{2}\int_{\varphi=0}^{\pi/2} -\rho^3\cos\varphi\, d\rho\, d\varphi$$

$$= -\frac{\rho^4}{4}\Big|_0^2 \times \sin\varphi\Big|_0^{\pi/2}$$

$$= -4\times 1 = -4 \quad \mu C$$

$\Psi_3 = \int_{S_3} \underline{D}\cdot d\underline{S} = \int_{S_3} \underline{D}\cdot(\rho\, d\rho\, d\varphi\, \underline{a}_z)$, $z = 4$

$$= \int_{\rho=0}^{2}\int_{\varphi=0}^{\pi/2} \rho^3\cos\varphi\, d\rho\, d\varphi$$

$$= +4 \quad \mu C$$

$\Psi_4 = \int_{S_4} \underline{D}\cdot d\underline{S} = \int_{S_4} \underline{D}\cdot(d\rho\, dz\, \underline{a}_\varphi)$, $\varphi = \pi/2$

$$= \int_{\rho=0}^{2}\int_{z=0}^{4} -\rho(z+1)\sin\varphi\, d\rho\, dz$$

4.22

c) $\Psi_4 = -\frac{\rho^2}{2}\Big|_0^2 \times \left(\frac{z^2}{2}+z\right)\Big|_0^4 \times \sin\varphi\Big|_{\varphi=\pi/2}$

$= -2 \times 12 \times 1 = -24 \ \mu C$

$\Psi_5 = \int_{S_5} \underline{D}\cdot d\underline{S} = \int_{S_5} \underline{D}\cdot\left(d\rho\, dz\,(-\underline{a}_\varphi)\right) \quad , \varphi = 0$

$= \int_{\rho=0}^{2}\int_{z=0}^{4} \rho\,(z+1)\underbrace{\sin\varphi}_{=0}\, d\rho\, d\varphi\Big|_{\varphi=0}$

$= 0$

$\Psi = \sum_{k=1}^{5} \Psi_k = 96 - 4 + 4 - 24 + 0 = 72 \ \mu C$

$\Psi = 72 \ \mu C = Q \quad \checkmark$

4.25 $\rho_v = \begin{cases} \frac{10}{r^2} \ mC/m^3 & 1 < r < 4 \\ 0 & r > 4, 0 < r < 1 \end{cases}$

a) $\Psi = ?$ at $r = 2m$ & $r = 6m$

$$\Psi = Q_{enclosed} = \iiint \rho_v \, dv$$

@ $r = 2 \Rightarrow \Psi = ?$

$$\Psi = Q = \int_{r=1}^{2} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{10}{r^2} \times r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= 10 \times r \Big|_1^2 \times (-\cos\theta)\Big|_0^{\pi} \times \varphi \Big|_0^{2\pi} = 10 \times 1 \times 2 \times 2\pi \ mC$$

$\Rightarrow \Psi = 40\pi \ mC = 125.6637 \ mC$

@ $r = 6 \Rightarrow \Psi = ?$

$$\Psi = Q = \int_{r=1}^{4} \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{10}{r^2} r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= 10 \times r \Big|_1^4 \times (-\cos\theta)\Big|_0^{\pi} \times \varphi \Big|_0^{2\pi} = 10 \times 3 \times 2 \times 2\pi \ mC$$

$\Rightarrow \Psi = 120\pi \ mC = 376.9911 \ mC$

b) $\underline{D} = ?$ at $r = 1 m$ & $r = 5 m$

$$\Psi = Q_{enclosed} = \oint \underline{D} \cdot d\underline{S} = D_r \oint ds = D_r \times 4\pi r^2$$

Due to symmetry : $\underline{D} = D_r \underline{a_r}$

4.25

b) $\underline{D} = \frac{Q_{enclosed}}{4\pi r^2} \underline{a}_r$

@ $r=1 \Rightarrow \underline{D} = ?$

$Q_{enclosed} = 0 \Rightarrow \underline{D} = \underline{0}$

@ $r=5 \Rightarrow \underline{D} = ?$

$Q_{enclosed} = 120\pi \Rightarrow \underline{D} = \frac{120\pi \underline{a}_r}{4\pi \times 5^2} = \frac{120\pi}{100\pi} \underline{a}_r$

$\Rightarrow \underline{D} = 1.2 \underline{a}_r \; mC/m^2$

4.28 $\begin{cases} Q_1 = 2\,nC & \text{at } (1,0,3) \\ Q_2 = -4\,nC & \text{at } (-2,1,5) \end{cases}$

$V_p = ?$ at $P(1,-2,3)$

$$\begin{cases} \underline{r} = (1,-2,3) \\ \underline{r}_1' = (1,0,3) \\ \underline{r}_2' = (-2,1,5) \end{cases} \Rightarrow \begin{cases} |\underline{r}-\underline{r}_1'| = (0^2+(-2)^2+0^2)^{1/2} = 2 \\ |\underline{r}-\underline{r}_2'| = (3^2+(-3)^2+(-2)^2)^{1/2} = \sqrt{22} \end{cases}$$

$$V_p = \frac{Q_1}{4\pi\varepsilon_0|\underline{r}-\underline{r}_1'|} + \frac{Q_2}{4\pi\varepsilon_0|\underline{r}-\underline{r}_2'|}$$

$$= \frac{10^{-9}}{4\pi \times \frac{10^{-9}}{36\pi}}\left[\frac{2}{2} + \frac{-4}{\sqrt{22}}\right] = 9\left(1 - \frac{4}{\sqrt{22}}\right)$$

$$= 1.3248\ V$$

4.30 $\begin{cases} Q_1 = 1 \text{ mC} & \text{at } (0,0,4) \\ Q_2 = -2 \text{ mC} & \text{at } (-2,5,1) \\ Q_3 = 3 \text{ mC} & \text{at } (3,-4,6) \end{cases}$

a) $V_P = ?$ at $P(-1,1,2)$

$$V_P = \sum_{k=1}^{3} \frac{Q_k}{4\pi\varepsilon_0 |\underline{r} - \underline{r}'_k|}$$

$$\begin{cases} |\underline{r} - \underline{r}'_1| = (1+1+4)^{1/2} = \sqrt{6} \\ |\underline{r} - \underline{r}'_2| = (1+16+1)^{1/2} = \sqrt{18} \\ |\underline{r} - \underline{r}'_3| = (16+25+16)^{1/2} = \sqrt{57} \end{cases}$$

$$V_P = \frac{10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi}} \left[\frac{1}{\sqrt{6}} + \frac{-2}{\sqrt{18}} + \frac{3}{\sqrt{57}} \right] = 9\times10^{6} \left[\frac{1}{\sqrt{6}} - \frac{2}{\sqrt{18}} + \frac{3}{\sqrt{57}} \right]$$

$$\Rightarrow V_P = 3.0078 \times 10^{6} \text{ V}$$

b) $Q(1,2,3) \Rightarrow V_{PQ} = ?$

for $Q(1,2,3)$:

$$V_{PQ} = V_Q - V_P$$

$$\begin{cases} |\underline{r} - \underline{r}'_1| = (1+4+1)^{1/2} = \sqrt{6} \\ |\underline{r} - \underline{r}'_2| = (9+9+4)^{1/2} = \sqrt{22} \\ |\underline{r} - \underline{r}'_3| = (4+36+9)^{1/2} = 7 \end{cases}$$

$$V_Q = \frac{10^{-3}}{4\pi \times \frac{10^{-9}}{36\pi}} \left[\frac{1}{\sqrt{6}} + \frac{-2}{\sqrt{22}} + \frac{3}{7} \right] = 9\times10^{6} \left[\frac{1}{\sqrt{6}} - \frac{2}{\sqrt{22}} + \frac{3}{7} \right]$$

$$\Rightarrow V_Q = 3.6938 \times 10^{6} \text{ V}$$

$$V_{PQ} = V_Q - V_P = 685.9333 \text{ KV}$$

4.33 $\rho_v = 1 - \left(\frac{r}{a}\right)^2 \quad 0<r<a$

$\underline{E} = ?$ at $r = 5a$

$$Q = \oint_S \underline{D} \cdot d\underline{S} = D_r \oint ds = D_r \times 4\pi r^2$$

$$\Rightarrow \underline{D} = \frac{Q}{4\pi r^2}\underline{a}_r \quad \Rightarrow \underline{E} = \frac{\underline{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}\underline{a}_r \quad , r = 5a$$

$$Q = \iiint \rho_v \, dv = \int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\left[1-\left(\frac{r}{a}\right)^2\right] r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]\Bigg|_0^a \times \underbrace{(-\cos\theta)\Big|_0^{\pi} \times \varphi\Big|_0^{2\pi}}_{=4\pi}$$

$$= \left(\frac{a^3}{3} - \frac{a^5}{5a^2}\right) \times 4\pi$$

$$= 4\pi a^3\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{8}{15}\pi a^3$$

$$\Rightarrow \underline{E} = \frac{\frac{8}{15}\pi a^3}{4\pi \times \frac{10^{-9}}{36\pi} \times (5a)^2}\underline{a}_r = \frac{72}{15}\pi a \times \frac{10^9}{25}\underline{a}_r = \frac{24}{125}\pi a \times 10^9 \underline{a}_r$$

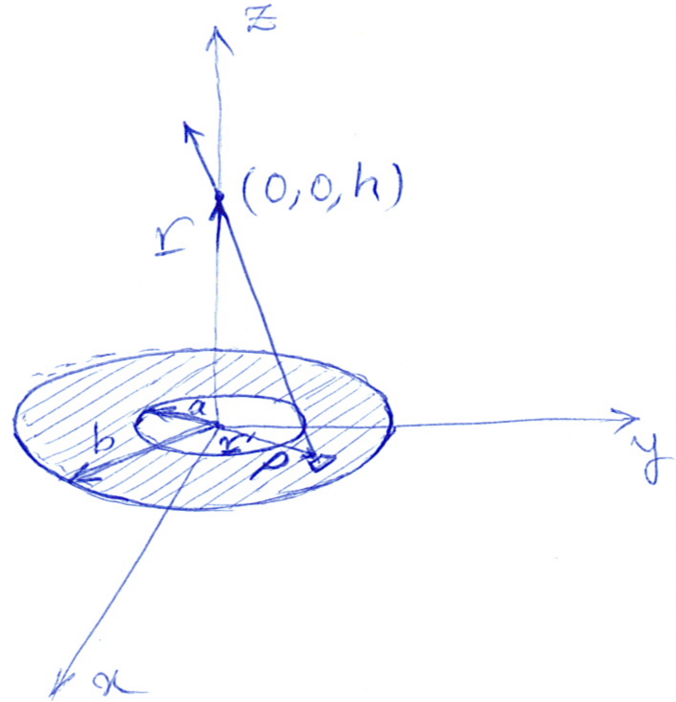
$$\Rightarrow \underline{E} = 0.192\pi a \times 10^9 \underline{a}_r \ \text{V/m}$$

$$= 0.6032 a \times 10^9 \underline{a}_r \ \text{V/m}$$

4.35 $a < \rho < b$

$$\rho_s = \frac{Q}{S} = \frac{Q}{\pi(b^2 - a^2)}$$

$\underline{D} = ?$ at $(0,0,h)$



$$\underline{D} = \int_S \frac{\rho_s ds (\underline{r} - \underline{r}')}{4\pi |\underline{r} - \underline{r}'|^3}$$

$$= \int_{\rho=a}^{b} \int_{\varphi=0}^{2\pi} \frac{\rho_s (h\underline{a}_z - \rho \underline{a}_\rho)\rho d\rho d\varphi}{4\pi (h^2 + \rho^2)^{3/2}} \begin{cases} \underline{r} - \underline{r}' = h\underline{a}_z - \rho \underline{a}_\rho \\ |\underline{r} - \underline{r}'| = \sqrt{h^2 + \rho^2} \end{cases}$$

$$ds = \rho d\rho d\varphi$$

Due to symmetry : $D_\rho = 0 \Rightarrow \underline{D} = D_z \underline{a}_z$

$$D_z = \int_{\rho=a}^{b} \int_{\varphi=0}^{2\pi} \frac{\rho_s h \rho d\rho d\varphi}{4\pi (h^2 + \rho^2)^{3/2}} = \frac{\rho_s h}{4\pi} \int_{\rho=a}^{b} \frac{\rho d\rho}{(h^2 + \rho^2)^{3/2}} \times \underbrace{\int_{\varphi=0}^{2\pi} d\varphi}_{=2\pi}$$

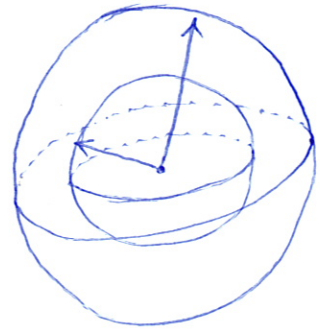
$$= \frac{\rho_s h}{2} \int_{\rho=a}^{b} \frac{\rho}{(h^2 + \rho^2)^{3/2}} d\rho = \frac{\rho_s h}{2} \times \frac{-1}{\sqrt{h^2 + \rho^2}} \Big|_a^b$$

$$= \frac{\rho_s h}{2} \left[\frac{1}{\sqrt{h^2 + a^2}} - \frac{1}{\sqrt{h^2 + b^2}} \right]$$

$$\Rightarrow \underline{D} = \frac{Qh}{2\pi (b^2 - a^2)} \left[\frac{1}{\sqrt{a^2 + h^2}} - \frac{1}{\sqrt{b^2 + h^2}} \right] \underline{a}_z$$

4.37 $\begin{cases} Q_1 = 10\ nC & \text{on } r = 3\ cm \\ Q_2 = -5\ nC & \text{on } r = 5\ cm \end{cases}$

$$\underline{D} = \begin{cases} ? & r < 3\ cm \\ ? & 3 < r < 5\ cm \\ ? & r > 5\ cm \end{cases}$$



$$r < 3\ cm \Rightarrow Q_{enc} = 0$$

$$\Rightarrow \underline{D} = \underline{0}$$

$$3^{cm} < r < 5^{cm} \Rightarrow Q_{enc} = 10\ nC$$

$$= \oint_S \underline{D} . d\underline{S} = D_r \times 4\pi r^2$$

$$\Rightarrow \underline{D} = \frac{10}{4\pi r^2} \underline{a}_r \ nC/m^2$$

$$r > 5^{cm} \Rightarrow Q_{enc} = 10 - 5 = 5\ nC$$

$$\Rightarrow \underline{D} = \frac{5}{4\pi r^2} \underline{a}_r \ nC/m^2$$

$$\Rightarrow \underline{D} = \begin{cases} \underline{0} & r < 3^{cm} \\ \frac{10}{4\pi r^2} \underline{a}_r \ nC/m^2 & 3^{cm} < r < 5^{cm} \\ \frac{5}{4\pi r^2} \underline{a}_r \ nC/m^2 & 5^{cm} < r \end{cases}$$

4.40 $\underline{E} = 2\rho \sin\varphi \, \underline{a}_\rho + \rho \cos\varphi \, \underline{a}_\varphi$ V/m

$Q = 20\ \mu C \quad \Rightarrow W = ?$

$$W = -Q \int \underline{E} \cdot d\underline{\ell}$$

$$d\underline{\ell} = d\rho \, \underline{a}_\rho + \rho d\varphi \, \underline{a}_\varphi + dz \, \underline{a}_z$$

$$\underline{E} \cdot d\underline{\ell} = 2\rho \sin\varphi \, d\rho + \rho^2 \cos\varphi \, d\varphi$$

$$\begin{cases} A(\rho_1, \varphi_1, z_1) \longrightarrow A'(\rho_2, \varphi_1, z_1) \longrightarrow B'(\rho_2, \varphi_2, z_1) \longrightarrow B \\ B(\rho_2, \varphi_2, z_2) \end{cases}$$

$$W = -Q \int_A^B \underline{E} \cdot d\underline{\ell} = -Q \int_{\rho=\rho_1}^{\rho_2} 2\rho \sin\varphi \, d\rho \Big|_{\varphi=\varphi_1} - Q \int_{\varphi=\varphi_1}^{\varphi_2} \rho^2 \cos\varphi \, d\varphi \Big|_{\rho=\rho_2}$$

$$-Q \int_{z=z_1}^{z_2} 0 \, dz$$

$$\Rightarrow W = -Q\left(\rho^2 \Big|_{\rho_1}^{\rho_2}\right) \sin\varphi_1 - Q \rho_2^2 \left(\sin\varphi \Big|_{\varphi_1}^{\varphi_2}\right) - 0$$

$$= -Q\left[(\rho_2^2 - \rho_1^2)\sin\varphi_1 + \rho_2^2(\sin\varphi_2 - \sin\varphi_1)\right]$$

$$= -Q\left[\cancel{\rho_2^2 \sin\varphi_1} - \rho_1^2 \sin\varphi_1 + \rho_2^2 \sin\varphi_2 - \cancel{\rho_2^2 \sin\varphi_1}\right]$$

$$\Rightarrow W = -Q\left[\rho_2^2 \sin\varphi_2 - \rho_1^2 \sin\varphi_1\right]$$

4.42 $\underline{E} = \frac{10}{r^2}\underline{a_r}$ V/m

$$\begin{cases} A\left(1, \frac{\pi}{4}, \frac{\pi}{2}\right) \\ B(5, \pi, 0) \end{cases} \Rightarrow V_{AB} = ?$$

$$V_{AB} = V_B - V_A = -\int_A^B \underline{E} \cdot \underline{dl}$$

$$= -\int_{r=1}^{5} E_r \, dr$$

$$= -\int_{r=1}^{5} \frac{10}{r^2} dr$$

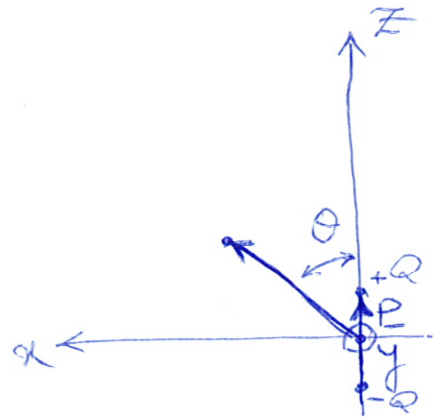
$$= \left.\frac{10}{r}\right|_1^5 = 2 - 10$$

$$= -8 \text{ V}$$

$\Rightarrow V_{AB} = -8$ V

4.50 Electric Dipole $\underline{P} = p\,\underline{a}_z$ C.m at $(x, z) = (0, 0)$

V at $(0, 1\,nm) = 9\,V \Rightarrow V$ at $(1\,nm, 1\,nm) = ?$



$$V = \frac{\underline{P} \cdot \underline{a}_r}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\underset{\uparrow\,x}{(0,}\ \underset{\uparrow\,z}{1\,nm)} \equiv (1\,nm, 0^\circ, 0^\circ)$$

$$\Rightarrow V = \frac{p\,\overbrace{\cos 0^\circ}^{=1}}{4\pi\varepsilon_0 \times (10^{-9})^2} = 9\,V \Rightarrow \frac{p}{4\pi\varepsilon_0 \times 10^{-18}} = 9\,V$$

$$(1\,nm, 1\,nm) \equiv (\sqrt{2}\,nm, 45^\circ, 0^\circ)$$

$$\Rightarrow V = \frac{p\cos 45^\circ}{4\pi\varepsilon_0 \times 2 \times 10^{-18}} = \underbrace{\frac{p}{4\pi\varepsilon_0 \times 10^{-18}}}_{=9} \times \frac{\frac{\sqrt{2}}{2}}{2} = 9\frac{\sqrt{2}}{4}\,V$$

$$\Rightarrow V = 3.1820\,V$$

4.53

$V = 2x^2 + 6y^2$ V free space

$\begin{cases} -1 \leqslant x \leqslant 1 \\ -1 \leqslant y \leqslant 1 \\ -1 \leqslant z \leqslant 1 \end{cases} \Rightarrow W_E = ?$

$$W_E = \frac{1}{2}\varepsilon_0 \iiint_v |\underline{E}|^2 dv$$

$$\underline{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\underline{a}_x + \frac{\partial V}{\partial y}\underline{a}_y + \frac{\partial V}{\partial z}\underline{a}_z\right)$$

$$= -4x\underline{a}_x - 12y\underline{a}_y$$

$$|\underline{E}|^2 = 16x^2 + 144y^2$$

$$\Rightarrow W_E = \frac{1}{2}\varepsilon_0 \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1}(16x^2 + 144y^2)\,dx\,dy\,dz$$

$$= \frac{1}{2}\varepsilon_0 \left[2\frac{16}{3}x^3\Big|_{-1}^{1} + 2\frac{144}{3}y^3\Big|_{-1}^{1}\right] \times z\Big|_{-1}^{1}$$

$$= \frac{1}{2}\frac{10^{-9}}{36\pi}\left[\frac{16\times 8}{3} + \frac{144\times 8}{3}\right] = \frac{1280}{2\times 3} \times \frac{10^{-9}}{36\pi}\ J$$

$$\Rightarrow W_E = 1.8863\ nJ$$