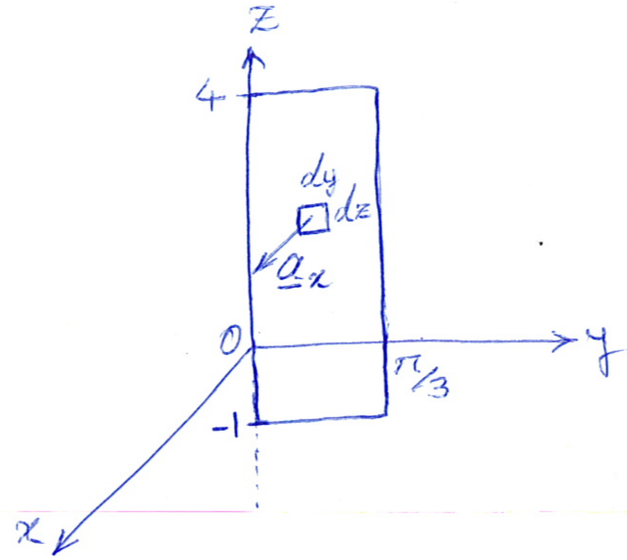
5.1 $\underline{J} = 50 \sin(4y)\, \underline{a}_x \quad A/m^2$

$$\begin{cases} x = 0 \\ 0 \leqslant y \leqslant \pi/3 \\ -1 \leqslant z \leqslant z \end{cases} \Rightarrow I = ?$$



$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$d\underline{s} = dy\,dz\, \underline{a}_x$$

$$I = \iint_S \left(50 \sin(4y)\, \underline{a}_x\right) \cdot \left(dy\,dz\, \underline{a}_x\right)$$

$$= \int_{y=0}^{\pi/3} \int_{z=-1}^{4} 50 \sin(4y)\, dy\, dz$$

$$= 50 \times \int_{y=0}^{\pi/3} \sin(4y)\, dy \times \int_{z=-1}^{4} dz$$

$$= 50 \times \left.\frac{-\cos(4y)}{4}\right|_{y=0}^{\pi/3} \times \left. z \right|_{z=-1}^{4}$$

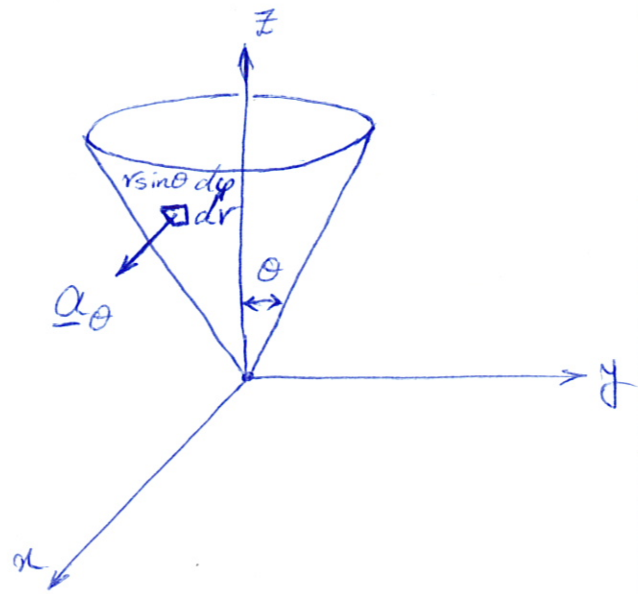
$$= 50 \times \frac{(-(-0.5) - (-1))}{4} \times (4 - (-1))$$

$$= 93.75 \text{ A}$$

$$\Rightarrow I = 93.75 \text{ A}$$

5.2 $\quad \underline{J} = 3r^2\cos\theta\,\underline{a}_r - r^2\sin\theta\,\underline{a}_\theta \quad A/m^2$

a) $\begin{cases} \theta = 30^\circ \\ 0 < \varphi < 2\pi \\ 0 < r < 2\,m \end{cases} \Rightarrow I = ?$



$$I = \iint_S \underline{J}\cdot d\underline{s}$$

$$d\underline{s} = r\sin\theta\, d\varphi\, dr\, \underline{a}_\theta$$

$$I = \iint_S (3r^2\cos\theta\,\underline{a}_r - r^2\sin\theta\,\underline{a}_\theta)\cdot(r\sin\theta\, dr\, d\varphi\,\underline{a}_\theta)$$

$$= \int_{r=0}^{2}\int_{\varphi=0}^{2\pi} -r^3\sin^2\theta\, dr\, d\varphi$$

$$= -\sin^2\theta\Big|_{\theta=30^\circ} \times \int_{r=0}^{2} r^3\, dr \times \int_{\varphi=0}^{2\pi} d\varphi$$

$$= -\sin^2\theta\Big|_{\theta=30^\circ} \times \frac{r^4}{4}\Big|_{r=0}^{2} \times \varphi\Big|_{\varphi=0}^{2\pi}$$

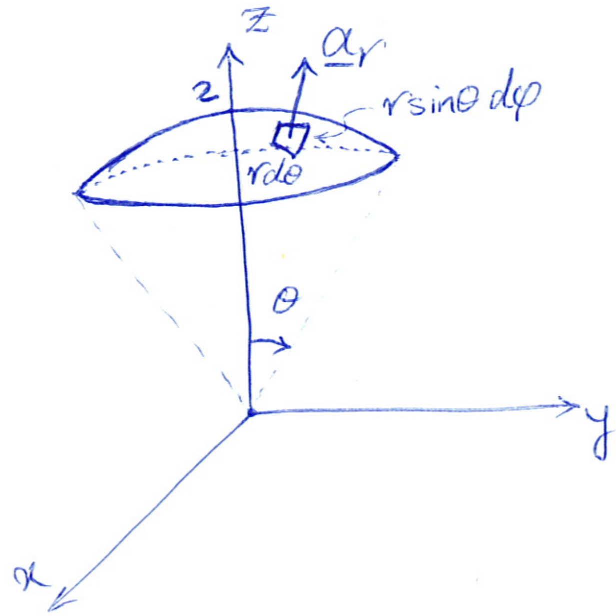
$$= -\sin^2(30^\circ) \times \frac{2^4}{4} \times 2\pi$$

$$= -\frac{1}{4} \times 4 \times 2\pi = -2\pi \ A$$

$$\Rightarrow I = -2\pi\ A = -6.2832\ A$$

5.2

b) $\begin{cases} r=2 \\ 0<\theta<30^\circ \\ 0<\varphi<2\pi \end{cases} \Rightarrow I=?$



$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$d\underline{s} = r^2 \sin\theta \, d\theta \, d\varphi \, \underline{a}_r$$

$$I = \iint_S (3r^2 \cos\theta \, \underline{a}_r - r^2 \sin\theta \, \underline{a}_\theta) \cdot (r^2 \sin\theta \, d\theta \, d\varphi \, \underline{a}_r)$$

$$= \int_{\theta=0}^{30^\circ} \int_{\varphi=0}^{2\pi} 3r^4 \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$= 3r^4 \Big|_{r=2} \times \int_{\theta=0}^{30^\circ} \cos\theta \sin\theta \, d\theta \times \int_{\varphi=0}^{2\pi} d\varphi$$

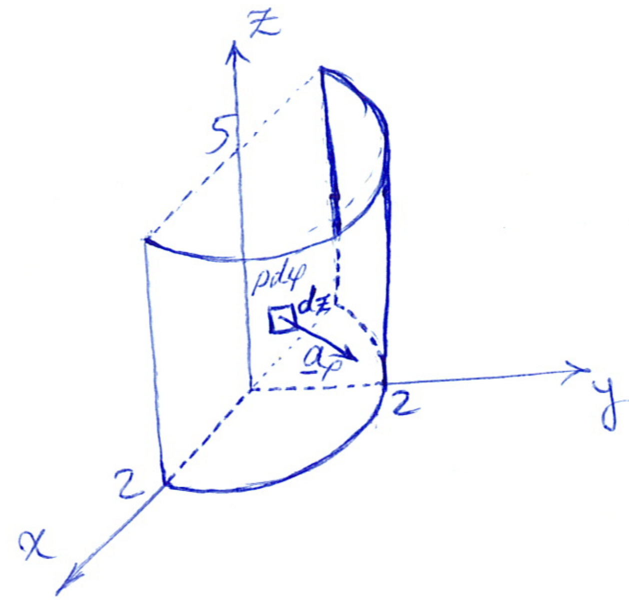
$$= 3r^4 \Big|_{r=2} \times \frac{\sin^2\theta}{2} \Big|_{\theta=0}^{30^\circ} \times \varphi \Big|_{\varphi=0}^{2\pi}$$

$$= 3 \times 2^4 \times \frac{1/4}{2} \times 2\pi$$

$$= 12\pi \ \text{A}$$

$$\Rightarrow I = 12\pi \ \text{A} = 37.6991 \ \text{A}$$

5.3 $\underline{J} = \frac{10}{\rho} \sin\varphi \, \underline{a}_\rho \quad A/m^2$



$$\begin{cases} \rho = 2 \\ 0 < \varphi < \pi \\ 0 < z < 5\,m \end{cases} \Rightarrow I = ?$$

$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$d\underline{s} = \rho \, d\varphi \, dz \, \underline{a}_\rho$$

$$I = \iint_S \left(\frac{10}{\rho} \sin\varphi \, \underline{a}_\rho \right) \cdot \left(\rho \, d\varphi \, dz \, \underline{a}_\rho \right)$$

$$= \int_{\varphi=0}^{\pi} \int_{z=0}^{5} 10 \sin\varphi \, d\varphi \, dz$$

$$= 10 \times \int_{\varphi=0}^{\pi} \sin\varphi \, d\varphi \times \int_{z=0}^{5} dz$$

$$= 10 \times (-\cos\varphi) \Big|_{\varphi=0}^{\pi} \times z \Big|_{z=0}^{5}$$

$$= 10 \times 2 \times 5 = 100 \text{ A}$$

$$\Rightarrow I = 100 \text{ A}$$

5.5 $Q = 10^{-4} e^{-3t}$ C from a sphere through a wire

$$\begin{cases} I \text{ at } t=0 = ? \\ I \text{ at } t=2.5^s = ? \end{cases}$$

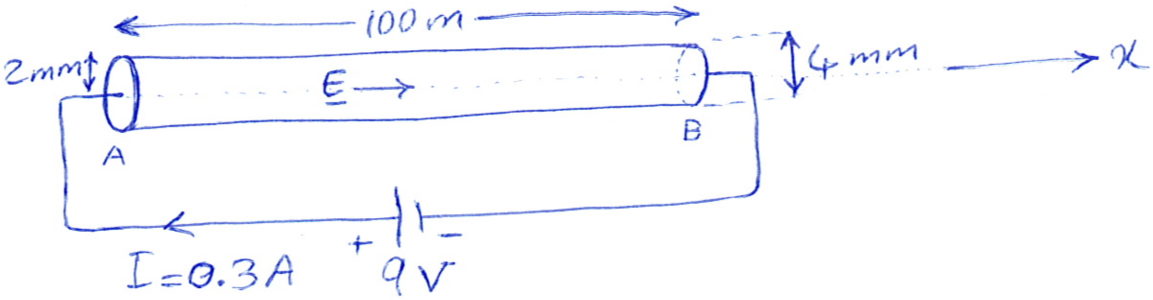
$$I = -\frac{dQ}{dt}$$

$$= -(-3) \times 10^{-4} e^{-3t}$$

$$\Rightarrow \quad I = 3\times 10^{-4} e^{-3t}$$

$$\begin{cases} I(t=0) = 3\times 10^{-4} \text{ A} = 0.3 \text{ mA} \\ I(t=2.5^s) = 3\times 10^{-4} e^{-3\times 2.5} = 0.1659 \ \mu\text{A} \end{cases}$$

5.8



a) $\underline{E} = ? \quad = E\underline{a}_x$

$$V_A - V_B = \int_A^B \underline{E}\cdot d\underline{l} = E \times l$$

$$\Rightarrow \quad 9 = E \times 100$$

$$\Rightarrow \quad E = 0.09 \ \text{V/m}$$

b) $\sigma = ?$

$$R = \frac{V}{I} = \frac{l}{\sigma S}$$

$$\Rightarrow \sigma = \frac{l}{RS} = \frac{l}{S} \times \frac{I}{V}$$

$$= \frac{100}{\pi(2\times10^{-3})^2} \times \frac{0.3}{9}$$

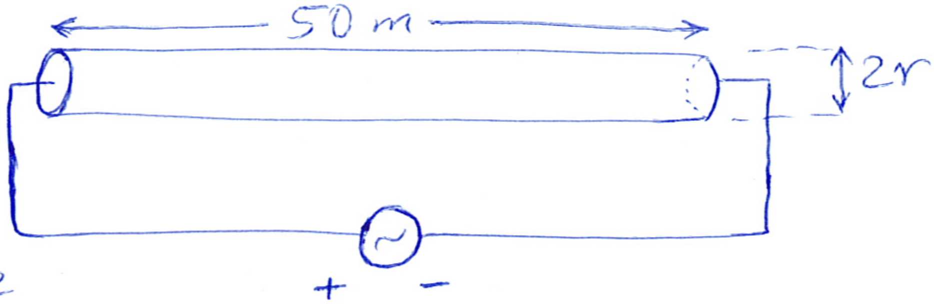
$$= \frac{10^8}{4\pi} \times \frac{0.3}{9} = \frac{10^8}{120\pi}$$

$$\Rightarrow \sigma = 2.6526 \times 10^5 \ \text{S/m}$$

5.10

$$\begin{cases} P = 2\ \text{KW} \\ V = 120\ V_{rms} @ 60\text{Hz} \\ \sigma = 1.5 \times 10^{7}\ S/m \end{cases} \Rightarrow r = ?$$

$l = 50\ m$



$$P = \frac{V^2}{R} = RI^2$$

$$\Rightarrow R = \frac{V^2}{P} = \frac{120^2}{2 \times 10^3} = 7.2\ \Omega$$

Also, $R = \dfrac{l}{\sigma S} = \dfrac{l}{\sigma \pi r^2}$

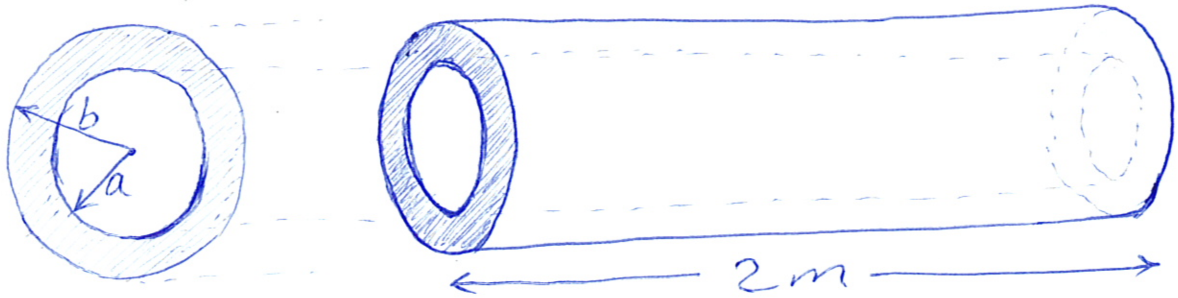
$$\Rightarrow r = \sqrt{\frac{l}{\sigma \pi R}}$$

$$= \sqrt{\frac{50}{1.5 \times 10^7 \times \pi \times 7.2}}$$

$$= \sqrt{0.1474 \times 10^{-6}}$$

$$\Rightarrow r = 0.3839\ mm$$

5.13



$$\begin{cases} \sigma = 3\times 10^{4}\ S/m \\ l = 2m \\ a = 3\ cm \\ b = 5\ cm \end{cases} \Rightarrow R = ?$$

$$R = \frac{l}{\sigma S} = \frac{l}{\sigma \underbrace{\pi(b^2-a^2)}_{S \rightarrow \text{shaded area}}}$$

$$= \frac{2}{3\times 10^4 \times \pi(5^2\times 10^{-4} - 3^2\times 10^{-4})}$$

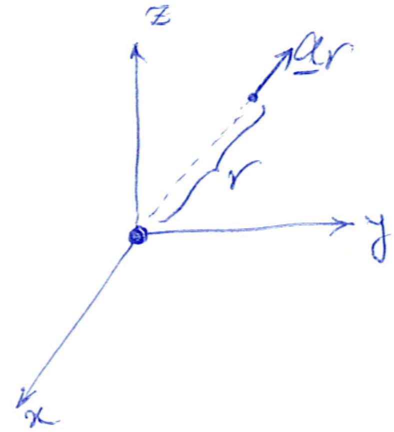
$$= \frac{2}{3\pi(25-9)} = \frac{1}{24\pi}$$

$$\Rightarrow R = 13.2629\ m\Omega$$

5.16 $Q = 10\ mC$ point charge in Wood $\rightarrow$ at Origin

$\varepsilon_r = 4.0$

$\underline{P}$ at $(r = 1\ m) = ?$



$$\underline{P} = \chi_e \varepsilon_0 \underline{E}$$

$$\underline{E} = \frac{Q}{4\pi \underbrace{\varepsilon_r \varepsilon_0}_{\varepsilon} r^2} \underline{a_r}$$

$$\Rightarrow \underline{P} = \chi_e \varepsilon_0 \underline{E} = \frac{\chi_e \cancel{\varepsilon_0} Q \underline{a_r}}{4\pi \varepsilon_r \cancel{\varepsilon_0} r^2} = \frac{\chi_e Q \underline{a_r}}{4\pi \varepsilon_r r^2}$$

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e)\varepsilon_0 \Rightarrow \begin{cases} \varepsilon_r = 4 \\ \chi_e = \varepsilon_r - 1 = 3 \end{cases}$$

$$\Rightarrow \underline{P} \text{ at } (r = 1\ m) = \frac{3 \times 10 \times 10^{-3}}{4\pi \times 4 \times 1^2} \underline{a_r} = 0.5968 \times 10^{-3} \underline{a_r}$$

$$= 0.5968\ mC/m^2\ \underline{a_r}$$

5.17 $\begin{cases} \underline{P} = \dfrac{100}{\rho}\underline{a}_\rho \ nC/m^2 \\ \varepsilon_r = 3.5 \end{cases} \Rightarrow \begin{cases} \underline{E} \text{ at}(\rho=2m) = ? \\ \underline{D} \text{ at}(\rho=2m) = ? \end{cases}$

$$\underline{P} = \chi_e \varepsilon_0 \underline{E} \Rightarrow \underline{E} = \frac{1}{\chi_e \varepsilon_0}\underline{P} = \frac{1}{(\varepsilon_r - 1)\varepsilon_0} \times \frac{100}{\rho} \times 10^{-9}\underline{a}_\rho$$

$$\Rightarrow \underline{E}(\rho=2) = \frac{1}{2.5 \times \frac{10^{-9}}{36\pi}} \times \frac{100 \times 10^{-9}}{2}\underline{a}_\rho = \frac{36\pi \times 10^{9} \times 100 \times 10^{-9}}{5}\underline{a}_\rho$$

$$= 2.2619 \ \underline{a}_\rho \ KV/m$$

$$\underline{D} = \varepsilon_r \varepsilon_0 \underline{E} = \frac{\varepsilon_r \varepsilon_0}{(\varepsilon_r - 1)\varepsilon_0}\underline{P} = \frac{\varepsilon_r}{\varepsilon_r - 1}\underline{P} = \frac{\varepsilon_r}{\varepsilon_r - 1} \times \frac{100}{\rho} \times 10^{-9}\underline{a}_\rho$$

$$\Rightarrow \underline{D}(\rho=2) = \frac{3.5}{2.5} \times \frac{100}{2} \times 10^{-9}\underline{a}_\rho$$

$$= 70 \ \underline{a}_\rho \ nC/m^2$$

5.18 $\begin{cases} \text{Teflon}: \varepsilon = 2.1\varepsilon_0 \Rightarrow \varepsilon_r = 2.1 \\ \underline{E} = 6\underline{a}_x + 12\underline{a}_y - 20\underline{a}_z \text{ V/m} \end{cases} \Rightarrow \begin{cases} \underline{D} = ? \\ \underline{P} = ? \end{cases}$

$$\underline{D} = \varepsilon_r \varepsilon_0 \underline{E}$$

$$= 2.1 \times \frac{10^{-9}}{36\pi}(6\underline{a}_x + 12\underline{a}_y - 20\underline{a}_z)$$

$$= 0.1114\,\underline{a}_x + 0.2228\,\underline{a}_y - 0.3714\,\underline{a}_z \text{ nC/m}^2$$

$$\underline{P} = \chi_e \varepsilon_0 \underline{E} = (\varepsilon_r - 1)\varepsilon_0 \underline{E}$$

$$= 1.1 \times \frac{10^{-9}}{36\pi}(6\underline{a}_x + 12\underline{a}_y - 20\underline{a}_z)$$

$$= 0.0584\,\underline{a}_x + 0.1167\,\underline{a}_y - 0.1945\,\underline{a}_z \text{ nC/m}^2$$

5.19 $\varepsilon = 5\varepsilon_0 \Rightarrow \varepsilon_r = 5$

$V = 10x^2yz - 5z^2$ V

a) $\underline{E} = ?$

$$= -\nabla V = -\frac{\partial V}{\partial x}\underline{a}_x - \frac{\partial V}{\partial y}\underline{a}_y - \frac{\partial V}{\partial z}\underline{a}_z$$

$$= -20xyz\,\underline{a}_x - 10x^2z\,\underline{a}_y - 10(x^2y - z)\underline{a}_z \text{ V/m}$$

b) $\underline{D} = ?$

$$= \varepsilon\underline{E} = \varepsilon_r\varepsilon_0\underline{E}$$

$$= 5 \times \frac{10^{-9}}{36\pi}\left(-20xyz\,\underline{a}_x - 10x^2z\,\underline{a}_y - 10(x^2y - z)\underline{a}_z\right)$$

$$= -0.8842xyz\,\underline{a}_x - 0.4421x^2z\,\underline{a}_y - 0.4421(x^2y - z)\underline{a}_z \text{ nC/m}^2$$

c) $\underline{P} = ?$

$$= \chi_e\varepsilon_0\underline{E} = (\varepsilon_r - 1)\varepsilon_0\underline{E}$$

$$= 4 \times \frac{10^{-9}}{36\pi}\left(-20xyz\,\underline{a}_x - 10x^2z\,\underline{a}_y - 10(x^2y - z)\underline{a}_z\right)$$

$$= -0.7074xyz\,\underline{a}_x - 0.3537x^2z\,\underline{a}_y - 0.3537(x^2y - z)\underline{a}_z \text{ nC/m}^2$$

5.19

d) $\rho_v = ?$

$$= \nabla \cdot \underline{D} = \varepsilon \nabla \cdot \underline{E} = \varepsilon_r \varepsilon_0 \nabla \cdot \underline{E}$$

$$= 5 \times \frac{10^{-9}}{36\pi} \left[\frac{\partial}{\partial x}(-20xyz) + \frac{\partial}{\partial y}(-10x^2 z) + \frac{\partial}{\partial z}(10z - 10x^2 y) \right]$$

$$= 5 \times \frac{10^{-9}}{36\pi} \left[-20yz + 0 + 10 \right]$$

$$= (0.4421 - 0.8842\, yz)\ nC/m^3$$

5.22 $\begin{cases} D = 350\ nC/m^2 \\ \varepsilon_r = 6.4 \end{cases} \Rightarrow \begin{cases} \chi_e = ? \\ E = ? \\ P = ? \end{cases}$

$$\chi_e = \varepsilon_r - 1 = 6.4 - 1 = 5.4$$

$$D = \varepsilon E = \varepsilon_r \varepsilon_0 E$$

$$\Rightarrow E = \frac{1}{\varepsilon_r \varepsilon_0} D = \frac{350 \times 10^{-9}}{6.4 \times \frac{10^{-9}}{36\pi}} = \frac{350 \times 36\pi}{6.4} = 6185.01\ V/m$$

$$P = \chi_e \varepsilon_0 E = \frac{\chi_e \cancel{\varepsilon_0}}{\varepsilon_r \cancel{\varepsilon_0}} D$$

$$\Rightarrow P = \frac{5.4}{6.4} \times 350 \times 10^{-9} = 295.3125\ nC/m^2$$

5.25 $D = 5P \Rightarrow \varepsilon_r = ?$

$$D = \varepsilon_r \varepsilon_0 E$$

$$P = \chi_e \varepsilon_0 E = (\varepsilon_r - 1)\varepsilon_0 E$$

$$D = \varepsilon_r \varepsilon_0 E = 5P$$

$$= 5(\varepsilon_r - 1)\varepsilon_0 E$$

$$\Rightarrow \quad \varepsilon_r = 5\varepsilon_r - 5$$

$$\Rightarrow \quad 4\varepsilon_r = 5$$

$$\Rightarrow \quad \varepsilon_r = 1.25$$

5.27

$$\begin{cases} \underline{D} \triangleq D_x \underline{a}_x + D_y \underline{a}_y + D_z \underline{a}_z \\ \underline{E} \triangleq E_x \underline{a}_x + E_y \underline{a}_y + E_z \underline{a}_z \end{cases}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\underline{E} = E_0 (\underline{a}_x + \underline{a}_y - \underline{a}_z) \ \text{V/m} \Rightarrow \underline{D} = ?$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} E_0 \\ E_0 \\ -E_0 \end{bmatrix}$$

$$= \varepsilon_0 E_0 \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \varepsilon_0 E_0 \begin{bmatrix} 4+1-1 \\ 1+3-1 \\ 1+1-2 \end{bmatrix} = \varepsilon_0 E_0 \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$$\Rightarrow \underline{D} = \varepsilon_0 E_0 (4 \underline{a}_x + 3 \underline{a}_y) \ \text{C/m}^2$$

5.28 Which current densities are static (time-independent) ?

static (time-independent) $\Rightarrow \dfrac{\partial \rho_v}{\partial t} = 0$

And we know that $\nabla \cdot \underline{J} = -\dfrac{\partial \rho_v}{\partial t}$

if $\dfrac{\partial \rho_v}{\partial t} = 0 \Rightarrow \nabla \cdot \underline{J} = 0$

a) $\underline{J} = 2x^3 y \, \underline{a}_x + 4x^2 z^2 \, \underline{a}_y - 6x^2 yz \, \underline{a}_z$

$$\nabla \cdot \underline{J} = \frac{\partial}{\partial x}(2x^3 y) + \frac{\partial}{\partial y}(4x^2 z^2) + \frac{\partial}{\partial z}(-6x^2 yz)$$

$$= 6x^2 y + 0 + (-6x^2 y) = 0$$

$\Rightarrow$ It is Static.

b) $\underline{J} = xy \, \underline{a}_x + y(z+1) \, \underline{a}_y + 2y \, \underline{a}_z$

$$\nabla \cdot \underline{J} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(y(z+1)) + \frac{\partial}{\partial z}(2y)$$

$$= y + (z+1) + 0$$

$$= y + z + 1 \neq 0$$

$\Rightarrow$ It is NOT static.

5.28

c) $\underline{J} = \frac{z^2}{\rho}\underline{a}_\rho + z\cos\varphi\,\underline{a}_z$

$$\nabla\cdot\underline{J} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\times\frac{z^2}{\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}(0) + \frac{\partial}{\partial z}(z\cos\varphi)$$

$$= 0 + 0 + \cos\varphi$$

$$= \cos\varphi \neq 0$$

$\Rightarrow$ It is NOT Static.

d) $\underline{J} = \frac{\sin\theta}{r^2}\underline{a}_r$

$$\nabla\cdot\underline{J} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\times\frac{\sin\theta}{r^2}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(0\times\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(0)$$

$$= 0 + 0 + 0$$

$$= 0$$

$\Rightarrow$ It is Static.

5.29 $\quad \underline{J} = \frac{100}{\rho^2} \underline{a}_\rho \quad A/m^2$

a) $\frac{\partial \rho_v}{\partial t} = ?$

$$\nabla \cdot \underline{J} = -\frac{\partial \rho_v}{\partial t}$$

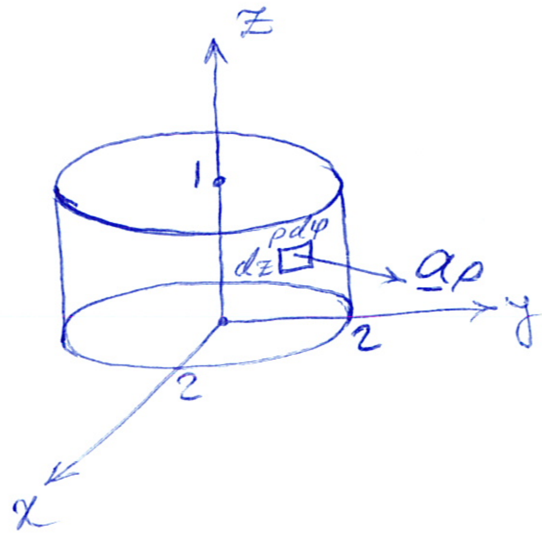
$$\Rightarrow \frac{\partial \rho_v}{\partial t} = -\nabla \cdot \underline{J} = -\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \times \frac{100}{\rho^2}\right) + \frac{1}{\rho}\frac{\partial}{\partial \varphi}(0) + \frac{\partial}{\partial z}(0)\right]$$

$$= -\left[\frac{1}{\rho} \times \frac{-100}{\rho^2} + 0 + 0\right]$$

$$= \frac{100}{\rho^3}$$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} = \frac{100}{\rho^3}$$

b) $\begin{cases} \rho = 2 \\ 0 < z < 1 \\ 0 < \varphi < 2\pi \end{cases} \Rightarrow I = ?$



$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$d\underline{s} = \rho \, d\varphi \, dz \, \underline{a}_\rho$$

5.29

b) $$I = \iint_S \underline{J} \cdot d\underline{S}$$

$$= \int_{\varphi=0}^{2\pi} \int_{z=0}^{1} \left(\frac{100}{\rho^2} \underline{a}_\rho \right) \cdot \left(\rho \, d\varphi \, dz \, \underline{a}_\rho \right)$$

$$= \int_{\varphi=0}^{2\pi} \int_{z=0}^{1} \frac{100}{\rho^2} \rho \, d\varphi \, dz$$

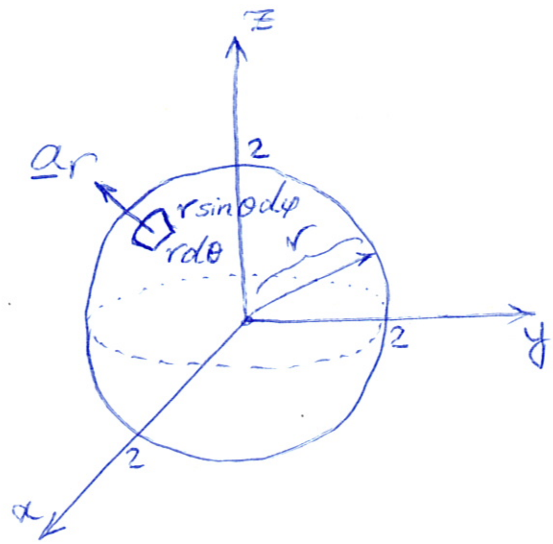
$$= \frac{100}{\rho} \Big|_{\rho=2} \times \int_{\varphi=0}^{2\pi} d\varphi \times \int_{z=0}^{1} dz$$

$$= \frac{100}{2} \times 2\pi \times 1$$

$$= 100\pi = 314.1593 \text{ A}$$

$$\Rightarrow I = 314.1593 \text{ A}$$

5.30 $\begin{cases} \underline{J} = \dfrac{5e^{-10^4 t}}{r} \underline{a_r} \quad A/m^2 \\ \text{at } t = 0.1 \text{ ms} \end{cases}$



a) $r = 2m \Rightarrow I = ?$

$$I = \iint_S \underline{J} \cdot d\underline{s}$$

$$d\underline{s} = r^2 \sin\theta \, d\theta \, d\varphi \, \underline{a_r}$$

$$I = \iint_S \left(\frac{5e^{-10^4 t}}{r} \underline{a_r} \right) \cdot \left(r^2 \sin\theta \, d\theta \, d\varphi \, \underline{a_r} \right)$$

$$= \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{5e^{-10^4 t}}{r} \times r^2 \sin\theta \, d\theta \, d\varphi$$

$$= 5e^{-10^4 t} r \Big|_{r=2} \times \int_{\theta=0}^{\pi} \sin\theta \, d\theta \times \int_{\varphi=0}^{2\pi} d\varphi$$

$$= 10e^{-10^4 t} \times (-\cos\theta) \Big|_{\theta=0}^{\pi} \times \varphi \Big|_{\varphi=0}^{2\pi}$$

$$= 10e^{-10^4 t} \times 2 \times 2\pi = 40\pi e^{-10^4 t}$$

at $t = 0.1 \text{ ms} = 10^{-4}$

5.30

a) at $t = 10^{-4}$

$$I = 40\pi e^{-10^4 \times 10^{-4}} = 40\pi e^{-1} = \frac{40\pi}{e}$$

$$= 46.2291 \text{ A}$$

b) $\begin{cases} \text{at } r = 2\text{ m} \\ \text{at } t = 0.1\text{ ms} \end{cases} \Rightarrow \rho_v = ?$

$$-\frac{\partial \rho_v}{\partial t} = \nabla \cdot \underline{J} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{5e^{-10^4 t}}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(0 \times \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(0)$$

$$= \frac{1}{r^2} \times 5e^{-10^4 t} + 0 + 0$$

$$= \frac{5e^{-10^4 t}}{r^2}$$

$$\frac{\partial \rho_v}{\partial t} = -\nabla \cdot \underline{J} = \frac{-5e^{-10^4 t}}{r^2}$$

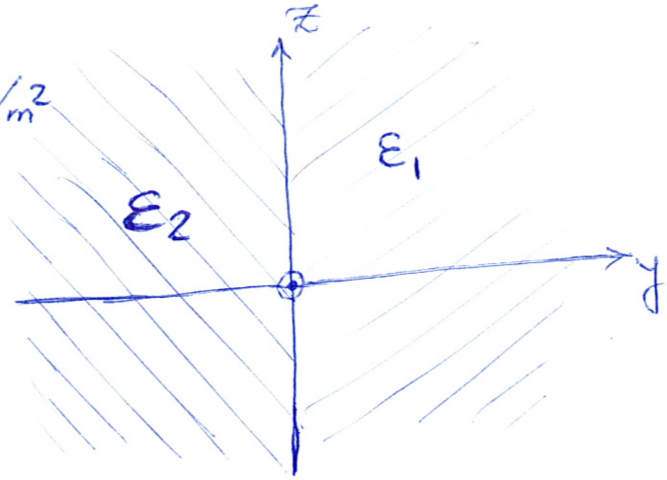
$$\rho_v = \int_t \frac{\partial \rho_v}{\partial t} = \int_t \frac{-5e^{-10^4 t}}{r^2} dt = \frac{-5}{r^2}\int_t e^{-10^4 t} dt$$

$$= \frac{-5}{r^2} \times \frac{e^{-10^4 t}}{-10^4} = \frac{5e^{-10^4 t}}{10^4 r^2}$$

$$\rho_v\left(\begin{matrix} t = 10^{-4} \\ r = 2 \end{matrix}\right) = \frac{5e^{-10^4 \times 10^{-4}}}{10^4 \times 4} = \frac{5}{4e \times 10^4} = 45.9849 \ \mu\text{C/m}^3$$

5.35

$$\underline{D}_1 = 4\underline{a}_x - 6\underline{a}_y + 8\underline{a}_z \ nC/m^2$$



$$\begin{cases} y>0 \Rightarrow \varepsilon_1 = 2.5\,\varepsilon_0 \\ y<0 \Rightarrow \varepsilon_2 = 8.1\,\varepsilon_0 \end{cases}$$

$$\begin{cases} \underline{D}_2 = ? \\ \underline{E}_2 = ? \end{cases}$$

$$\begin{cases} \underline{E}_{1t} = \underline{E}_{2t} \Rightarrow \dfrac{\underline{D}_{1t}}{\varepsilon_1} = \dfrac{\underline{D}_{2t}}{\varepsilon_2} \quad (1) \\ \rho_s = 0 \Rightarrow \underline{D}_{1n} = \underline{D}_{2n} \quad (2) \end{cases}$$

$$\underline{D}_1 = 4\underline{a}_x - 6\underline{a}_y + 8\underline{a}_z \Rightarrow \begin{cases} \underline{D}_{1t} = 4\underline{a}_x + 8\underline{a}_z \ nC/m^2 \\ \underline{D}_{1n} = -6\underline{a}_y \ nC/m^2 \end{cases}$$

$$\begin{cases} \text{From } (1) \Rightarrow \underline{D}_{2t} = \dfrac{\varepsilon_2}{\varepsilon_1}\underline{D}_{1t} = \dfrac{8.1\cancel{\varepsilon_0}}{2.5\cancel{\varepsilon_0}}(4\underline{a}_x + 8\underline{a}_z) \\ \qquad = 12.96\,\underline{a}_x + 25.92\,\underline{a}_z \ nC/m^2 \\ \text{From } (2) \Rightarrow \underline{D}_{2n} = \underline{D}_{1n} = -6\underline{a}_y \ nC/m^2 \end{cases}$$

$$\Rightarrow \underline{D}_2 = \underline{D}_{2t} + \underline{D}_{2n}$$

$$= 12.96\,\underline{a}_x - 6\underline{a}_y + 25.92\,\underline{a}_z \ nC/m^2$$

5.35

$$\underline{E}_2 = \frac{\underline{D}_2}{\varepsilon_2} = \frac{1}{8.1 \times \frac{10^{-9}}{36\pi}} \left(12.96\,\underline{a}_x - 6\underline{a}_y + 25.92\,\underline{a}_z\right) \times 10^{-9}$$

$$= \frac{36\pi}{8.1}\left(12.96\,\underline{a}_x - 6\underline{a}_y + 25.92\,\underline{a}_z\right)$$

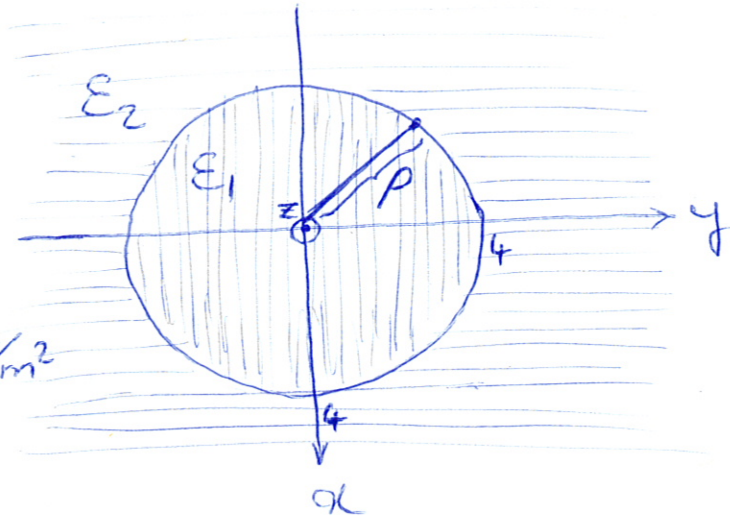
$$= 180.9557\,\underline{a}_x - 83.7758\,\underline{a}_y + 361.9115\,\underline{a}_z$$

V/m

5.38

$\rho \leqslant 4^{cm} \left\{ \begin{array}{l} \varepsilon_{r_1} = 3.5 \Rightarrow \varepsilon_1 = 3.5\,\varepsilon_0 \\ \varepsilon_{r_2} = 1.5 \Rightarrow \varepsilon_2 = 1.5\,\varepsilon_0 \end{array} \right.$

$\rho \geqslant 4^{cm}$



$$\underline{D}_2 = 12\,\underline{a}_\rho - 6\,\underline{a}_\varphi + 9\,\underline{a}_z \;\; nC/m^2$$

a) $\left\{ \begin{array}{l} \underline{D}_1 = ? \\ \underline{E}_1 = ? \end{array} \right.$

$$\left\{ \begin{array}{lll} \underline{E}_{1t} = \underline{E}_{2t} & \Rightarrow \dfrac{\underline{D}_{1t}}{\varepsilon_1} = \dfrac{\underline{D}_{2t}}{\varepsilon_2} & (1) \\ \rho_s = 0 & \Rightarrow \underline{D}_{1n} = \underline{D}_{2n} & (2) \end{array} \right.$$

$$\underline{D}_2 = 12\,\underline{a}_\rho - 6\,\underline{a}_\varphi + 9\,\underline{a}_z \Rightarrow \left\{ \begin{array}{l} \underline{D}_{2t} = -6\,\underline{a}_\varphi + 9\,\underline{a}_z \\ \underline{D}_{2n} = 12\,\underline{a}_\rho \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{from (1)} \Rightarrow \underline{D}_{1t} = \dfrac{\varepsilon_1}{\varepsilon_2}\underline{D}_{2t} = \dfrac{3.5\cancel{\varepsilon_0}}{1.5\cancel{\varepsilon_0}}(-6\,\underline{a}_\varphi + 9\,\underline{a}_z) = -14\,\underline{a}_\varphi + 21\,\underline{a}_z \;\; nC/m^2 \\ \text{from (2)} \Rightarrow \underline{D}_{1n} = \underline{D}_{2n} = 12\,\underline{a}_\rho \;\; nC/m^2 \end{array} \right.$$

$$\Rightarrow \underline{D}_1 = 12\,\underline{a}_\rho - 14\,\underline{a}_\varphi + 21\,\underline{a}_z \;\; nC/m^2$$

$$\underline{E}_1 = \frac{\underline{D}_1}{\varepsilon_1} = \frac{1}{3.5 \times \frac{10^{-9}}{36\pi}} \left(12\,\underline{a}_\rho - 14\,\underline{a}_\varphi + 21\,\underline{a}_z\right) \times 10^{-9}$$

$$= 387.7623\,\underline{a}_\rho - 452.3893\,\underline{a}_\varphi + 678.5840\,\underline{a}_z \;\; V/m$$

5.38

b) $\begin{cases} \underline{P}_2 = ? \\ \rho_{pv2} = ? \end{cases}$

$$\underline{P}_2 = \chi_{e2}\,\varepsilon_0\,\underline{E}_2 = (\varepsilon_{r2}-1)\,\varepsilon_0\,\underline{E}_2 = \frac{(\varepsilon_{r2}-1)\cancel{\varepsilon_0}}{\varepsilon_{r2}\cancel{\varepsilon_0}}\,\underline{D}_2$$

$$= \frac{0.5}{1.5}\left(12\,\underline{a}_\rho - 6\,\underline{a}_\varphi + 9\,\underline{a}_z\right)\,nC/m^2$$

$$= 4\,\underline{a}_\rho - 2\,\underline{a}_\varphi + 3\,\underline{a}_z \quad nC/m^2$$

$$\rho_{pv2} = \nabla\cdot\underline{P}_2 = \nabla\cdot\left(4\,\underline{a}_\rho - 2\,\underline{a}_\varphi + 3\,\underline{a}_z\right)\times 10^{-9}$$

$$= 0$$

$$\Rightarrow \rho_{pv2} = 0$$

c) $\begin{cases} w_{E1} = ? \\ w_{E2} = ? \end{cases}$ Energy density

$$w_{E1} = \frac{1}{2}\,\underline{D}_1\cdot\underline{E}_1 = \frac{1}{2\varepsilon_1}\,|\underline{D}_1|^2$$

$$= \frac{1}{2\times 3.5\times\frac{10^{-9}}{36\pi}}\left(\underbrace{12^2+(-14)^2+21^2}_{=781}\right)\times 10^{-18}$$

$$= \frac{36\pi\times 781\times 10^{-9}}{7} = 12.6184\times 10^{-6}\ J/m^3$$

$$\Rightarrow w_{E1} = 12.6184\ \mu J/m^3$$

5.38

c) $w_{E2} = \frac{1}{2\varepsilon_2} |\underline{D}_2|^2$

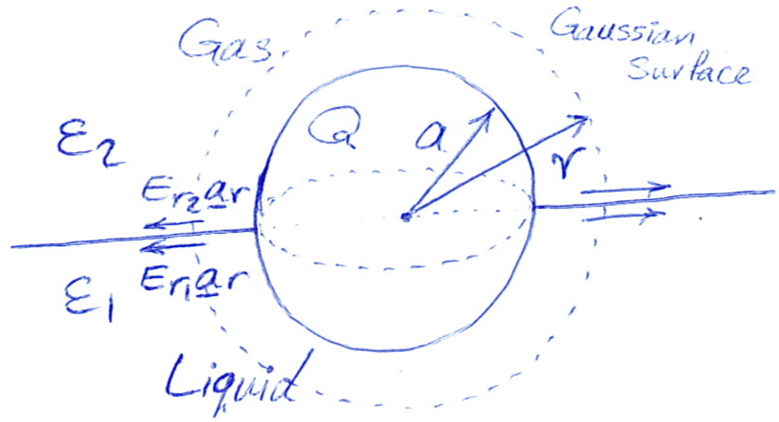
$$= \frac{1}{2 \times 1.5 \times \frac{10^{-9}}{36\pi}} \Big(\underbrace{12^2 + (-6)^2 + 9^2}_{=261} \Big) \times 10^{-18}$$

$$= \frac{36\pi \times 261 \times 10^{-9}}{3} = 9.8395 \times 10^{-6} \ J/m^3$$

$\Rightarrow w_{E2} = 9.8395 \ \mu J/m^3$

5.39

$\underline{E} = ?$



sphere is conductor $\Rightarrow$

$r < a \Rightarrow \underline{E} = \underline{0}$

For $r > a$:

$$Q_{enc} = \oint_S \underline{D} \cdot d\underline{s} = \oint_S \varepsilon \underline{E} \cdot d\underline{s}$$

$$\left\{ \begin{array}{l} E_{1t} = E_{2t} \\ \quad \Downarrow \\ E_{r_1} = E_{r2} \\ \quad \triangleq E_r \end{array} \right.$$

$$\left\{ \begin{array}{l} d\underline{s} = r^2 \sin\theta \, d\theta \, d\varphi \, \underline{a}_r \\ \underline{E} = E_r \, \underline{a}_r \end{array} \right.$$ $\leftarrow$ symmetry of problem

$$\Rightarrow Q_{enc} = Q = \oint_S \underline{D} \cdot d\underline{s}$$

$$= \int_{S_1} \underline{D}_1 \cdot d\underline{s} + \int_{S_2} \underline{D}_2 \cdot d\underline{s}$$

$$= \varepsilon_1 E_{r_1} \int_{S_1} ds + \varepsilon_2 E_{r_2} \int_{S_2} ds$$

$$= \varepsilon_1 E_r \frac{4\pi r^2}{2} + \varepsilon_2 E_r \frac{4\pi r^2}{2}$$

5.39

$$Q = E_r(\varepsilon_1+\varepsilon_2)\frac{4\pi r^2}{2}$$

$$\Rightarrow \quad E_r = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)r^2}$$

$$\Rightarrow \quad \underline{E} = \begin{cases} \underline{0} & r<a \\ \dfrac{Q}{2\pi(\varepsilon_1+\varepsilon_2)r^2}\underline{a}_r & r>a \end{cases}$$