REVIEW QUESTIONS

6.1 Equation $\nabla \cdot (-\varepsilon \nabla V) = \rho_v$ may be regarded as Poisson's equation for an inhomogeneous medium.

(a) True (b) False

6.2 In cylindrical coordinates, the equation

$$\frac{\partial^2 \psi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \psi}{\partial z^2} + 10 = 0$$

is called

(a) Maxwell's equation
(b) Laplace's equation
(c) Poisson's equation
(d) Helmholtz's equation
(e) Lorentz's equation

6.3 Two potential functions V_1 and V_2 satisfy Laplace's equation within a closed region and assume the same values on its surface. V_1 must be equal to V_2.

(a) True
(b) False
(c) Not necessarily

6.4 Which of the following potentials does not satisfy Laplace's equation?

(a) $V = 2x + 5$
(b) $V = 10\,xy$
(c) $V = r \cos \phi$
(d) $V = \dfrac{10}{r}$
(e) $V = \rho \cos \phi + 10$

6.5 Which of the following is not true?

(a) $-5 \cos 3x$ is a solution to $\phi''(x) + 9\phi(x) = 0$
(b) $10 \sin 2x$ is a solution to $\phi''(x) - 4\phi(x) = 0$
(c) $-4 \cosh 3y$ is a solution to $R''(y) - 9R(y) = 0$
(d) $\sinh 2y$ is a solution to $R''(y) - 4R(y) = 0$
(e) $\dfrac{g''(x)}{g(x)} = -\dfrac{h''(y)}{h(y)} = f(z) = -1$ where $g(x) = \sin x$ and $h(y) = \sinh y$

6.6 If $V_1 = X_1 Y_1$ is a product solution of Laplace's equation, which of these are not solutions of Laplace's equation?

(a) $-10X_1Y_1$
(b) $X_1Y_1 + 2xy$
(c) $X_1Y_1 - x + y$
(d) $X_1 + Y_1$
(e) $(X_1 - 2)(Y_1 + 3)$

6.7 The capacitance of a capacitor filled by a linear dielectric is independent of the charge on the plates and the potential difference between the plates.

(a) True (b) False

6.8 A parallel-plate capacitor connected to a battery stores twice as much charge with a given dielectric as it does with air as dielectric. The susceptibility of the dielectric is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

6.9 A potential difference V_o is applied to a mercury column in a cylindrical container. The mercury is now poured into another cylindrical container of half the radius and the same potential difference V_o applied across the ends. As a result of this change of space, the resistance will be increased

(a) 2 times
(b) 4 times
(c) 8 times
(d) 16 times

6.10 Two conducting plates are inclined at an angle 30° to each other with a point charge between them. The number of image charges is

(a) 12
(b) 11
(c) 6
(d) 5
(e) 3

Answers: 6.1a, 6.2c, 6.3a, 6.4c, 6.5b, 6.6d,e, 6.7a, 6.8b, 6.9d, 6.10b.

PROBLEMS

Section 6.2—Poisson's and Laplace's Equations

6.1 Given $V = 5x^3y^2z$ and $\varepsilon = 2.25\varepsilon_o$, find (a) **E** at point $P(-3, 1, 2)$, (b) ρ_v at P.

6.2 Conducting sheets are located at $y = 1$ and $y = 3$ planes. The space between them is filled with a nonuniform charge distribution $\rho_v = \dfrac{y}{4\pi}$ nC/m³ and $\varepsilon = 4\varepsilon_o$. Assuming that $V(y = 1) = 0$ and $V(y = 3) = 50$ V, find $V(y = 2)$.

6.3 Let $V = \dfrac{\sin 3\phi}{\rho}$ V in a dielectric material for which $\varepsilon = 2.8\,\varepsilon_0$.

(a) Find **E** and **P** at point $A(1, 20°, 4)$.
(b) Calculate the volume charge density at A.

6.4 A charge is uniformly distributed over $0 < x < 1$ such that $\rho_v = -6\varepsilon$. Find the potential $V(x)$ assuming that $V(0) = 0$ and $V(1) = 10$.

6.5 A certain material occupies the space between two conducting slabs located at $y = \pm 2$ cm. When heated, the material emits electrons such that $\rho_v = 50(1 - y^2)\ \mu$C/m³. If the slabs are both held at 30 kV, find the potential distribution within the slabs. Take $\varepsilon = 3\varepsilon_o$.

6.6 An infinitely long coaxial cylindrical structure has an inner conductor of radius 2 mm and an outer conductor of radius 4.5 mm. The space between the conductor is filled with an electron cloud with $\rho_v = \dfrac{10\varepsilon_o}{\rho}$ C/m^3 and $\varepsilon = \varepsilon_o$. If the inner conductor is grounded, and the outer conductor is maintained at 40 V, determine the potential distribution for 2 mm $< \rho <$ 4.5 mm.

6.7 Show that each of the following potentials satisfies Laplace's equation.

(a) $V_1 = e^{-y} \sin x$

(b) $V_2 = V_o \sin\left(\dfrac{\pi x}{a}\right) \sinh\left(\dfrac{\pi y}{a}\right)$

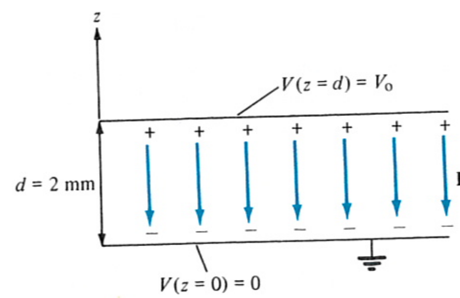


FIGURE 6.28 For Problem 6.11.

(c) $V_3 = \dfrac{\cos \phi}{\rho}$

(d) $V_4 = \dfrac{10 \cos \theta}{r^2}$

6.8 Show that $\mathbf{E} = (E_x, E_y, E_z)$ satisfies Laplace's equation.

6.9 Let $V = 10\rho \cos \phi$. Show that V satisfies Laplace's equation.

6.10 The potential field $V = 2x^2yz - y^3z$ exists in a dielectric medium having $\varepsilon = 2\varepsilon_o$. (a) Does V satisfy Laplace's equation? (b) Calculate the total charge within the unit cube $0 < x < 1$ m, $0 < y < 1$ m, $0 < z < 1$ m.

6.11 Consider the conducting plates shown in Figure 6.28. If $V(z = 0) = 0$ and $V(z = 2\text{ mm}) = 50$ V, determine V, $\mathbf{E}$, and $\mathbf{D}$ in the dielectric region ($\varepsilon_r = 1.5$) between the plates and ρ_S on the plates.

6.12 The cylindrical capacitor whose cross section is in Figure 6.29 has inner and outer radii of 5 mm and 15 mm, respectively. If $V(\rho = 5\text{ mm}) = 100$ V and $V(\rho = 15\text{ mm}) = 0$ V, calculate V, $\mathbf{E}$, and $\mathbf{D}$ at $\rho = 10$ mm and ρ_S on each plate. Take $\varepsilon_r = 2.0$.

6.13 Two conducting planes are located at $x = 0$ and $x = 50$ mm. The zero voltage reference is at $x = 20$ mm. Given that $\mathbf{E} = -110\mathbf{a}_x$ V/m, calculate the conductor voltages.

6.14 Two semi-infinite planes are located in free space at $\phi = 0$ and $\phi = \pi/4$. The planes are seperated by a thin insulating strip along the z-axis. If the planes at $\phi = 0$ is maintained at 50 V, while the place at $\phi = \pi/4$ is maintained at -50 V, find V and $\mathbf{E}$ at $\rho = 1$ cm, $\phi = \pi/6$.

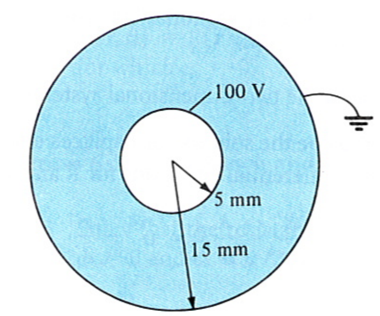


FIGURE 6.29 Cylindrical capacitor of Problem 6.12.

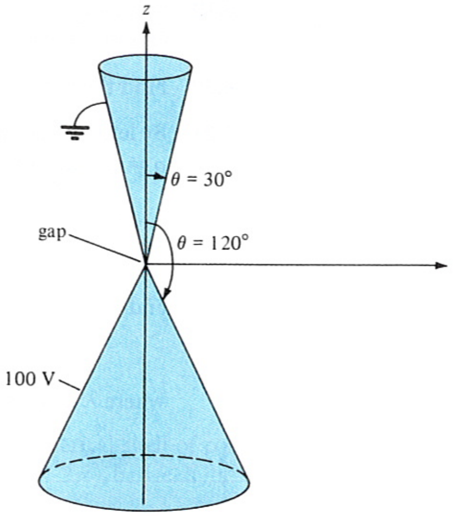


FIGURE 6.30 Conducting cones of Problem 6.16.

6.15 The region between concentric spherical conducting shells $r = 0.5$ m and $r = 1$ m is charge free. If $V(r = 0.5) = -50$ V and $V(r = 1) = 50$ V, determine the potential distribution and the electric field strength in the region between the shells.

6.16 Find V and $\mathbf{E}$ at $(3, 0, 4)$ due to the two conducting cones of infinite extent shown in Figure 6.30.

***6.17** The inner and outer electrodes of a diode are coaxial cylinders of radii $a = 0.6$ mm and $b = 30$ mm, respectively. The inner electrode is maintained at 70 V, while the outer electrode is grounded. (a) Assuming that the length of the electrodes $\ell \gg a, b$ and ignoring the effects of space charge, calculate the potential at $\rho = 15$ mm. (b) If an electron is injected radially through a small hole in the inner electrode with velocity 10^7 m/s, find its velocity at $\rho = 15$ mm.

6.18 An electrode with a hyperbolic shape $(xy = 4)$ is placed above a grounded right-angle corner as in Figure 6.31. Calculate V and $\mathbf{E}$ at point $(1, 2, 0)$ when the electrode is connected to a 20 V source.

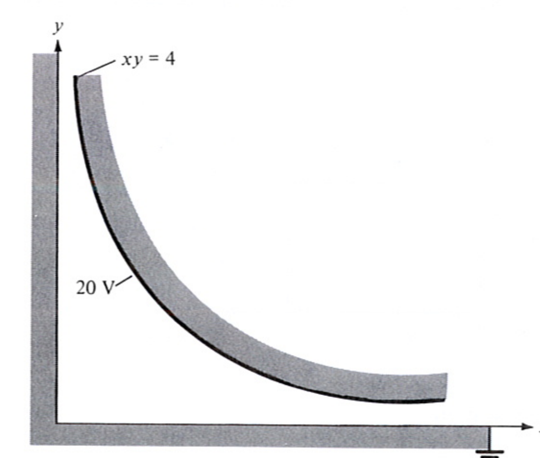


FIGURE 6.31 For Problem 6.18.

***6.19** Solve Laplace's equation for the two-dimensional electrostatic systems of Figure 6.32 and find the potential $V(x, y)$.

***6.20** Find the potential $V(x, y)$ due to the two-dimensional systems of Figure 6.33.

6.21 By letting $V(\rho, \phi) = R(\rho)\Phi(\phi)$ be the solution of Laplace's equation in a region where $\rho \neq 0$, show that the separated differential equations for R and Φ are

$$R'' + \frac{R'}{\rho} - \frac{\lambda}{\rho^2}R = 0$$

and

$$\Phi'' + \lambda\Phi = 0$$

where λ is the separation constant.

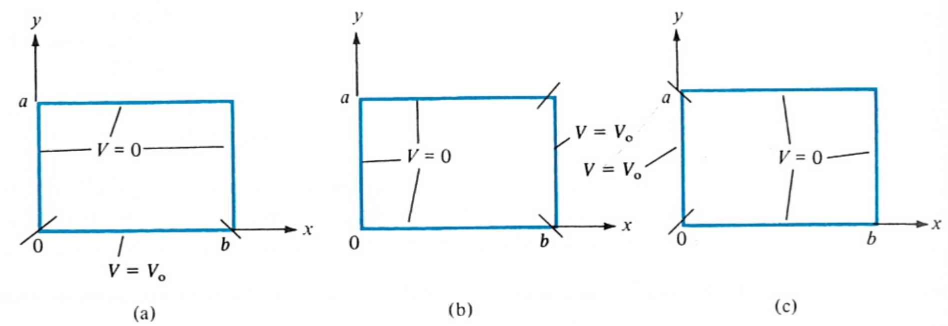


FIGURE 6.32 For Problem 6.19.

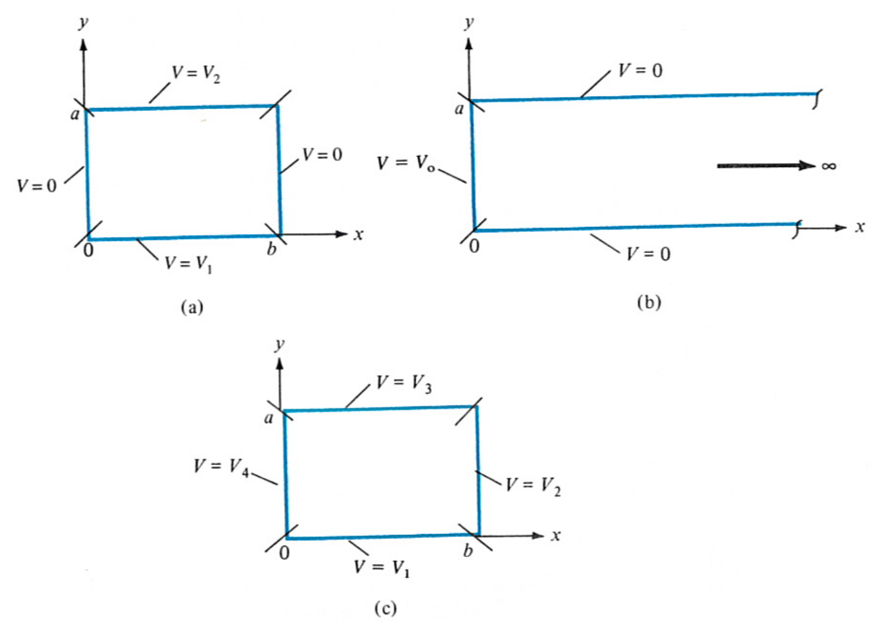


FIGURE 6.33 For Problem 6.20.

6.22 A potential in spherical coordinates is a function of r and θ but not ϕ. Assuming that $V(r, \theta) = R(r)F(\theta)$, obtain the separated differential equations for R and F in a region for which $\rho_v = 0$.

Section 6.5—Resistance and Capacitance

6.23 Show that the resistance of the bar of Figure 6.17 between the vertical ends located at $\phi = 0$ and $\phi = \pi/2$ is

$$R = \frac{\pi}{2\sigma t \ln \dfrac{b}{a}}$$

***6.24** Show that the resistance of the sector of a spherical shell of conductivity σ, with cross section shown in Figure 6.34 (where $0 \leq \phi < 2\pi$), between its base (i.e., from $r = a$ to $r = b$) is

$$R = \frac{1}{2\pi\sigma(1 - \cos\alpha)}\left[\frac{1}{a} - \frac{1}{b}\right]$$

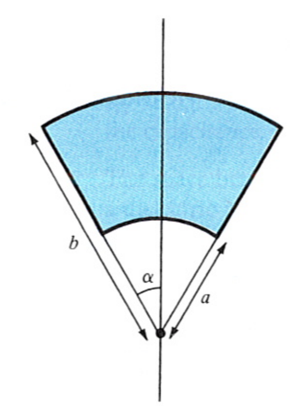
***6.25** A hollow conducting hemisphere of radius a is buried with its flat face lying flush with the earth's surface, thereby serving as an earthing electrode. If the conductivity of earth is σ, show that the leakage conductance between the electrode and earth is $2\pi a\sigma$.

6.26 Another method of finding the capacitance of a capacitor is by using energy considerations, that is,

$$C = \frac{2W_E}{V_o^2} = \frac{1}{V_o^2}\int \varepsilon|\mathbf{E}|^2\, dv$$

Using this approach, derive eqs. (6.22), (6.28), and (6.32).

FIGURE 6.34 For Problem 6.24.



6.27 In an integrated circuit, a capacitor is formed by growing a silicon dioxide layer ($\varepsilon_r = 4$) of thickness 1 μm over the conducting silicon substrate and covering it with a metal electrode of area S. Determine S if a capacitance of 2 nF is desired.

6.28 Calculate the capacitance of the parallel-plate capacitor shown in Figure 6.35.

6.29 Evaluate the capacitance of the parallel-plate capacitor shown in Figure 6.36.

6.30 The parallel-plate capacitor of Figure 6.37 is quarter-filled with mica ($\varepsilon_r = 6$). Find the capacitance of the capacitor.

6.31 To appreciate the physical size of 1-F capacitor, consider a parallel-plate capacitor filled with air and with separation distance of 1 mm. Find the area of the plates to provide a capacitance of 1 F.

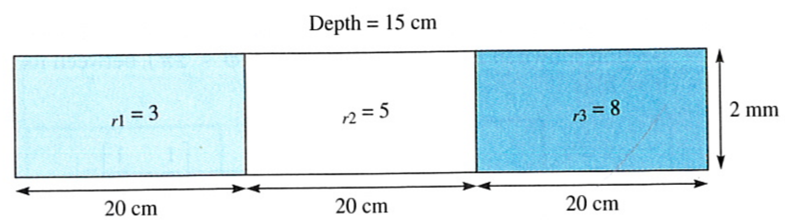


FIGURE 6.35 For Problem 6.28.

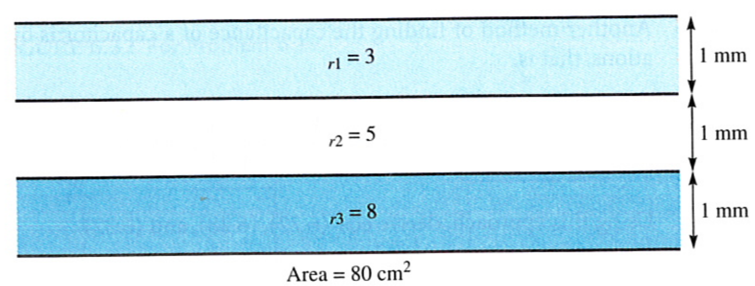


FIGURE 6.36 For Problem 6.29.

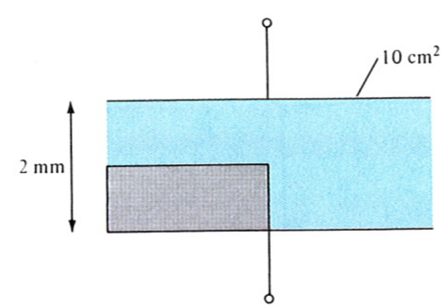


FIGURE 6.37 For Problem 6.30.

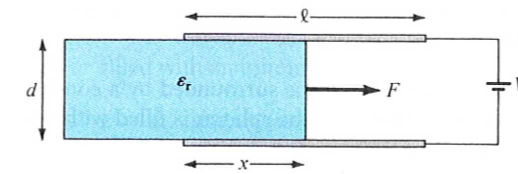


FIGURE 6.38 For Problem 6.32.

***6.32** An air-filled parallel plate capacitor of length L, width a, and plate separation d has its plates maintained at constant potential difference V_o. If a dielectric slab of dielectric constant ε_r is slid between the plates and is withdrawn until only a length x remains between the plates as in Figure 6.38, show that the force tending to restore the slab to its original position is

$$F = \frac{\varepsilon_o(\varepsilon_r - 1)\, a\, V_o^2}{2d}$$

6.33 A parallel-plate capacitor has plate area 200 cm^2 and plate separation of 3 mm. The charge density is 1 μC/m^2 and air is the dielectric. Find

(a) The capacitance of the capacitor

(b) The voltage between the plates

(c) The force with which the plates attract each other

6.34 A parallel-plate capacitor has surface area of 8 cm^2 and seperation distance of 1 cm. The plates are separated by a dielectric slab with $\varepsilon_r = 3.5$. Calculate the charge per plate when the voltage across it is 20 V.

6.35 A parallel-plate capacitor consists of two circular plates of radius 0.3 m and a 4 mm gap between them. The dielectric material between the plates has $\varepsilon_r = 8.0$. If the potential difference between the plates is 6 V, calculate the charge on each plate and the capacitance.

6.36 A parallel-plate capacitor with plate area S and spacing d has a charge Q on each plate. Assuming that the space between the plates is filled with dielectric $\varepsilon_r = \varepsilon_r \varepsilon_o$, determine the energy stored when the plate spacing is: (a) doubled, (b) halved.

6.37 A parallel-plate capacitor has separation 5 mm and area 0.4 m^2. If the space between the plates is filled with dielectric with $\varepsilon_{r1} = 2.5$, $0 < d < 1.5$ mm, dielectric with $\varepsilon_{r2} = 5.6$, 1.5 mm $< d <$ 3 mm, and dielectric with $\varepsilon_{r3} = 8.1$, 3 mm $< d <$ 5 mm, calculate the capacitance.

6.38 The space between spherical conducting shells $r = 5$ cm and $r = 10$ cm is filled with a dielectric material for which $\varepsilon = 2.25\varepsilon_o$. The two shells are maintained at a potential difference of 80 V. (a) Find the capacitance of the system. (b) Calculate the charge density on shell $r = 5$ cm.

6.39 A spherical capacitor has inner radius d and outer radius a. Concentric with the spherical conductors and lying between them is a spherical shell of outer radius c and inner radius b. If the regions $d < r < c$, $c < r < b$, and $b < r < a$ are filled with materials with permittivites ε_1, ε_2, and ε_3, respectively, determine the capacitance of the system.

6.40 Determine the capacitance of a conducting sphere of radius 5 cm deeply immersed in seawater ($\varepsilon_r = 80$).

6.41 A conducting sphere of radius 1 cm is surrounded by a concentric conducting sphere of radius 2 cm. If the space between the spheres is filled with polypropylene ($\varepsilon_r = 2.25$), calculate the capacitance of the system.

***6.42** In an ink-jet printer the drops are charged by surrounding the jet of radius 20 μm with a concentric cylinder of radius 600 μm as in Figure 6.39. Calculate the minimum voltage required to generate a charge 50 fC on the drop if the length of the jet inside the cylinder is 100 μm. Take $\varepsilon = \varepsilon_o$.

6.43 The capacitance per unit length of a two-wire transmission line shown in Figure 6.40 is given by

$$C = \frac{\pi\varepsilon}{\cosh^{-1}\left[\dfrac{d}{2a}\right]}$$

Determine the conductance per unit length.

***6.44** A spherical capacitor has an inner conductor of radius a carrying charge Q and is maintained at zero potential. If the outer conductor contracts from a radius b to c under internal forces, prove that the work performed by the electric field as a result of the contraction is

$$W = \frac{Q^2(b - c)}{8\pi\varepsilon bc}$$

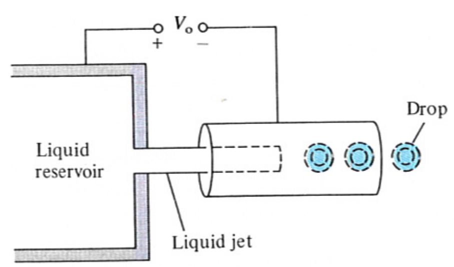


FIGURE 6.39 Simplified geometry of an ink-jet printer; for Problem 6.42.

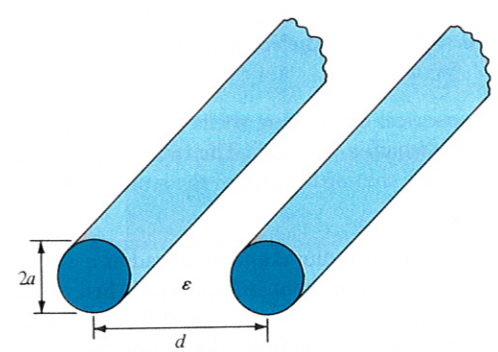


FIGURE 6.40 For Problem 6.43.

***6.45** A parallel-plate capacitor has its plates at $x = 0, d$ and the space between the plates is filled with an inhomogeneous material with permittivity $\varepsilon = \varepsilon_o\left(1 + \dfrac{x}{d}\right)$. If the plate at $x = d$ is maintained at V_o while the plate at $x = 0$ is grounded, find:

(a) V and **E**

(b) **P**

(c) ρ_{ps} at $x = 0, d$

(d) the capacitance, assuming that each plate has area S

6.46 A spherical capacitor has inner radius a and outer radius b and is filled with an inhomogeneous dielectric with $\varepsilon = \varepsilon_o k/r^2$. Show that the capacitance of the capacitor is

$$C = \frac{4\pi\varepsilon_o k}{b - a}$$

6.47 A cylindrical capacitor with inner radius a and outer radius b is filled with an inhomogeneous dielectric having $\varepsilon = \varepsilon_o k/\rho$, where k is a constant. Calculate the capacitance per unit length of the capacitor.

6.48 If the earth is regarded a spherical capacitor, what is its capacitance? Assume the radius of the earth to be approximately 6370 km.

6.49 A capacitor is formed by two coaxial metal cylinders of radii $a = 1$ mm and $b = 5$ mm. If the space between the cylinders is filled with a dielectric having $\varepsilon_r = 3(1 + \rho)$, $a < \rho < b$, and ρ is in millimeters, determine the capacitance per meter.

Section 6.6—Method of Images

6.50 A grounded metal sheet is located in the $z = 0$ plane, while a point charge Q is located at $(0, 0, a)$. Find the force acting on a point charge $-Q$ placed at $(a, 0, a)$.

6.51 Grounded conducting sheets are situated at $x = 0$ and $y = 0$, while a point charge Q is placed at $(a, a, 0)$. Determine the potential for $x > 0, y > 0$.

6.52 Two point charges of 3 nC and −4 nC are placed, respectively, at $(0, 0, 1\text{ m})$ and $(0, 0, 2\text{ m})$ while an infinite conducting plane is at $z = 0$. Determine

(a) The total charge induced on the plane

(b) The magnitude of the force of attraction between the charges and the plane

***6.53** A point charge of 10 μC is located at $(1, 1, 1)$, and the positive portions of the coordinate planes are occupied by three mutually perpendicular plane conductors maintained at zero potential. Find the force on the charge due to the conductors.

6.54 A point charge Q is placed between two earthed intersecting conducting planes that are inclined at 45° to each other. Determine the number of image charges and their locations.

6.55 Infinite line $x = 3$, $z = 4$ carries 16 nC/m and is located in free space above the conducting plane $z = 0$. (a) Find **E** at $(2, -2, 3)$. (b) Calculate the induced surface charge density on the conducting plane at $(5, -6, 0)$.

6.56 In free space, infinite planes $y = 4$ and $y = 8$ carry charges 20 nC/m^2 and 30 nC/m^2, respectively. If plane $y = 2$ is grounded, calculate **E** at $P(0, 0, 0)$ and $Q(-4, 6, 2)$.

PART

3

MAGNETOSTATICS