7.7 Which of these statements is not characteristic of a static magnetic field?

(a) It is solenoidal.
(b) It is conservative.
(c) It has no sinks or sources.
(d) Magnetic flux lines are always closed.
(e) The total number of flux lines entering a given region is equal to the total number of flux lines leaving the region.

7.8 Two identical coaxial circular coils carry the same current I but in opposite directions. The magnitude of the magnetic field **B** at a point on the axis midway between the coils is

(a) Zero
(b) The same as that produced by one coil
(c) Twice that produced by one coil
(d) Half that produced by one coil.

7.9 Which one of these equations is not Maxwell's equation for a static electromagnetic field in a linear homogeneous medium?

(a) $\nabla \cdot \mathbf{B} = 0$
(b) $\nabla \times \mathbf{D} = 0$
(c) $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_o I$
(d) $\oint \mathbf{D} \cdot d\mathbf{S} = Q$
(e) $\nabla^2 \mathbf{A} = \mu_o \mathbf{J}$

7.10 Two bar magnets with their north poles having strength $Q_{m1} = 20$ A · m and $Q_{m2} = 10$ A · m (magnetic charges) are placed inside a volume as shown in Figure 7.26. The magnetic flux leaving the volume is

(a) 200 Wb
(b) 30 Wb
(c) 10 Wb
(d) 0 Wb
(e) −10 Wb

Answers: 7.1c, 7.2c, 7.3 (a)-(ii), (b)-(vi), (c)-(i), (d)-(v), (e)-(iii), 7.4d, 7.5a, 7.6 (a) 10 A, (b) −20 A, (c) 0, (d) −10 A, 7.7b, 7.8a, 7.9e, 7.10d.

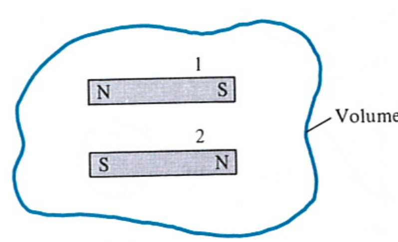


FIGURE 7.26 For Review Question 7.10.

PROBLEMS

Section 7.2—Biot–Savart's Law

7.1 (a) State Biot–Savart's law.
(b) The y- and z-axes, respectively, carry filamentary currents 10 A along $\mathbf{a}_y$ and 20 A along $-\mathbf{a}_z$. Find **H** at $(-3, 4, 5)$.

7.2 Two infinitely long filaments are placed parallel to the x-axis as shown in Figure 7.27. (a) Find **H** at the origin. (b) Determine **H** at $(-1, 2, 2)$.

7.3 A current element $I\,d\mathbf{l} = 4\mathbf{a}_x$ A · m is located at the origin. Determine its contribution to the magnetic field intensity at (a) (1, 0, 0), (b) (0, 1, 0), (c) (0, 0, 1), (d) (1, 1, 1).

7.4 A current filament is 5 mm long and is situated at $x = 0, y = 4, z = -3$ parallel to the x-axis. If the filament carries a current of 3 A along $\mathbf{a}_x$, find **H** at the origin.

7.5 A conducting filament carries current I from point $A(0, 0, a)$ to point $B(0, 0, b)$. Show that at point $P(x, y, 0)$,

$$\mathbf{H} = \frac{I}{4\pi\sqrt{x^2 + y^2}}\left[\frac{b}{\sqrt{x^2 + y^2 + b^2}} - \frac{a}{\sqrt{x^2 + y^2 + a^2}}\right]\mathbf{a}_\phi$$

7.6 Consider AB in Figure 7.28 as part of an electric circuit. Find **H** at the origin due to AB.

7.7 A finite-length line is shown in Figure 7.29. Show that **H** at point P is given by

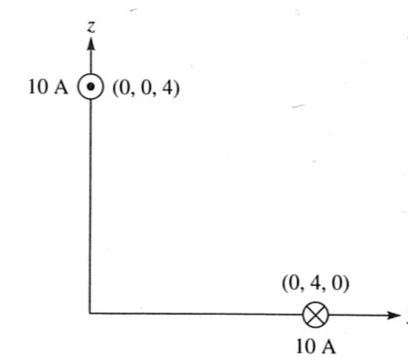
$$\mathbf{H} = \frac{I}{4\pi\rho}(\sin\beta_2 - \sin\beta_1)\mathbf{a}_\phi$$

7.8 The L-shaped conductor shown in Figure 7.30 is part of a circuit. Find **H** at (4, 8, 0).

7.9 Line $x = 0, y = 0, 0 \le z \le 10$ m carries current 2 A along $\mathbf{a}_z$. Calculate **H** at points

(a) (5, 0, 0)
(b) (5, 5, 0)
(c) (5, 15, 0)
(d) (5, −15, 0)

FIGURE 7.27 For Problem 7.2.



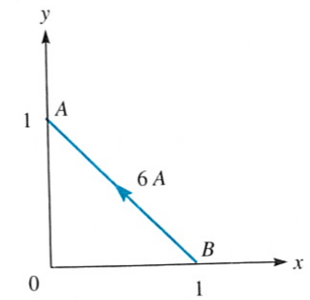


FIGURE 7.28 For Problem 7.6.

*7.10 (a) Find **H** at $(0, 0, 5)$ due to side 2 of the triangular loop in Figure 7.6(a).
(b) Find **H** at $(0, 0, 5)$ due to the entire loop.

7.11 An infinitely long conductor is bent into an L shape as shown in Figure 7.31. If a direct current of 5 A flows in the current, find the magnetic field intensity at (a) $(2, 2, 0)$, (b) $(0, -2, 0)$, and (c) $(0, 0, 2)$.

7.12 Find **H** at the center C of an equilateral triangular loop of side 4 m carrying 5 A of current as in Figure 7.32.

7.13 A rectangular loop carrying 10 A of current is placed on $z = 0$ plane as shown in Figure 7.33. Evaluate **H** at

(a) $(2, 2, 0)$
(b) $(4, 2, 0)$
(c) $(4, 8, 0)$
(d) $(0, 0, 2)$

7.14 A square conducting loop of side $2a$ lies in the $z = 0$ plane and carries a current I in the counterclockwise direction. Show that at the center of the loop

$$\mathbf{H} = \frac{\sqrt{2}I}{\pi a}\mathbf{a}_z$$

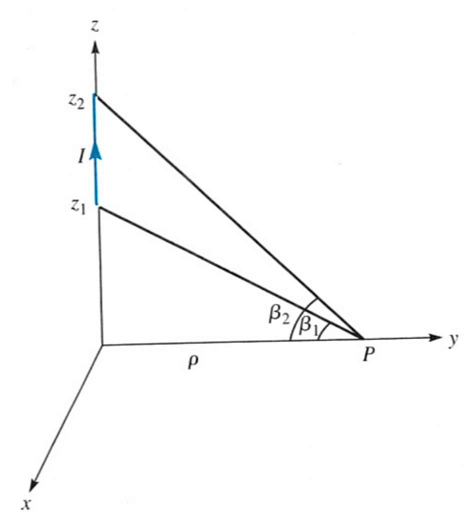


FIGURE 7.29 For Problem 7.7.

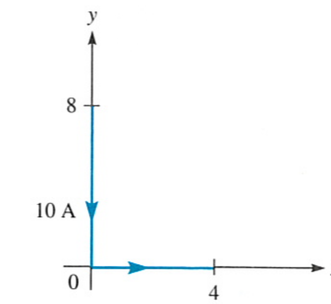


FIGURE 7.30 For Problem 7.8.

*7.15 (a) A filamentary loop carrying current I is bent to assume the shape of a regular polygon of n sides. Show that at the center of the polygon

$$H = \frac{nI}{2\pi r}\sin\frac{\pi}{n}$$

where r is the radius of the circle circumscribed by the polygon.

(b) Apply this for the cases of $n = 3$ and $n = 4$ and see if your results agree with those for the triangular loop of Problem 7.12 and the square loop of Problem 7.14, respectively.

(c) As n becomes large, show that the result of part (a) becomes that of the circular loop of Example 7.3.

7.16 For the filamentary loop shown in Figure 7.34, find the magnetic field strength at O.

7.17 Two identical current loops have their centers at $(0, 0, 0)$ and $(0, 0, 4)$ and their axes the same as the z-axis (so that the "Helmholtz coil" is formed). If each loop has a radius of 2 m and carries a current of 5 A in $\mathbf{a}_\phi$, calculate **H** at

(a) $(0, 0, 0)$
(b) $(0, 0, 2)$

7.18 A solenoid of radius 4 mm and length 2 cm has 150 turns/m and carries a current of 500 mA. Find (a) $|\mathbf{H}|$ at the center, (b) $|\mathbf{H}|$ at the ends of the solenoid.

7.19 Plane $x = 10$ carries a current of 100 mA/m along $\mathbf{a}_z$, while line $x = 1$, $y = -2$ carries a filamentary current of 20π mA along $\mathbf{a}_z$. Determine **H** at $(4, 3, 2)$.

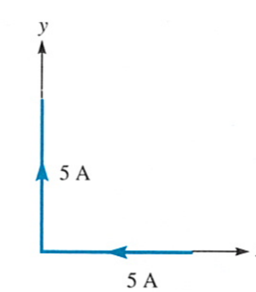


FIGURE 7.31 Current filament for Problem 7.11.

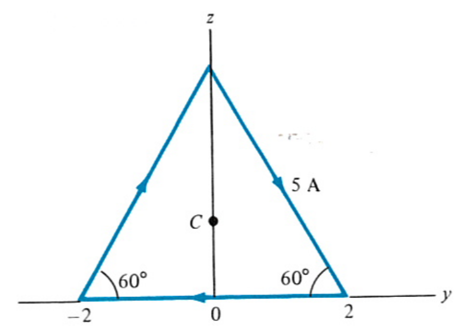


FIGURE 7.32 Equilateral triangular loop for Problem 7.12.

Section 7.3—Ampère's Circuit Law

7.20 (a) State Ampère's circuit law.

(b) A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find **H** everywhere.

7.21 Current sheets of $20\mathbf{a}_x$ A/m and $-20\mathbf{a}_x$ A/m are located at $y = 1$ and $y = -1$, respectively. Find **H** in region $-1 < y < 1$.

7.22 The $z = 0$ plane carries current $\mathbf{K} = 10\mathbf{a}_x$ A/m, while current filament situated at $y = 0, z = 6$ carries current I along $\mathbf{a}_x$. Find I such that $\mathbf{H}(0, 0, 3) = 0$.

7.23 (a) An infinitely long solid conductor of radius a is placed along the z-axis. If the conductor carries current I in the $+z$ direction, show that

$$\mathbf{H} = \frac{I\rho}{2\pi a^2}\mathbf{a}_\phi$$

within the conductor. Find the corresponding current density.

(b) If $I = 3$ A and $a = 2$ cm in part (a), find **H** at $(0, 1 \text{ cm}, 0)$ and $(0, 4 \text{ cm}, 0)$.

7.24 An infinitely long cylindrical conductor of radius a is placed along the z-axis. If the current density is $\mathbf{J} = \frac{J_o}{\rho}\mathbf{a}_z$, where J_o is constant, find **H** everywhere.

7.25 Let $\mathbf{H} = k_o\left(\frac{\rho}{a}\right)\mathbf{a}_\phi$, $\rho < a$, where k_o is a constant. (a) Find **J** for $\rho < a$. (b) Find **H** for $\rho > a$.

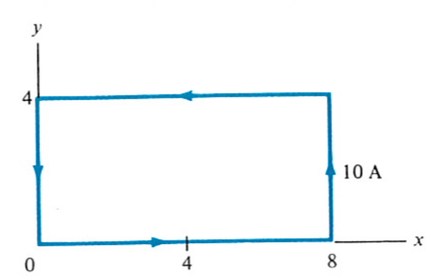


FIGURE 7.33 Rectangular loop of Problem 7.13.

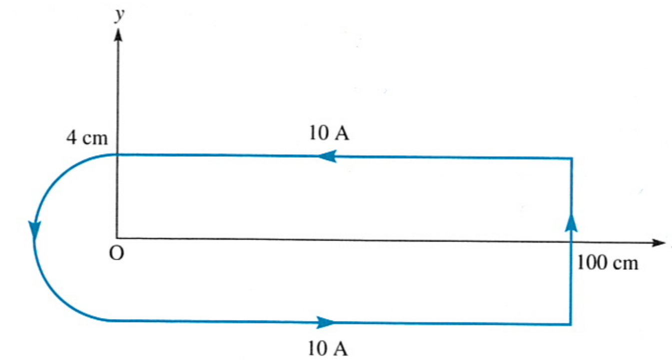


FIGURE 7.34 Filamentary loop of Problem 7.16 not drawn to scale.

7.26 An infinitely long filamentary wire carries a current of 2 A along the z-axis in the $+z$-direction. Calculate the following:

(a) **B** at $(-3, 4, 7)$

(b) The flux through the square loop described by $2 \le \rho \le 6, 0 \le z \le 4, \phi = 90°$.

7.27 The electric motor shown in Figure 7.35 has field

$$\mathbf{H} = \frac{10^6}{\rho}\sin 2\phi\, \mathbf{a}_\rho \text{ A/m}$$

Calculate the flux per pole passing through the air gap if the axial length of the pole is 20 cm.

7.28 Consider the two-wire transmission line whose cross section is illustrated in Figure 7.36. Each wire is of radius 2 cm, and the wires are separated 10 cm. The wire centered at $(0, 0)$ carries a current of 5 A while the other centered at $(10 \text{ cm}, 0)$ carries the return current. Find **H** at

(a) $(5 \text{ cm}, 0)$

(b) $(10 \text{ cm}, 5 \text{ cm})$

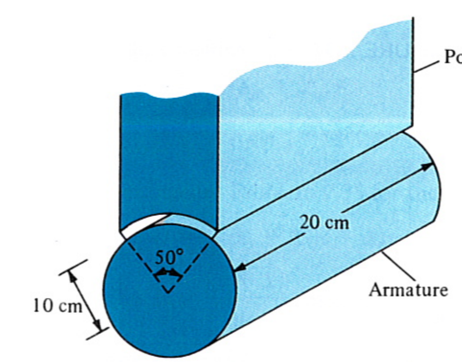


FIGURE 7.35 Electric motor pole of Problem 7.27.

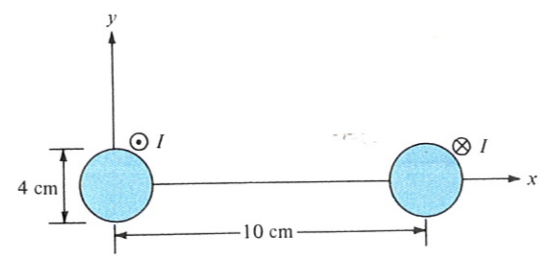


FIGURE 7.36 Two-wire line of Problem 7.28.

7.29 An electron beam forms a current of density

$$\mathbf{J} = \begin{cases} J_o(1-\rho^2/a^2)\mathbf{a}_z, & \rho < a \\ 0, & \rho > a \end{cases}$$

(a) Determine the total current.

(b) Find the magnetic field intensity everywhere.

Section 7.5—Magnetic Flux Density

7.30 Determine the magnetic flux through a rectangular loop $(a \times b)$ due to an infinitely long conductor carrying current I as shown in Figure 7.37. The loop and the straight conductors are separated by distance d.

7.31 A long, straight wire of radius a has current density $\mathbf{J} = J_o e^{-\beta(a-\rho)}\mathbf{a}_z$ $\rho < a$, where β is a constant. Determine $\mathbf{B}$ inside and outside the wire.

7.32 A semicircular loop of radius a in free space carries a current I. Determine the magnetic flux density at the center of the loop.

7.33 In free space, the magnetic flux density is

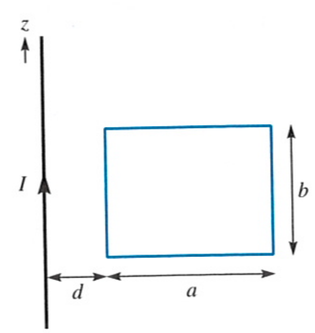
$$B = y^2\mathbf{a}_x + z^2\mathbf{a}_y + x^2\mathbf{a}_z \text{ Wb/m}^2$$

(a) Show that B is a magnetic field

(b) Find the magnetic flux through $x = 1, 0 < y < 1, 1 < z < 4$.

(c) Calculate $\mathbf{J}$.

FIGURE 7.37 For Problem 7.30.



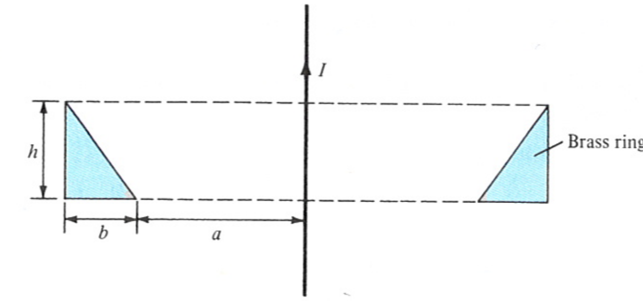


FIGURE 7.38 Cross section of a brass ring enclosing a long straight wire; for Problem 7.34.

***7.34** A brass ring with triangular cross section encircles a very long straight wire concentrically as in Figure 7.38. If the wire carries a current I, show that the total number of magnetic flux lines in the ring is

$$\Psi = \frac{\mu_o I h}{2\pi b}\left[b - a \ln \frac{a+b}{b}\right]$$

Calculate Ψ if $a = 30$ cm, $b = 10$ cm, $h = 5$ cm, and $I = 10$ A.

7.35 In free space, let $\mathbf{H} = \frac{10^6}{\rho} \sin\phi \, \mathbf{a}_\rho$ A/m. Calculate the flux crossing the surface defined by $0 < \phi < \pi/3, 0 < z < 2$ m.

7.36 In free space, $\mathbf{B} = \frac{5}{\rho}\cos\phi \, \mathbf{a}_z$ Wb/m^2. Determine the magnetic flux crossing the strip $z = 0, 1 < \rho < 2$ m, $0 < \phi < \pi/4$.

Section 7.6—Maxwell's Equations

7.37 Consider the following arbitrary fields. Find out which of them can possibly represent an electrostatic or magnetostatic field in free space.

(a) $\mathbf{A} = y\cos ax\mathbf{a}_x + (y + e^{-x})\mathbf{a}_z$

(b) $\mathbf{B} = \frac{20}{\rho}\mathbf{a}_\rho$

(c) $\mathbf{C} = r^2 \sin\theta \, \mathbf{a}_\phi$

7.38 Reconsider Problem 7.35 for the following fields.

(a) $\mathbf{D} = y^2 z\mathbf{a}_x + 2(x+1)yz\mathbf{a}_y - (x+1)z^2\mathbf{a}_z$

(b) $\mathbf{E} = \frac{(z+1)}{\rho}\cos\phi \, \mathbf{a}_\rho + \frac{\sin\phi}{\rho}\mathbf{a}_z$

(c) $\mathbf{F} = \frac{1}{r^2}(2\cos\theta \, \mathbf{a}_r + \sin\theta \, \mathbf{a}_\theta)$

Section 7.7—Magnetic Scalar and Vector Potentials

7.39 A current element of length L carries current I in the z direction. Show that at a very distant point,

$$\mathbf{A} = \frac{\mu_o IL}{4\pi r}\mathbf{a}_z$$

Find **B**.

7.40 Given the magnetic vector potential

$$\mathbf{A} = e^x \sin y\mathbf{a}_x + (y + \cos y)\mathbf{a}_z \text{ Wb/m}$$

determine the magnetic flux density at the origin.

7.41 For a current distribution in free space,

$$\mathbf{A} = (2x^2y + yz)\mathbf{a}_x + (xy^2 - xz^3)\mathbf{a}_y - (6xyz - 2x^2y^2)\mathbf{a}_z \text{ Wb/m}$$

(a) Calculate **B**.

(b) Find the magnetic flux through a loop described by $x = 1, 0 < y < 2, 0 < z < 2$.

(c) Show that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \cdot \mathbf{B} = 0$.

7.42 The magnetic vector potential of a current distribution in free space is given by

$$\mathbf{A} = 15e^{-\rho} \sin \phi \, \mathbf{a}_z \text{ Wb/m}$$

Find **H** at $(3, \pi/4, -10)$. Calculate the flux through $\rho = 5, 0 \leq \phi \leq \pi/2, 0 \leq z \leq 10$.

7.43 An infinitely long conductor of radius a carries a uniform current with $\mathbf{J} = J_o\mathbf{a}_z$. Show that the magnetic vector potential for $\rho < a$ is

$$\mathbf{A} = -\frac{1}{4}\mu_o J_o \rho^2 \mathbf{a}_z$$

7.44 Find the **B**- field corresponding to the magnetic vector potential

$$\mathbf{A} = \sin\frac{\pi x}{2}\cos\frac{\pi y}{2}\mathbf{a}_z$$

7.45 The magnetic vector potential of two parallel infinite straight current filaments in free space carrying equal current I in the opposite direction is

$$\mathbf{A} = \frac{\mu I}{2\pi}\ln\frac{d - \rho}{\rho}\mathbf{a}_z$$

where d is the separation distance between the filaments (with one filament placed along the z-axis). Find the corresponding magnetic flux density **B**.

7.46 The magnetic field intensity in a certain conducting medium is

$$\mathbf{H} = xy^2\mathbf{a}_x + x^2z\mathbf{a}_y - y^2z\mathbf{a}_z \text{ A/m}$$

(a) Calculate the current density at point $P(2, -1, 3)$.

(b) What is $\frac{\partial \rho_v}{\partial t}$ at P?

7.47 Let $\mathbf{A} = 10\rho^2\mathbf{a}_z \; \mu\text{Wb/m}$.

(a) Find **H** and **J**

(b) Determine the total current crossing the surface $z = 1, 0 \leq \rho \leq 2, 0 \leq \phi \leq 2\pi$.

7.48 Prove that the magnetic scalar potential at $(0, 0, z)$ due to a circular loop of radius a shown in Figure 7.8(a) is

$$V_m = \frac{I}{2}\left[1 - \frac{z}{[z^2 + a^2]^{1/2}}\right]$$

*7.49 A coaxial transmission line is constructed such that the radius of the inner conductor is a and the outer conductor has radii $3a$ and $4a$. Find the vector magnetic potential within the outer conductor. Assume $A_z = 0$ for $\rho = 3a$.

7.50 The z-axis carries a filamentary current 12 A along $\mathbf{a}_z$. Calculate V_m at $(4, 30°, -2)$ if V_m=0 at $(10, 60°, 7)$.

7.51 Plane $z = -2$ carries a current of $50\mathbf{a}_y$ A/m. If $V_m = 0$ at the origin, find V_m at

(a) $(-2, 0, 5)$

(b) $(10, 3, 1)$

7.52 Prove in cylindrical coordinates that

(a) $\nabla \times (\nabla V) = 0$

(b) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

*7.53 If $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $R = |\mathbf{R}|$, show that

$$\nabla\frac{1}{R} = -\nabla'\frac{1}{R} = -\frac{\mathbf{R}}{R^3}$$

where ∇ and ∇' are del operators with respect to (x, y, z) and (x', y', z'), respectively.