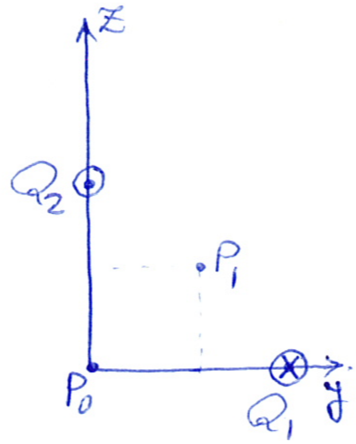
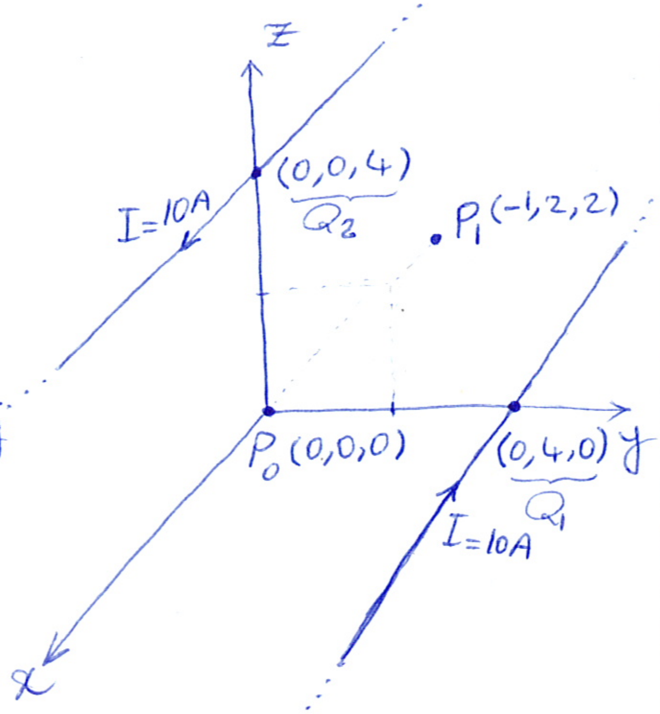
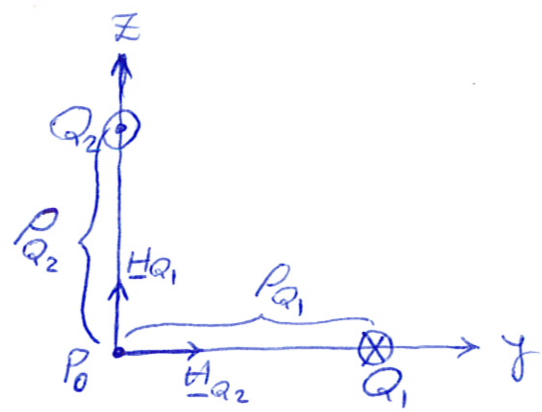
7.2





a) $\underline{H}$ at P_0 = ?



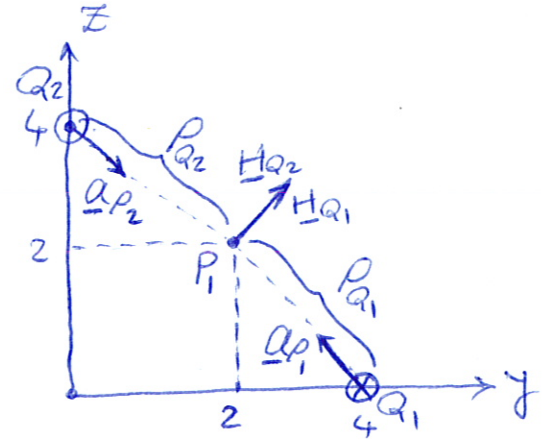
$$\underline{H} = \underline{H}_{Q_1} + \underline{H}_{Q_2}$$

$$\underline{H}_{Q_1} = \frac{I_{Q_1}}{2\pi\rho_{Q_1}}\underline{a}_z = \frac{10}{2\pi\times 4}\underline{a}_z$$

$$\underline{H}_{Q_2} = \frac{I_{Q_2}}{2\pi\rho_{Q_2}}\underline{a}_y = \frac{10}{2\pi\times 4}\underline{a}_y$$

$$\Rightarrow \underline{H} = \frac{10}{8\pi}(\underline{a}_y + \underline{a}_z)\ A/m = 0.3979\,(\underline{a}_y + \underline{a}_z)\ A/m$$

b) $\underline{H}$ at P_1 = ?



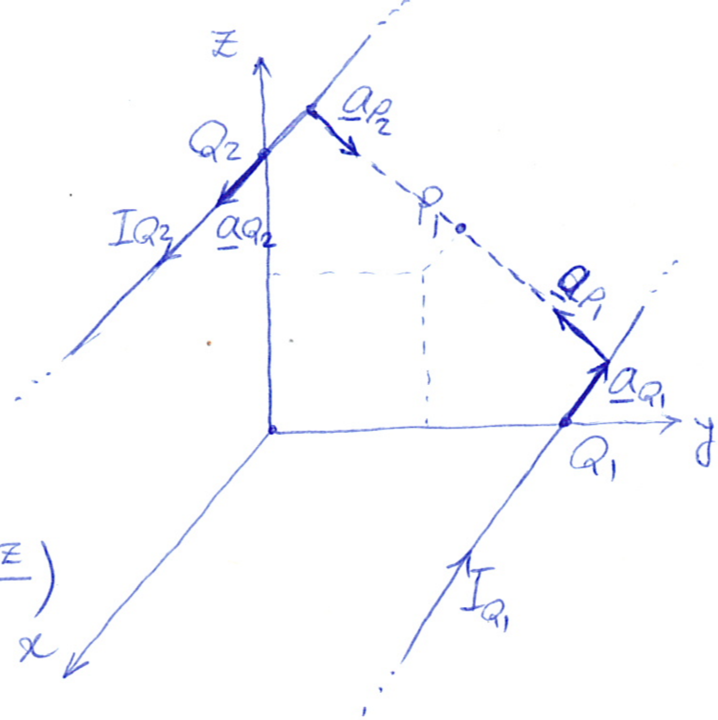
$$\underline{H} = \underline{H}_{Q_1} + \underline{H}_{Q_2}$$

$$\underline{H}_{Q_1} = \frac{I_{Q_1}}{2\pi\rho_{Q_1}}(\underline{a}_{Q_1} \times \underline{a}_{\rho_1})$$

7.2

b)

$$\begin{cases} \underline{a}_{Q_1} = -\underline{a}_x \\ \underline{a}_{\rho_1} = \dfrac{-2\underline{a}_y + 2\underline{a}_z}{\sqrt{8}} \\ \rho_{Q_1} = \sqrt{8} \end{cases}$$



$$\underline{H}_{Q_1} = \frac{10}{2\pi\sqrt{8}}(-\underline{a}_x)\times\left(\frac{-2\underline{a}_y+2\underline{a}_z}{\sqrt{8}}\right)$$

$$= \frac{10}{2\pi \times 8}\left(2\underline{a}_z + 2\underline{a}_y\right)$$

$$= \frac{10}{8\pi}\left(\underline{a}_y + \underline{a}_z\right) \ \text{A/m}$$

$$\underline{H}_{Q_2} = \frac{I_{Q_2}}{2\pi\rho_{Q_2}}\left(\underline{a}_{Q_2} \times \underline{a}_{\rho_2}\right)$$

$$\begin{cases} \underline{a}_{Q_2} = \underline{a}_x \\ \underline{a}_{\rho_2} = \dfrac{2\underline{a}_y - 2\underline{a}_z}{\sqrt{8}} \\ \rho_{Q_2} = \sqrt{8} \end{cases}$$

$$\underline{H}_{Q_2} = \frac{10}{2\pi\sqrt{8}}(\underline{a}_x)\times\left(\frac{2\underline{a}_y-2\underline{a}_z}{\sqrt{8}}\right)$$

$$= \frac{10}{2\pi \times 8}\left(2\underline{a}_z + 2\underline{a}_y\right)$$

$$= \frac{10}{8\pi}\left(\underline{a}_y + \underline{a}_z\right) \ \text{A/m}$$

7.2

b) $\underline{H} = \underline{H}_{Q_1} + \underline{H}_{Q_2}$

$$= \frac{10}{8\pi}(\underline{a}_y + \underline{a}_z) + \frac{10}{8\pi}(\underline{a}_y + \underline{a}_z)$$

$$= \frac{10}{4\pi}(\underline{a}_y + \underline{a}_z) \text{ A/m}$$

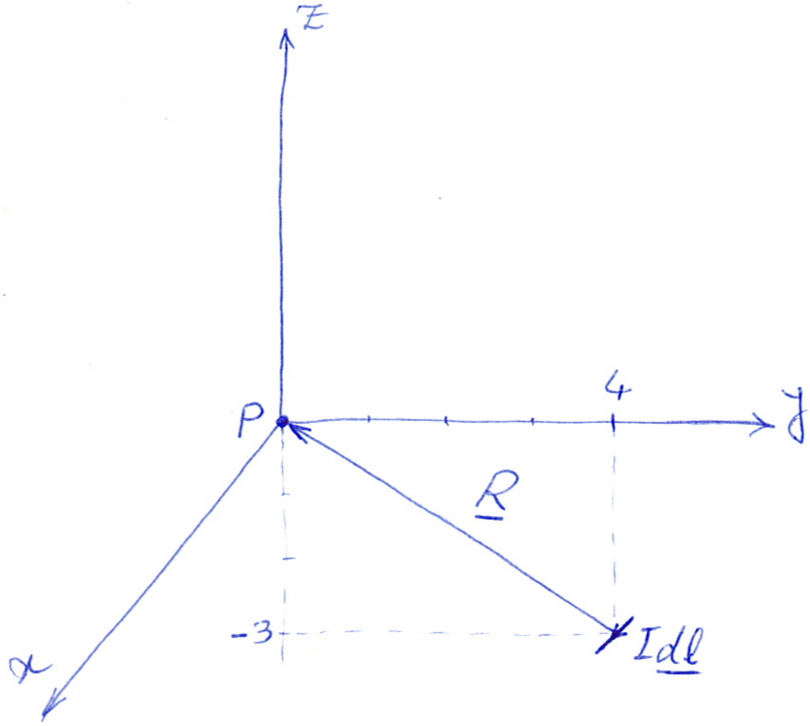
$$= 0.7958(\underline{a}_y + \underline{a}_z) \text{ A/m}$$

7.4

$I = 3\,A$

$\underline{dl} = 5\times10^{-3}\,\underline{a}_x$

$\underline{H}$ at $P = ?$



$$\underline{H} = d\underline{H} = \frac{I\underline{dl}\times\underline{R}}{4\pi|\underline{R}|^3}$$

$$\underline{R} = (0,0,0) - (0,4,-3) = -4\underline{a}_y + 3\underline{a}_z$$

$$|\underline{R}| = \sqrt{16+9} = 5$$

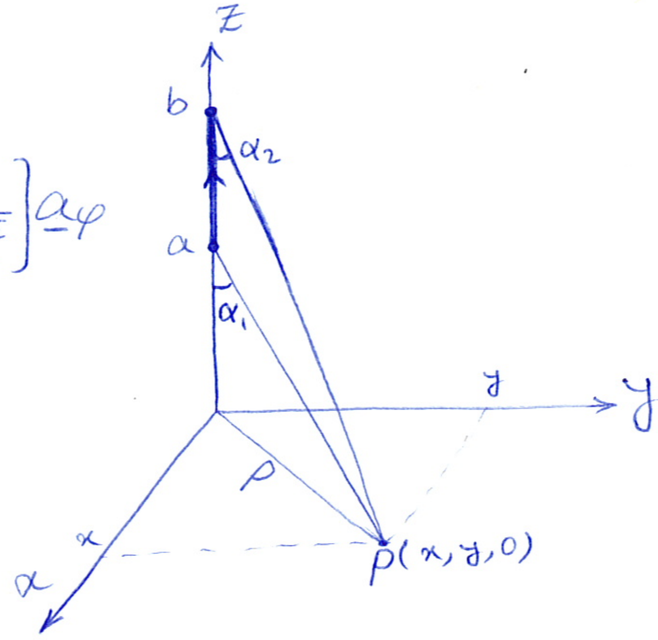
$$\Rightarrow \underline{H} = \frac{3\times(5\times10^{-3}\,\underline{a}_x)\times(-4\underline{a}_y + 3\underline{a}_z)}{4\pi\times5^3}$$

$$= \frac{3\times5\times10^{-3}}{4\pi\times125}(-4\underline{a}_z - 3\underline{a}_y)$$

$$= \frac{3}{\pi}\times10^{-5}(-3\underline{a}_y - 4\underline{a}_z) \quad A/m$$

$$= -28.6479\,\underline{a}_y - 38.1972\,\underline{a}_z \quad \mu A/m$$

7.5



$$\underline{H} \text{ at } P \stackrel{?}{=} \frac{I}{4\pi\sqrt{x^2+y^2}}\left[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}\right]\underline{a}_\varphi$$

$$\underline{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\underline{a}_\ell \times \underline{a}_\rho$$

$$\rho = \sqrt{x^2+y^2}$$

$$\cos\alpha_2 = \frac{b}{\sqrt{\rho^2+b^2}} = \frac{b}{\sqrt{x^2+y^2+b^2}}$$

$$\cos\alpha_1 = \frac{a}{\sqrt{\rho^2+a^2}} = \frac{a}{\sqrt{x^2+y^2+a^2}}$$

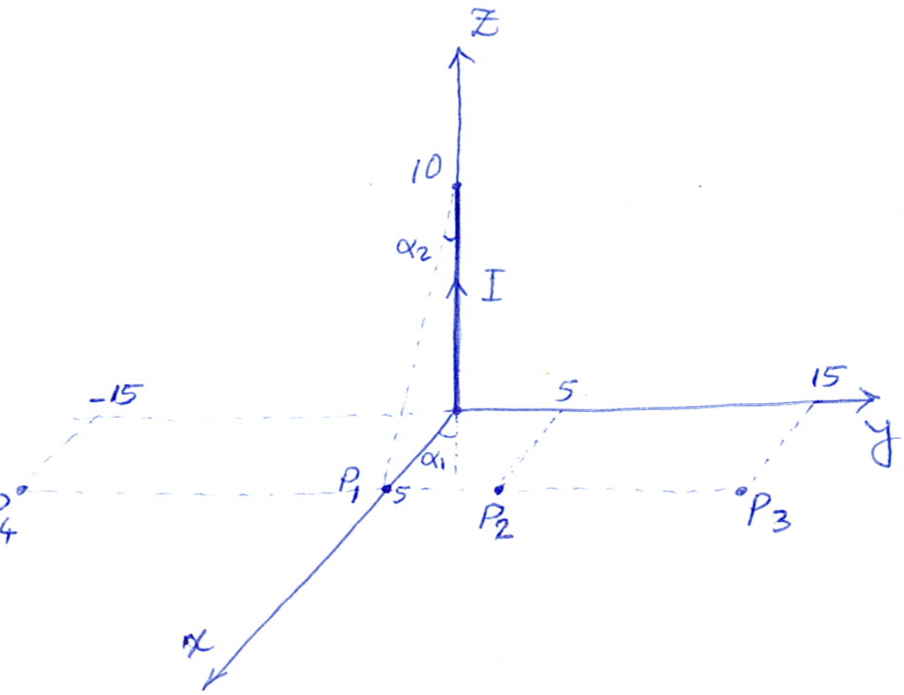
$$\underline{a}_\ell \times \underline{a}_\rho = \underline{a}_z \times \underline{a}_\rho = \underline{a}_\varphi$$

$$\Rightarrow \underline{H} = \frac{I}{4\pi\sqrt{x^2+y^2}}\left[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}\right]\underline{a}_\varphi$$

7.9

$$I = 2A \qquad \begin{cases} x = 0 \\ y = 0 \\ 0 \leq z \leq 10 \end{cases}$$

$$\underline{a}_l = \underline{a}_z$$



$$\underline{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\underline{a}_l \times \underline{a}_\rho$$

a) $\underline{H}$ at $P_1 = ?$

$P_1 = (5,0,0)$

$\Rightarrow \rho = 5$

$\cos\alpha_1 = \cos(90^\circ) = 0$

$$\cos\alpha_2 = \frac{10}{\sqrt{100+25}} = \frac{2}{\sqrt{5}}$$

$\underline{a}_l \times \underline{a}_\rho = \underline{a}_z \times \underline{a}_x = \underline{a}_y$

$$\underline{H} = \frac{2}{4\pi \times 5}\left(\frac{2}{\sqrt{5}} - 0\right)\underline{a}_y = \frac{1}{5\sqrt{5} \times \pi}\underline{a}_y \ \ A/m$$

$= 28.4705$ mA/m

b) $\underline{H}$ at $P_2 = ?$

$P_2 = (5, 5, 0)$

$\Rightarrow \rho = \sqrt{25+25} = 5\sqrt{2}$

$\cos\alpha_1 = 0$

$$\cos\alpha_2 = \frac{10}{\sqrt{25+25+100}} = \frac{10}{\sqrt{150}}$$

$$\underline{a}_l \times \underline{a}_\rho = \underline{a}_z \times \left(\frac{5\underline{a}_x + 5\underline{a}_y}{5\sqrt{2}}\right) = \frac{1}{\sqrt{2}}(\underline{a}_y - \underline{a}_x)$$

7.9

b)

$$\underline{H} = \frac{2}{4\pi \times 5\sqrt{2}}\left(\frac{10}{\sqrt{150}} - 0\right)\left(\frac{-\underline{a}_x + \underline{a}_y}{\sqrt{2}}\right) = \frac{-\underline{a}_x + \underline{a}_y}{2\pi\sqrt{150}} \quad A/m$$

$$= 12.9949\,(-\underline{a}_x + \underline{a}_y) \quad mA/m$$

c) $\underline{H}$ at $P_3 = ?$

$$P_3 = (5, 15, 0)$$

$$\Rightarrow \rho = \sqrt{25 + 225} = 5\sqrt{10}$$

$$\cos\alpha_1 = 0$$

$$\cos\alpha_2 = \frac{10}{\sqrt{25+225+100}} = \frac{10}{\sqrt{350}}$$

$$\underline{a}_\ell \times \underline{a}_\rho = \underline{a}_z \times \left(\frac{5\underline{a}_x + 15\underline{a}_y}{5\sqrt{10}}\right) = \frac{\underline{a}_y - 3\underline{a}_x}{\sqrt{10}}$$

$$\underline{H} = \frac{2}{4\pi \times 5\sqrt{10}}\left(\frac{10}{\sqrt{350}} - 0\right)\left(\frac{-3\underline{a}_x + \underline{a}_y}{\sqrt{10}}\right) = \frac{-3\underline{a}_x + \underline{a}_y}{10\pi \times \sqrt{350}} \quad A/m$$

$$= 1.7014\,(-3\underline{a}_x + \underline{a}_y) \quad mA/m$$

d) $\underline{H}$ at $P_4 = ?$

$$P_4 = (5, -15, 0)$$

$$\Rightarrow \rho = 5\sqrt{10}$$

$$\cos\alpha_1 = 0$$

$$\cos\alpha_2 = \frac{10}{\sqrt{350}}$$

$$\underline{a}_\ell \times \underline{a}_\rho = \underline{a}_z \times \left(\frac{5\underline{a}_x - 15\underline{a}_y}{5\sqrt{10}}\right) = \frac{3\underline{a}_x + \underline{a}_y}{\sqrt{10}}$$

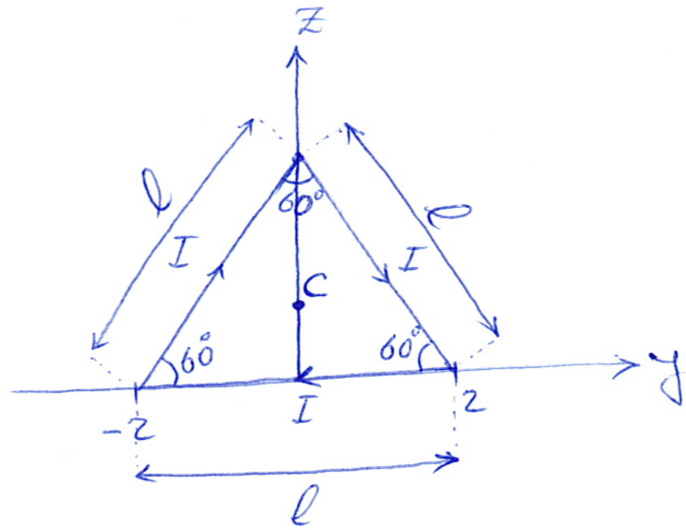
$$\underline{H} = \frac{3\underline{a}_x + \underline{a}_y}{10\pi \times \sqrt{350}} \quad A/m = 1.7014\,(3\underline{a}_x + \underline{a}_y) \quad mA/m$$

7.12

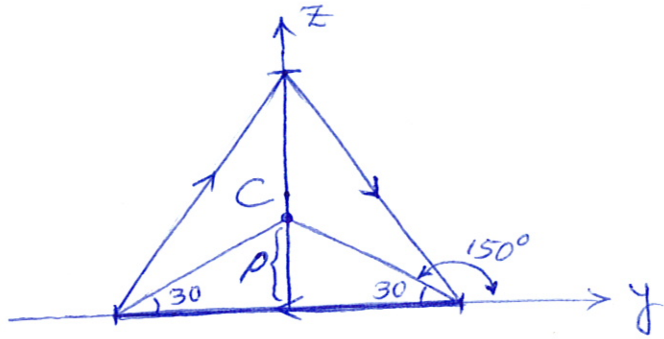
$I = 5\,A$

$l = 4\,m$

$\underline{H}$ at $C = ?$



Because of symmetry of problem, $\underline{H}$ is 3 times of $\underline{H}'$ which is the result of bottom side of triangle.



$\underline{H} = 3\underline{H}'$

$$\underline{H}' = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\underline{a}_l \times \underline{a}_\rho$$

$$\begin{cases} \tan 30° = \dfrac{\rho}{2} \Rightarrow \rho = 2\tan 30° = \dfrac{2}{\sqrt{3}} \\ \cos\alpha_1 = \cos 150° = -\dfrac{\sqrt{3}}{2} \\ \cos\alpha_2 = \cos 30° = \dfrac{\sqrt{3}}{2} \\ \underline{a}_l \times \underline{a}_\rho = (-\underline{a}_y) \times (\underline{a}_z) = -\underline{a}_x \end{cases}$$

$$\underline{H}' = \frac{5}{4\pi \times \frac{2}{\sqrt{3}}}\left(\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right)(-\underline{a}_x) = -\frac{15}{8\pi}\underline{a}_x \ A/m$$

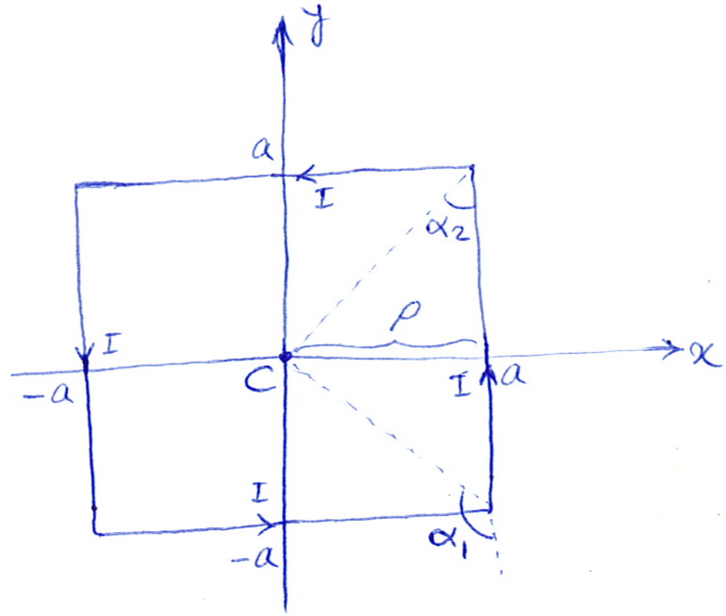
$$\Rightarrow \underline{H} = 3\underline{H}' = \frac{-45}{8\pi}\underline{a}_x \ A/m$$

$$= -1.7905\,\underline{a}_x \ A/m$$

7.14

$C = (0,0,0)$

$\underline{H}$ at C = ? $= \dfrac{\sqrt{2}\, I}{\pi a} \underline{a}_z$



Symmetry of problem $\Rightarrow$

$$\underline{H} = 4\underline{H}'$$

where $\underline{H}'$ is due to the right side of the loop.

$$\underline{H}' = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, \underline{a}_\ell \times \underline{a}_\rho$$

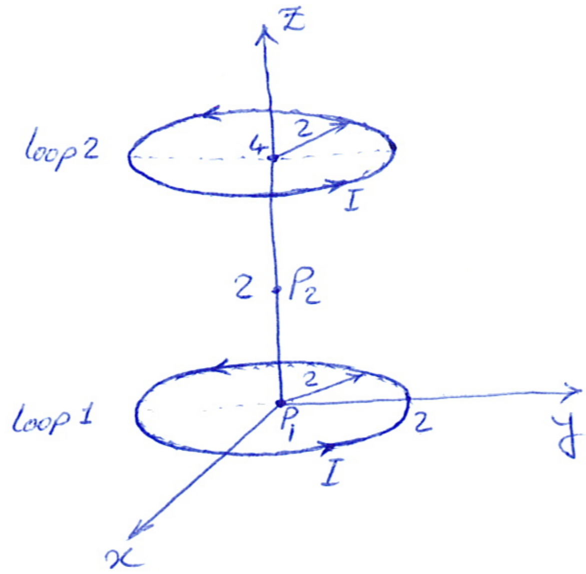
$$\begin{cases} \rho = a \\ \cos\alpha_1 = \cos 135° = -\dfrac{\sqrt{2}}{2} \\ \cos\alpha_2 = \cos 45° = \dfrac{\sqrt{2}}{2} \\ \underline{a}_\ell \times \underline{a}_\rho = \underline{a}_y \times (-\underline{a}_x) = \underline{a}_z \end{cases}$$

$$\underline{H}' = \frac{I}{4\pi a}\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)\underline{a}_z = \frac{I \times \sqrt{2}}{4\pi a}\underline{a}_z \ \text{A/m}$$

$$\Rightarrow \underline{H} = 4\underline{H}' = \frac{\sqrt{2}\, I}{\pi a}\underline{a}_z \ \text{A/m}$$

7.17

$I = 5\ A$ in $\underline{a}_\varphi$ direction



a) $\underline{H}$ at P_1 = ?

$$\underline{H}_1 = \frac{I \rho^2}{2(\rho^2 + h^2)^{3/2}} \underline{a}_z$$

loop 1 : $\rho = 2$ & $h = 0$

loop 2 : $\rho = 2$ & $h = 4$

$$\Rightarrow \underline{H} = \frac{I \times 2^2}{2(2^2+0^2)^{3/2}} \underline{a}_z + \frac{I \times 2^2}{2(2^2+4^2)^{3/2}} \underline{a}_z = \left(\frac{5\times4}{2\times8} + \frac{5\times4}{2(20)^{3/2}}\right) \underline{a}_z$$

$= 1.3618\ \underline{a}_z$ A/m

b) $\underline{H}$ at P_2 = ?

$P_2 = (0, 0, 2)$

loop 1 & 2 : $\rho = 2$ & $h = 2$

$\Rightarrow \underline{H}_1 = \underline{H}_2$

$$\Rightarrow \underline{H} = 2\underline{H}_1 = 2 \times \frac{5 \times 2^2}{2(2^2 + 2^2)^{3/2}} \underline{a}_z = \frac{20}{8\sqrt{8}} \underline{a}_z$$

$= 0.8839\ \underline{a}_z$ A/m

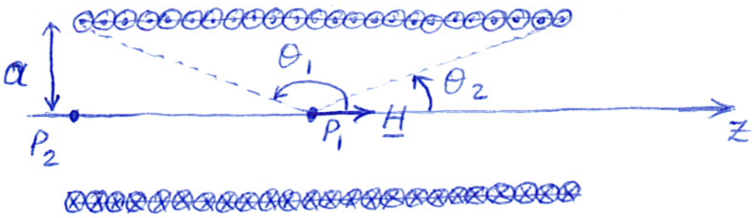
7.18 Solenoid

$a = 4\ mm$

$l = 2cm$

$n = 150 \text{ turns/m} = \frac{N}{l}$

$I = 500\ mA$



a) $|\underline{H}|$ at $P_1 = ?$

$$\underline{H} = \frac{nI}{2}(\cos\theta_2 - \cos\theta_1)\,\underline{a}_z$$

at the center of solenoid $\Rightarrow \cos\theta_1 = -\cos\theta_2$

$$\cos\theta_2 = \frac{l/2}{(a^2 + (\frac{l}{2})^2)^{1/2}}$$

$$\Rightarrow \underline{H} = \frac{nI}{2} \times 2\frac{l/2}{\sqrt{a^2 + l^2/4}}\,\underline{a}_z = \frac{nlI}{2\sqrt{a^2 + l^2/4}}\,\underline{a}_z$$

$$\Rightarrow |\underline{H}| = \frac{150 \times 2\times10^{-2} \times 500\times10^{-3}}{2\sqrt{16\times10^{-6} + 1\times10^{-4}}} = \frac{750\times10^{-3}}{10^{-3}\sqrt{116}}$$

$$= 69.6358\ A/m$$

b) $|\underline{H}|$ at $P_2 = ?$

$$\cos\theta_1 = \cos 90^\circ = 0$$

$$\cos\theta_2 = \frac{l}{\sqrt{a^2 + l^2}}$$

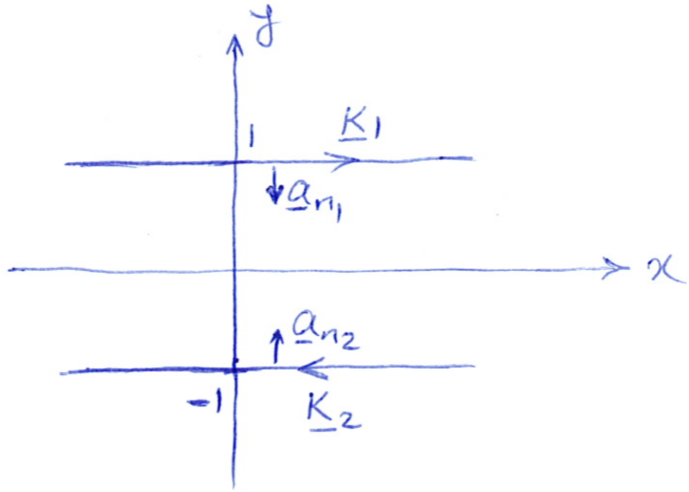
$$\Rightarrow \underline{H} = \frac{nI}{2}\left(\frac{l}{\sqrt{a^2 + l^2}} - 0\right)\underline{a}_z = \frac{nlI}{2\sqrt{a^2 + l^2}}\,\underline{a}_z$$

$$\Rightarrow |\underline{H}| = \frac{150 \times 2\times10^{-2} \times 500\times10^{-3}}{2\sqrt{16\times10^{-6} + 400\times10^{-6}}} = \frac{750}{\sqrt{416}} = 36.7718\ A/m$$

7.21

$\underline{K}_1 = 20\,\underline{a}_x$ A/m at $y=1$

$\underline{K}_2 = -20\,\underline{a}_x$ A/m at $y=-1$



$-1<y<1 \Rightarrow \underline{H} = ?$

$$\begin{cases} \underline{a}_{n_1} = -\underline{a}_y \\ \underline{a}_{n_2} = \underline{a}_y \end{cases}$$

$\underline{H} = \underline{H}_1 + \underline{H}_2$

$\underline{H}_1 = \frac{1}{2}\,\underline{K}_1 \times \underline{a}_{n_1}$

$= \frac{1}{2}(20\,\underline{a}_x)\times(-\underline{a}_y)$

$= -10\,\underline{a}_z$ A/m

$\underline{H}_2 = \frac{1}{2}\,\underline{K}_2 \times \underline{a}_{n_2}$

$= \frac{1}{2}(-20\,\underline{a}_x)\times(\underline{a}_y)$

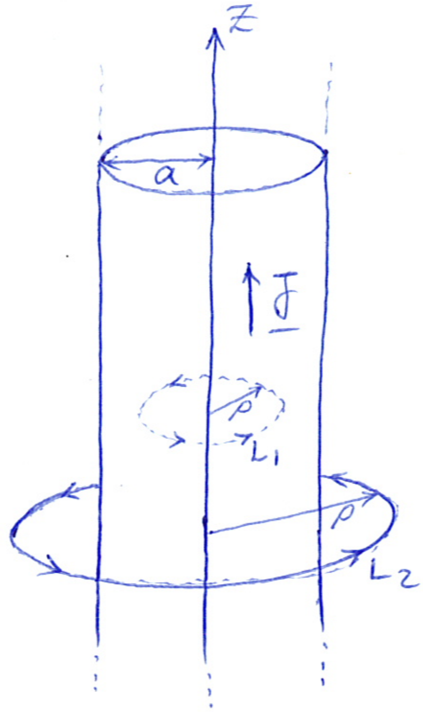
$= -10\,\underline{a}_z$ A/m

$\underline{H} = \underline{H}_1 + \underline{H}_2 = -20\,\underline{a}_z$ A/m

7.24

$\underline{J} = \frac{J_0}{\rho} \underline{a}_z$

$\underline{H} = ?$



$0 < \rho < a$

Symmetry $\Rightarrow \underline{H} = H_\varphi \underline{a}_\varphi$

$$\oint_{L_1} \underline{H} \cdot \underline{dl} = I_{enc} = \iint \underline{J} \cdot \underline{ds}$$

$$\oint_{L_1} \underline{H} \cdot \underline{dl} = H_\varphi \int_{L_1} dl = H_\varphi \times 2\pi\rho$$

$$I_{enc} = \int_{\rho=0}^{\rho} \int_{\varphi=0}^{2\pi} \left(\frac{J_0}{\rho} \underline{a}_z\right) \cdot \left(\rho \, d\rho \, d\varphi \, \underline{a}_z\right)$$

$$= \int_{\rho=0}^{\rho} \int_{\varphi=0}^{2\pi} J_0 \, d\rho \, d\varphi = J_0 \times 2\pi\rho$$

$$\Rightarrow H_\varphi \times 2\pi\rho = J_0 \times 2\pi\rho$$

$$\Rightarrow \underline{H} = J_0 \underline{a}_\varphi$$

$\rho > a$

with the same argument: $\underline{H} = H_\varphi \underline{a}_\varphi$

$$\oint_{L_2} \underline{H} \cdot \underline{dl} = H_\varphi \int_{L_2} dl = H_\varphi \times 2\pi\rho$$

$$I_{enc} = \int_{\rho=0}^{a} \int_{\varphi=0}^{2\pi} \underline{J} \cdot \underline{ds} = \int_{\rho=0}^{a} \int_{\varphi=0}^{2\pi} \frac{J_0}{\rho} \rho \, d\varphi \, d\rho$$

7.24

$$I_{enc} = J_0 \times 2\pi a$$

$$\Rightarrow H_\varphi \times 2\pi\rho = J_0 \times 2\pi a$$

$$\Rightarrow \underline{H} = \frac{J_0 a}{\rho} \underline{a}_\varphi$$

$$\underline{H} = \begin{cases} J_0 \underline{a}_\varphi & 0<\rho<a \\ \dfrac{J_0 a}{\rho} \underline{a}_\varphi & \rho > a \end{cases}$$

7.27

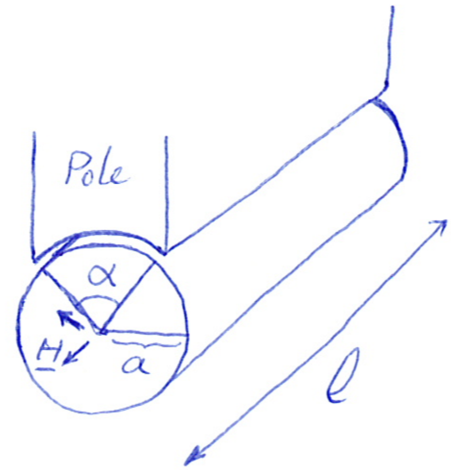
$$\underline{H} = \frac{10^6}{\rho} \sin(2\varphi)\, \underline{a}_\rho \quad A/m$$

Flux per pole = ?

$\alpha = 50^\circ$

$a = 10$ cm

$l = 20$ cm



$$\Psi = \iint \underline{B} \cdot d\underline{s}$$

$$= \mu_0 \iint \underline{H} \cdot d\underline{s} \qquad d\underline{s} = \rho\, d\varphi\, dz\, \underline{a}_\rho$$

$$= \mu_0 \int_{\varphi=0}^{50^\circ} \int_{z=0}^{0.2} \frac{10^6}{\rho} \sin(2\varphi)\, \rho\, d\varphi\, dz$$

$$= 4\pi \times 10^{-7} \times 10^{+6} \times \left(\frac{-\cos(2\varphi)}{2}\right)\Bigg|_{\varphi=0}^{50^\circ} \times z \Bigg|_{z=0}^{0.2}$$

$$= \frac{4\pi}{10} \times \frac{1}{2}\left(-\cos(100^\circ) - (-1)\right) \times 0.2$$

$$= \frac{4\pi}{100}\left(1 - \cos(100^\circ)\right) \text{ Wb}$$

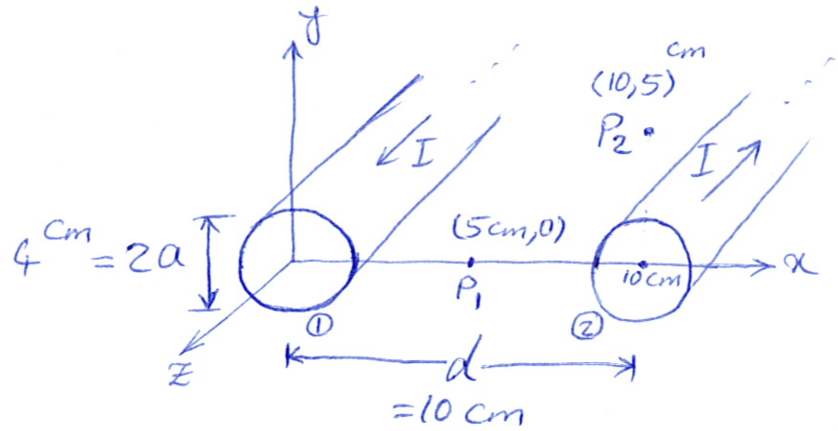
$$= 0.1475 \text{ Wb}$$

7.28

$a = 2\text{ cm}$

$d = 10\text{ cm}$

$I = 5\text{ A}$



a) $\underline{H}$ at $P_1 = ?$

$$\underline{H} = \underline{H}_1 + \underline{H}_2$$

wire ① $\Rightarrow \underline{H}_1$

wire ② $\Rightarrow \underline{H}_2$

$$\underline{H}_1 = \frac{I}{2\pi\rho}\,\underline{a}_\ell \times \underline{a}_\rho$$

$$= \frac{5}{2\pi \times 5 \times 10^{-2}}(\underline{a}_z \times \underline{a}_x) = \frac{50}{\pi}\underline{a}_y \text{ A/m}$$

$$\underline{H}_2 = \frac{I}{2\pi\rho}\,\underline{a}_\ell \times \underline{a}_\rho$$

$$= \frac{5}{2\pi \times 5 \times 10^{-2}}\left[(-\underline{a}_z) \times (-\underline{a}_x)\right] = \frac{50}{\pi}\underline{a}_y \text{ A/m}$$

$$\underline{H} = \underline{H}_1 + \underline{H}_2 = \frac{100}{\pi}\underline{a}_y \text{ A/m} = 31.831\underline{a}_y \text{ A/m}$$

b) $\underline{H}$ at $P_2 = ?$

$$\underline{H}_1 = \frac{I}{2\pi\rho}\,\underline{a}_\ell \times \underline{a}_\rho$$

$$= \frac{5}{2\pi\sqrt{100+25} \times 10^{-2}}\left[\underline{a}_z \times \left(\frac{10\underline{a}_x + 5\underline{a}_y}{\sqrt{125}}\right)\right]$$

$$= \frac{5}{2\pi \times 125 \times 10^{-2}}\left(10\underline{a}_y - 5\underline{a}_x\right) = \frac{10}{\pi}\left(-\underline{a}_x + 2\underline{a}_y\right) \text{ A/m}$$

$$\underline{H}_2 = \frac{I}{2\pi\rho}\,\underline{a}_\ell \times \underline{a}_\rho$$

7.28

b)

$$\underline{H}_2 = \frac{5}{2\pi \times 5 \times 10^{-2}} \left[(-\underline{a}_z) \times (\underline{a}_y) \right] = \frac{50}{\pi} \underline{a}_x \quad A/m$$

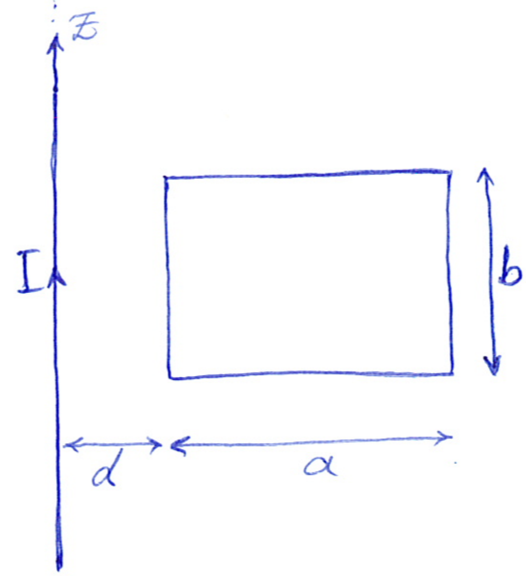
$$\underline{H} = \underline{H}_1 + \underline{H}_2 = \left(\frac{-10}{\pi} \underline{a}_x + \frac{20}{\pi} \underline{a}_y \right) + \left(\frac{50}{\pi} \underline{a}_x \right)$$

$$= \frac{40}{\pi} \underline{a}_x + \frac{20}{\pi} \underline{a}_y$$

$$= 12.7324 \, \underline{a}_x + 6.3662 \, \underline{a}_y \quad A/m$$

7.30

$\Psi = ?$



$$\underline{H} = \frac{I}{2\pi\rho}\,\underline{a}_\varphi$$

$$\underline{B} = \mu_0 \underline{H} = \frac{\mu_0 I}{2\pi\rho}\,\underline{a}_\varphi$$

$$\Psi = \iint_S \underline{B} \cdot d\underline{s} = \int_{\rho=d}^{d+a} \int_{z=0}^{b} \left(\frac{\mu_0 I}{2\pi\rho}\,\underline{a}_\varphi\right) \cdot \left(d\rho\, dz\, \underline{a}_\varphi\right)$$

$$= \int_{\rho=d}^{d+a} \int_{z=0}^{b} \frac{\mu_0 I}{2\pi\rho}\, d\rho\, dz$$

$$= \frac{\mu_0 I}{2\pi} \times \ln\rho \Big|_{\rho=d}^{d+a} \times z \Big|_{z=0}^{b}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{d}\right) \quad Wb$$

7.33 $\mu = \mu_0 \Leftarrow$ free space

$$\underline{B} = y^2 \underline{a}_x + z^2 \underline{a}_y + x^2 \underline{a}_z \ \ Wb/m^2$$

a) $\underline{B}$ is a magnetic field = ?

$\underline{B}$ should satisfy Maxwell's equation.

$$\Rightarrow \nabla . \underline{B} = 0$$

$$\nabla . \underline{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(z^2) + \frac{\partial}{\partial z}(x^2) = 0$$

b) $\begin{cases} x = 1 \\ 0 < y < 1 \\ 1 < z < 4 \end{cases} \Rightarrow \Psi = ?$

$$\Psi = \iint_S \underline{B} . d\underline{s} = \int_{y=0}^{1} \int_{z=1}^{4} \underline{B} . (dy dz \, \underline{a}_x)$$

$$= \int_{y=0}^{1} \int_{z=1}^{4} y^2 dy dz = \frac{1}{3} y^3 \Big|_{y=0}^{1} \times z \Big|_{z=1}^{4}$$

$$= \frac{1}{3} \times 3 = 1 \ Wb$$

c) $\nabla \times \underline{H} = \underline{J} \quad \Rightarrow \nabla \times \left(\frac{\underline{B}}{\mu_0}\right) = \underline{J}$

$\underline{J} = ?$

$$\Rightarrow \underline{J} = \frac{1}{\mu_0}(\nabla \times \underline{B})$$

7.33

c)

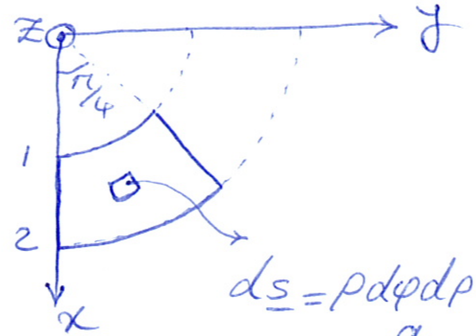
$$\nabla \times \underline{B} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & z^2 & x^2 \end{vmatrix} = (0-2z)\underline{a}_x - (2x-0)\underline{a}_y + (0-2y)\underline{a}_z$$

$$= -2z\underline{a}_x - 2x\underline{a}_y - 2y\underline{a}_z$$

$$\Rightarrow \underline{J} = \frac{-2}{\mu_0}(z\underline{a}_x + x\underline{a}_y + y\underline{a}_z) \quad A/m^2$$

7.36 $\quad \mu = \mu_0$ (free space)

$$\underline{B} = \frac{5}{\rho} \cos\varphi \, \underline{a}_z \quad Wb/m^2$$



$ds = \rho \, d\varphi \, d\rho \, \underline{a}_z$

$$\begin{cases} z = 0 \\ 1 < \rho < 2 \\ 0 < \varphi < \frac{\pi}{4} \end{cases} \Rightarrow \Psi = ?$$

$$\Psi = \iint_S \underline{B} \cdot d\underline{s} = \iint_S \left(\frac{5}{\rho} \cos\varphi \, \underline{a}_z \right) \cdot \left(\rho \, d\varphi \, d\rho \, \underline{a}_z \right)$$

$$= \int_{\rho=1}^{2} \int_{\varphi=0}^{\pi/4} \frac{5}{\cancel{\rho}} \cos\varphi \, \cancel{\rho} \, d\rho \, d\varphi$$

$$= 5 \times \sin\varphi \Big|_{\varphi=0}^{\pi/4} \times \rho \Big|_{\rho=1}^{2} = 5 \times \sin\frac{\pi}{4} \times (2-1)$$

$$= \frac{5\sqrt{2}}{2} \, Wb = 3.5355 \, Wb$$

7.41 $\quad \mu = \mu_0$ (free space)

$$\underline{A} = (2x^2y + yz)\underline{a}_x + (xy^2 - xz^3)\underline{a}_y - (6xyz - 2x^2y^2)\underline{a}_z \quad \text{Wb/m}$$

a) $\underline{B} = ?$

$$\underline{B} = \nabla \times \underline{A} = \begin{vmatrix} \underline{a}_x & \underline{a}_y & \underline{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x^2y + yz) & (xy^2 - xz^3) & (-6xyz + 2x^2y^2) \end{vmatrix}$$

$$= \left[(-6xz + 4x^2y) - (0 - 3xz^2)\right]\underline{a}_x - \left[(-6yz + 4xy^2) - (0 + y)\right]\underline{a}_y + \left[(y^2 - z^3) - (2x^2 + z)\right]\underline{a}_z$$

$$= (-6xz + 4x^2y + 3xz^2)\underline{a}_x + (6yz - 4xy^2 + y)\underline{a}_y + (y^2 - z^3 - 2x^2 - z)\underline{a}_z \quad \text{Wb/m}^2$$

b) $\begin{cases} x = 1 \\ 0 < y < 2 \\ 0 < z < 2 \end{cases} \Rightarrow \Psi = ?$

$$\Psi = \iint_S \underline{B} \cdot d\underline{s} \qquad d\underline{s} = dy\,dz\,\underline{a}_x$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} B_x\,dy\,dz = \int_{y=0}^{2}\int_{z=0}^{2} (-6xz + 4x^2y + 3xz^2)\,dy\,dz \Big|_{x=1}$$

$$= \int_{y=0}^{2}\int_{z=0}^{2} (-6z + 4y + 3z^2)\,dy\,dz$$

$$= \int_{y=0}^{2} (-3z^2 + 4yz + z^3)\Big|_{z=0}^{2}\,dy$$

7.41

b) $$\Psi = \int_{y=0}^{2} (-12 + 8y + 8)\,dy = \int_{y=0}^{2} (8y - 4)\,dy$$

$$= (4y^2 - 4y)\Big|_{y=0}^{2} = 4\times 2^2 - 4\times 2 = 8 \text{ Wb}$$

$$\Rightarrow \Psi = 8 \text{ Wb}$$

c) $$\left.\begin{array}{l} \nabla\cdot \underline{A} = 0 \\ \nabla\cdot \underline{B} = 0 \end{array}\right\}?$$

$$\nabla\cdot \underline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial}{\partial x}(2x^2y + yz) + \frac{\partial}{\partial y}(xy^2 - xz^3) + \frac{\partial}{\partial z}(-6xyz + 2x^2y^2)$$

$$= (4xy + 0) + (2xy - 0) + (-6xy + 0) = 0$$

$$\Rightarrow \nabla\cdot \underline{A} = 0$$

$$\nabla\cdot \underline{B} = \frac{\partial}{\partial x}(-6xz + 4x^2y + 3xz^2) + \frac{\partial}{\partial y}(6yz - 4xy^2 + y) + \frac{\partial}{\partial z}(y^2 - z^3 - 2x^2 - z)$$

$$= (-6z + 8xy + 3z^2) + (6z - 8xy + 1) + (0 - 3z^2 - 0 - 1)$$

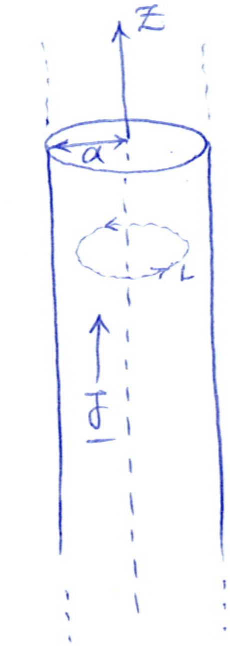
$$= 0$$

$$\Rightarrow \nabla\cdot \underline{B} = 0$$

7.43

uniform $\underline{J} = J_0 \underline{a}_z$

$\rho < a \Rightarrow \underline{A} \overset{?}{=} -\frac{1}{4}\mu_0 J_0 \rho^2 \underline{a}_z$



By applying Ampere's law:

$$\oint_L \underline{H} \cdot d\underline{l} = H_\varphi \int_L dl = H_\varphi \times 2\pi\rho = I_{enc}$$

$$= \iint_S \underline{J} \cdot d\underline{s} = \iint_S (J_0 \underline{a}_z) \cdot (\rho\, d\rho\, d\varphi\, \underline{a}_z)$$

$$= J_0 \times \pi \rho^2$$

$$\Rightarrow \underline{H} = H_\varphi \underline{a}_\varphi = \frac{J_0 \rho}{2} \underline{a}_\varphi$$

Also, we know $\underline{A}$ is in the same direction of $\underline{J}$, i.e., $\underline{A} = A_z \underline{a}_z$

$$\underline{A} = A_z \underline{a}_z$$

$$\nabla \times \underline{A} = \underline{B} = \mu_0 \underline{H}$$

$$\nabla \times \underline{A} = \frac{1}{\rho} \begin{vmatrix} \underline{a}_\rho & \rho \underline{a}_\varphi & \underline{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ 0 & \rho \times 0 & A_z \end{vmatrix} = \frac{1}{\rho}\left[\left(\frac{\partial A_z}{\partial \varphi} - 0\right)\underline{a}_\rho - \left(\frac{\partial A_z}{\partial \rho} - 0\right)\rho \underline{a}_\varphi + 0\right]$$

$$= \frac{1}{\rho}\frac{\partial A_z}{\partial \varphi}\underline{a}_\rho - \frac{\partial A_z}{\partial \rho}\underline{a}_\varphi$$

Due to symmetry $\Rightarrow$ A_z is NOT a function of φ

$$\Rightarrow \frac{\partial A_z}{\partial \varphi} = 0$$

$$\Rightarrow \nabla \times \underline{A} = -\frac{\partial A_z}{\partial \rho}\underline{a}_\varphi$$

7.43

$$\underline{B} = \mu_0 \underline{H} = \nabla \times \underline{A}$$

$$\Rightarrow \frac{\mu_0 J_0 \rho}{2} \underline{a}_\varphi = -\frac{\partial A_z}{\partial \rho} \underline{a}_\varphi$$

$$\Rightarrow \frac{\partial A_z}{\partial \rho} = -\frac{\mu_0 J_0 \rho}{2}$$

$$\Rightarrow A_z = \int -\frac{\mu_0 J_0 \rho}{2} d\rho = -\frac{\mu_0 J_0 \rho^2}{4}$$

$$\Rightarrow \underline{A} = -\frac{1}{4} \mu_0 J_0 \rho^2 \underline{a}_z$$