(b) Both $\mathbf{F}_e$ and $\mathbf{F}_m$ depend on the velocity of the charged particle.
(c) Both $\mathbf{F}_e$ and $\mathbf{F}_m$ can perform work.
(d) Both $\mathbf{F}_e$ and $\mathbf{F}_m$ are produced when a charged particle moves at a constant velocity.
(e) $\mathbf{F}_m$ is generally small in magnitude in comparison to $\mathbf{F}_e$.
(f) $\mathbf{F}_e$ is an accelerating force, whereas $\mathbf{F}_m$ is a purely deflecting force.

8.2 Two thin parallel wires carry currents along the same direction. The force experienced by one due to the other is
(a) Parallel to the lines
(b) Perpendicular to the lines and attractive
(c) Perpendicular to the lines and repulsive
(d) Zero

8.3 The force on differential length $d\mathbf{l}$ at point P in the conducting circular loop in Figure 8.32, is
(a) Outward along OP
(b) Inward along OP
(c) In the direction of the magnetic field
(d) Tangential to the loop at P

8.4 The resultant force on the circular loop in Figure 8.32 has the magnitude of
(a) $2\pi\rho_o IB$
(b) $\pi\rho_o^2 IB$
(c) $2\rho_o IB$
(d) Zero

8.5 What is the unit of magnetic charge?
(a) Ampere-meter squared
(b) Coulomb
(c) Ampere
(d) Ampere-meter

8.6 Which of these materials requires the least value of magnetic field strength to magnetize it?
(a) Nickel
(b) Silver

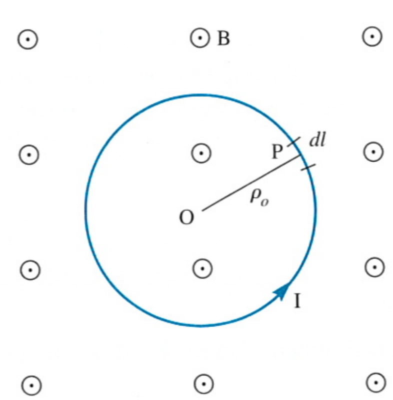


FIGURE 8.32 For Review Questions 8.3 and 8.4.

(c) Tungsten
(d) Sodium chloride

8.7 Identify the statement that is not true of ferromagnetic materials.
(a) They have a large χ_m.
(b) They have a fixed value of μ_r.
(c) Energy loss is proportional to the area of the hysteresis loop.
(d) They lose their nonlinearity property above the curie temperature.

8.8 Which of these formulas is wrong?
(a) $B_{1n} = B_{2n}$
(b) $B_2 = \sqrt{B_{2n}^2 + B_{2t}^2}$
(c) $H_1 = H_{1n} + H_{1t}$
(d) $\mathbf{a}_{n21} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{K}$, where $\mathbf{a}_{n21}$ is a unit vector normal to the interface and directed from region 2 to region 1.

8.9 Each of the following pairs consists of an electric circuit term and the corresponding magnetic circuit term. Which pairs are not corresponding?
(a) V and $\mathcal{F}$
(b) G and $\mathcal{P}$
(c) ε and μ
(d) IR and $H\mathcal{R}$
(e) $\sum I = 0$ and $\sum \Psi = 0$

8.10 A multilayer coil of 2000 turns of fine wire is 20 mm long and has a thickness 5 mm of winding. If the coil carries a current of 5 mA, the mmf generated is
(a) 10 A · t
(b) 500 A · t
(c) 2000 A · t
(d) None of the above

Answers: 8.1 b,c, 8.2b, 8.3a, 8.4d, 8.5, 8.6a, 8.7b, 8.8c, 8.9c,d, 8.10a.

PROBLEMS

Section 8.2—Forces Due to Magnetic Fields

8.1 An electron with velocity $\mathbf{u} = (3\mathbf{a}_x + 12\mathbf{a}_y - 4\mathbf{a}_z) \times 10^5$ m/s experiences no net force at a point in a magnetic field $\mathbf{B} = 10\mathbf{a}_x + 20\mathbf{a}_y + 30\mathbf{a}_z$ mWb/m². Find $\mathbf{E}$ at that point.

8.2 An electron ($m = 9.11 \times 10^{-31}$ kg) moves in a circular orbit of radius 0.4×10^{-10} m with an angular velocity of 2×10^{16} rad/s. Find the centripetal force required to hold the electron.

8.3 A charged particle ($m = 1.673 \times 10^{-27}$ kg) exists in a region with $\mathbf{E} = 10\mathbf{a}_x$ kV/m and $\mathbf{B} = \mathbf{a}_y$ Wb/m^2. If the particle moves without being deflected, calculate its kinetic energy.

***8.4** A particle with a mass of 1 kg and a charge of 2 C starts from rest at point $(2, 3, -4)$ in a region where $\mathbf{E} = -4\mathbf{a}_y$ V/m and $\mathbf{B} = 5\mathbf{a}_x$ Wb/m^2. Calculate

(a) The location of the particle at $t = 1$ s

(b) Its velocity and K.E. at that location

8.5 A -2 mC charge starts at point $(0, 1, 2)$ with a velocity of $5\mathbf{a}_x$ m/s in a magnetic field $\mathbf{B} = 6\mathbf{a}_y$ Wb/m^2. Determine the position and velocity of the particle after 10 s, assuming that the mass of the charge is 1 gram. Describe the motion of the charge.

***8.6** By injecting an electron beam normally to the plane edge of a uniform field $B_o\mathbf{a}_z$, electrons can be dispersed according to their velocity as in Figure 8.33.

(a) Show that the electrons would be ejected out of the field in paths parallel to the input beam as shown.

(b) Derive an expression for the exit distance d above entry point.

8.7 Two large conducting plates are 8 cm apart and have a potential difference 12kV. If a drop of oil with mass 0.4 g is suspended in space between the plates, find the charge on the drop.

8.8 Given that $\mathbf{B} = 4\mathbf{a}_x - 8\mathbf{a}_z$ Wb/m, find the force it exerts on a 0.2-m conductor on the y-axis with a current 2 A in the $-\mathbf{a}_y$ direction.

***8.9** Three infinite lines L_1, L_2, and L_3 defined by $x = 0, y = 0$; $x = 0, y = 4$; $x = 3, y = 4$, respectively, carry filamentary currents -100 A, 200 A, and 300 A along $\mathbf{a}_z$. Find the force per unit length on

(a) L_2 due to L_1

(b) L_1 due to L_2

(c) L_3 due to L_1

(d) L_3 due to L_1 and L_2. State whether each force is repulsive or attractive.

8.10 Two infinitely long parallel wires are separated by a distance of 20 cm. If the wires carry current of 10 A in opposite directions, calculate the force on the wires.

8.11 A conductor 2 m long carrying a current of 3 A is placed parallel to the z-axis at distance $\rho_o = 10$ cm as shown in Figure 8.34. If the field in the region is $\cos(\phi/3)\,\mathbf{a}_\rho$ Wb/m^2, how much work is required to rotate the conductor one revolution about the z-axis?

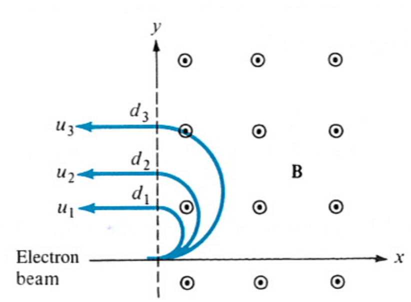


FIGURE 8.33 For Problem 8.5.

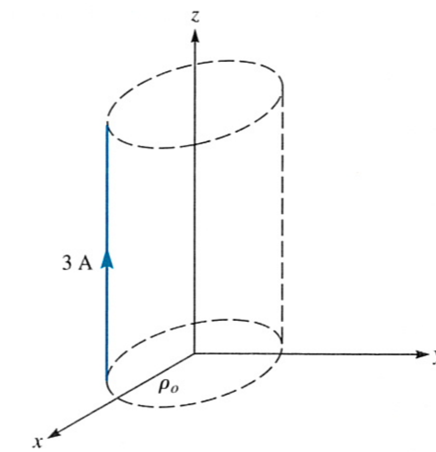


FIGURE 8.34 For Problem 8.11.

***8.12** A conducting triangular loop carrying a current of 2 A is located close to an infinitely long, straight conductor with a current of 5 A, as shown in Figure 8.35. Calculate (a) the force on side 1 of the triangular loop and (b) the total force on the loop.

***8.13** A three-phase transmission line consists of three conductors that are supported at points A, B, and C to form an equilateral triangle as shown in Figure 8.36. At one instant, conductors A and B both carry a current of 75 A while conductor C carries a return current of 150 A. Find the force per meter on conductor C at that instant.

8.14 A current sheet with $\mathbf{K} = 10\mathbf{a}_x$ A/m lies in free space in the $z = 2$ m plane. A filamentary conductor on the x-axis carries a current of 2.5 A in the $\mathbf{a}_x$-direction. Determine the force per unit length on the conductor.

8.15 The magnetic field in a certain region is $\mathbf{B} = 40\,\mathbf{a}_x$ mWb/m^2. A conductor that is 2 m in length lies in the z-axis and carries a current of 5 A in the $\mathbf{a}_z$-direction. Calculate the force on the conductor.

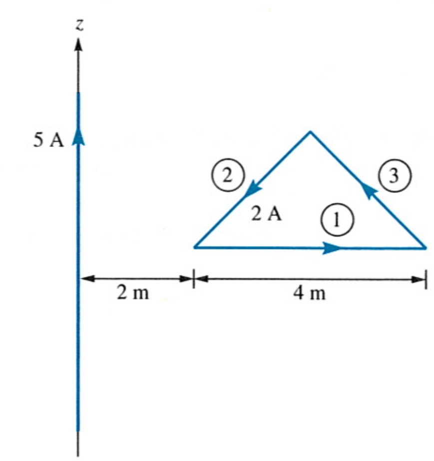


FIGURE 8.35 For Problem 8.12.

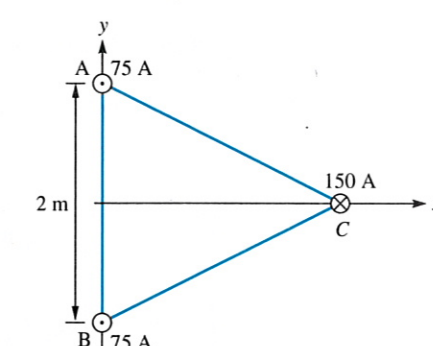


FIGURE 8.36 For Problem 8.13.

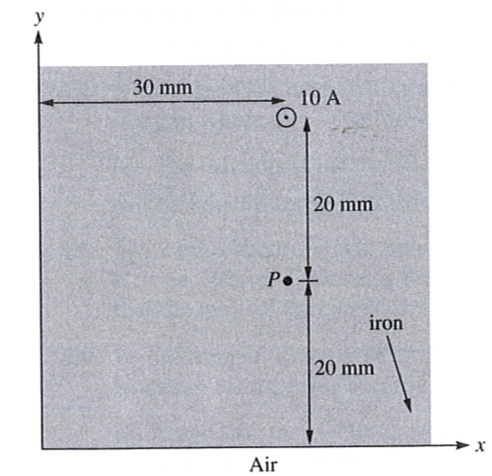


FIGURE 8.37 For Problem 8.16.

Sections 8.3 and 8.4—Magnetic Torque, Moments, and Dipole

***8.16** An infinitely long conductor is buried in but insulated from an iron mass ($\mu = 2000\mu_o$) as shown in Figure 8.37. Using image theory, estimate the magnetic flux density at point P.

8.17 A 60-turn coil carries a current of 2 A and lies in the plane $x + 2y - 5z = 12$ such that the magnetic moment **m** of the coil is directed away from the origin. Calculate **m**, assuming that the area of the coil is 8 cm^2.

8.18 A small magnet placed at the origin produces $\mathbf{B} = -0.5\mathbf{a}_z$ mWb/m^2 at $(10, 0, 0)$. Find **B** at

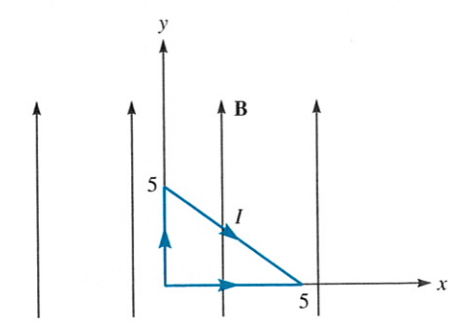
(a) $(0, 3, 0)$
(b) $(3, 4, 0)$
(c) $(1, 1, -1)$

8.19 A triangular loop is placed in the x-z plane, as shown in Figure 8.38. Assume that a dc current $I = 2$ A flows in the loop and that $\mathbf{B} = 30\mathbf{a}_z$ m Wb/m exists in the region. Find the forces and torque on the loop.

Section 8.5—Magnetization in Materials

8.20 A block of iron ($\mu = 5000\mu_o$) is placed in a uniform magnetic field with 1.5 Wb/m^2. If iron consists of 8.5×10^{28} atoms/m^3, calculate (a) the magnetization **M**, (b) the average dipole moment.

FIGURE 8.38 For Problem 8.19.



8.21 In a certain material, $\mu = 4.6\mu_o$ and $\mathbf{B} = 5x\mathbf{a}_z$ Wb/m^2. Determine: (a) χ_m, (b) **H**, (c) **M**.

8.22 In a ferromagnetic material ($\mu = 4.5\mu_o$),

$$\mathbf{B} = 4y\mathbf{a}_z \text{ mWb/m}^2$$

calculate: (a) χ_m, (b) **H**, (c) **M**, (d) $\mathbf{J}_b$.

8.23 The magnetic field intensity is $H = 1200$ A/m in a material when $B = 2$ Wb/m^2. When H is reduced to 400 A/m, $B = 1.4$ Wb/m^2. Calculate the change in the magnetization M.

8.24 An infinitely long cylindrical conductor of radius a and permeability $\mu_o\mu_r$ is placed along the z-axis. If the conductor carries a uniformly distributed current I along $\mathbf{a}_z$, find **M** and $\mathbf{J}_b$ for $0 < \rho < a$.

Section 8.7—Magnetic Boundary Conditions

***8.25** (a) For the boundary between two magnetic media such as is shown in Figure 8.16, show that the boundary conditions on the magnetization vector are

$$\frac{M_{1t}}{\chi_{m1}} - \frac{M_{2t}}{\chi_{m2}} = K \quad \text{and} \quad \frac{\mu_1}{\chi_{m1}} M_{1n} = \frac{\mu_2}{\chi_{m2}} M_{2n}$$

(b) If the boundary is not current free, show that instead of eq. (8.49), we obtain

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}\left[1 + \frac{K\mu_2}{B_2 \sin\theta_2}\right]$$

8.26 Region 1 ($z < 0$) is free space, while region 2 ($z < 0$) is a magnetic material for which $\mu_2 = 20\mu_o$. If $\mathbf{B}_1 = 20\mathbf{a}_x - 15\mathbf{a}_y + 30\mathbf{a}_z$ mWb/m^2, find $\mathbf{H}_2$.

8.27 Suppose space is divided into region 1 ($y < 0$, $\mu_1 = \mu_o\mu_{r1}$) and region 2 ($y < 0$, $\mu_2 = \mu_o\mu_{r2}$). If $\mathbf{H}_1 = \alpha\mathbf{a}_x + \beta\mathbf{a}_y + \delta\mathbf{a}_z$ A/m, find $\mathbf{H}_2$.

8.28 If $\mu_1 = 2\mu_o$ for region 1 ($0 < \phi < \pi$) and $\mu_2 = 5\mu_o$ for region 2 ($\pi < \phi < 2\pi$) and $\mathbf{B}_2 = 10\mathbf{a}_\rho + 15\mathbf{a}_\phi - 20\mathbf{a}_z$ mWb/m^2. Calculate (a) $\mathbf{B}_1$, (b) the energy densities in the two media.

8.29 The interface $2x + y = 8$ between two media carries no current. Medium 1 ($2x + y \geq 8$) is nonmagnetic with $\mathbf{H}_1 = -4\mathbf{a}_x + 3\mathbf{a}_y - \mathbf{a}_z$ A/m. Find (a) the magnetic energy density in medium 1, (b) $\mathbf{M}_2$ and $\mathbf{B}_2$ in medium 2 ($2x + y \leq 8$) with $\mu = 10\mu_o$, (c) the angles $\mathbf{H}_1$ and $\mathbf{H}_2$ make with the normal to the interface.

8.30 Inside a right circular cylinder, $\mu_1 = 800\,\mu_o$, while the exterior is free space. Given that $\mathbf{B}_1 = \mu_o(22\mathbf{a}_\rho + 45\mathbf{a}_\phi)$ Wb/m^2, determine $\mathbf{B}_2$ just outside the cylinder.

8.31 The plane $z = 0$ separates air ($z \geq 0$, $\mu = \mu_o$) from iron ($z \leq 0$, $\mu = 200\mu_o$). Given that

$$\mathbf{H} = 10\mathbf{a}_x + 15\mathbf{a}_y - 3\mathbf{a}_z \text{ A/m}$$

in air, find **B** in iron and the angle it makes with the interface.

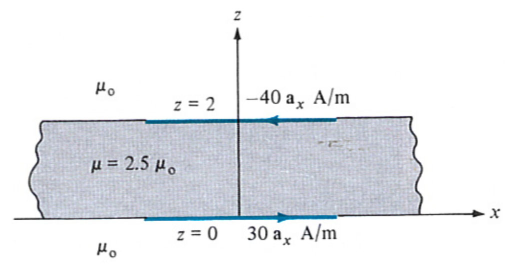


FIGURE 8.39 For Problem 8.32.

8.32 Region $0 \le z \le 2$ m is filled with an infinite slab of magnetic material ($\mu = 2.5\mu_o$). If the surfaces of the slab at $z = 0$ and $z = 2$, respectively, carry surface currents $30\mathbf{a}_x$ A/m and $-40\mathbf{a}_x$ A/m as in Figure 8.39, calculate **H** and **B** for

(a) $z < 0$

(b) $0 < z < 2$

(c) $z > 2$

Section 8.8—Inductors and Inductance

***8.33** (a) If the cross section of the toroid of Figure 7.15 is a square of side a, show that the self-inductance of the toroid is

$$L = \frac{\mu_o N^2 a}{2\pi} \ln\left[\frac{2\rho_o + a}{2\rho_o - a}\right]$$

(b) If the toroid has a circular cross section as in Figure 7.15, show that

$$L = \frac{\mu_o N^2 a^2}{2\rho_o}$$

where $\rho_o \gg a$.

8.34 A solenoid with length 10 cm and radius 1 cm has 450 turns. Calculate its inductance.

8.35 A square cross-section, air filled toroid has inner radius 3 cm, outer radius 5 cm, and height 2 cm. How many turns are required to produce an inductance of 45 μH?

8.36 A coaxial cable has an outer radius b and inner radius a. If the internal and external inductances are equal and $a = 8$ mm, find b.

8.37 Two parallel cylindrical conductors are separated by 1.2 m. If the conductors have an inductance per unit length of 1.37 μH/m, determine the conductor radius.

8.38 A coaxial cable has inner conductor of radius $a = 2.5$ mm and outer conductor of radius $b = 6$ mm. Assuming that the space between the conductors is filled with a nonmagnetic material, calculate the inductance per unit length.

8.39 Show that the mutual inductance between the rectangular loop and the infinite line current of Figure 8.4 is

$$M_{12} = \frac{\mu b}{2\pi} \ln\left[\frac{a + \rho_o}{\rho_o}\right]$$

Calculate M_{12} when $a = b = \rho_o = 1$ m.

***8.40** Prove that the mutual inductance between the closed wound coaxial solenoids of length ℓ_1 and ℓ_2 ($\ell_1 \gg \ell_2$), turns N_1 and N_2, and radii r_1 and r_2 with $r_1 \simeq r_2$ is

$$M_{12} = \frac{\mu N_1 N_2}{\ell_1} \pi r_1^2$$

Section 8.9—Magnetic Energy

8.41 A coaxial cable consists of an inner conductor of radius 1.2 cm and an outer conductor of radius 1.8 cm. The two conductors are separated by an insulating medium ($\mu = 4\mu_o$). If the cable is 3 m long and carries 25 mA current, calculate the energy stored in the medium.

8.42 In a certain region for which $\chi_m = 19$,

$$\mathbf{H} = 5x^2yz\mathbf{a}_x + 10xy^2z\mathbf{a}_y - 15xyz^2\mathbf{a}_z \text{ A/m}$$

How much energy is stored in $0 < x < 1, 0 < y < 2, -1 < z < 2$?

8.43 In a certain medium with $\mu = 4.5\,\mu_o$, $\mathbf{H} = 200\mathbf{a}_x + 500\mathbf{a}_y$ mA/m. Calculate the total energy stored in a $2 \times 2 \times 2$ cm cublical region centered at the origin.

Section 8.10—Magnetic Circuits

8.44 A cobalt ring ($\mu_r = 600$) has a mean radius of 30 cm. If a coil wound on the ring carries 12 A, calculate the number of turns required to establish an average magnetic flux density of 1.5 Wb/m² in the ring.

8.45 Refer to Figure 8.27. If the current in the coil is 0.5 A, find the mmf and the magnetic field intensity in the air gap. Assume that $\mu = 500\mu_o$ and that all branches have the same cross-sectional area of 10 cm².

8.46 The magnetic circuit of Figure 8.40 has a current of 10 A in the coil of 2000 turns. Assume that all branches have the same cross section of 2 cm² and that the material of the core is iron with $\mu_r = 1500$. Calculate R, $\mathcal{F}$, and Ψ for

(a) The core

(b) The air gap

8.47 Consider the magnetic circuit in Figure 8.41. Assuming that the core ($\mu = 1000\mu_o$) has a uniform cross section of 4 cm², determine the flux density in the air gap.

8.48 A ferromagnetic core with cross-section 40×60 mm experiences a flux $\Psi = 2.56$ mWb and an air gap that is 2.5 mm long. Determine the NI drop across the gap.

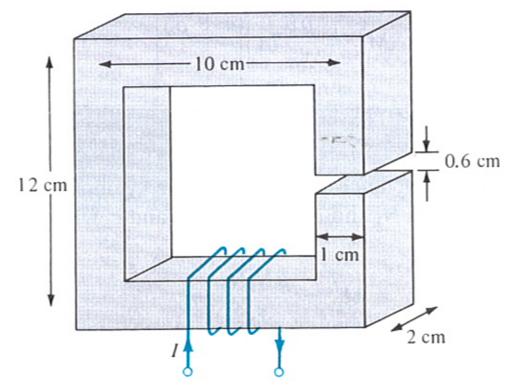


FIGURE 8.40 For Problem 8.44.

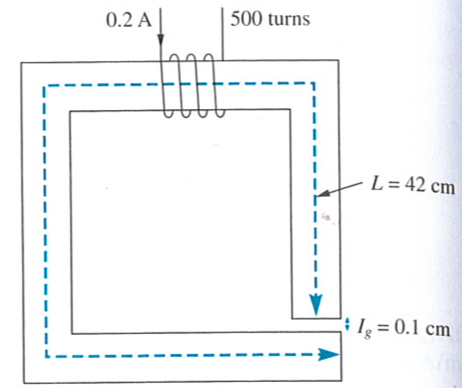


FIGURE 8.41 For Problem 8.45.

8.49 A ring of silicon steel is 1.5 cm wide and has a rectangular cross section with inner radius of 5 cm and outer radius of 6 cm. If a 500-turn coil produces a flux of 12 m Wb in the ring when the coil current is 2 mA, find **B** and μ.

Section 8.11—Force on Magnetic Materials

8.50 An electromagnetic relay is modeled as shown in Figure 8.42. What force is on the armature (moving part) of the relay if the flux in the air gap is 2 mWb? The area of the gap is 0.3 cm^2, and its length 1.5 mm.

8.51 A toroid with air gap, shown in Figure 8.43, has a square cross section. A long conductor carrying current I_2 is inserted in the air gap. If $I_1 = 200$ mA, $N = 750$, $\rho_o = 10$ cm, $a = 5$ mm, and $\ell_a = 1$ mm, calculate

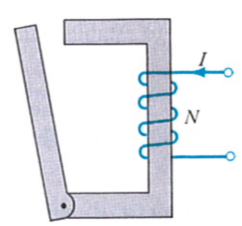


FIGURE 8.42 For Problem 8.50.

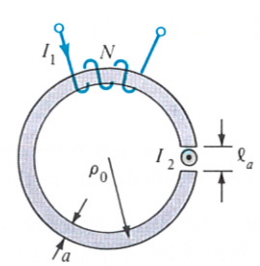


FIGURE 8.43 For Problem 8.51.

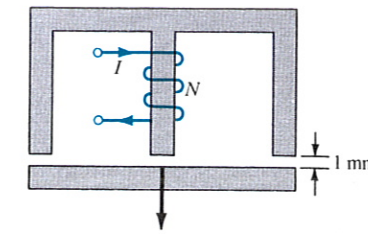


FIGURE 8.44 For Problem 8.52.

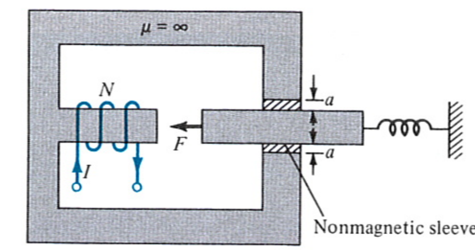


FIGURE 8.45 For Problem 8.53.

(a) The force across the gap when $I_2 = 0$ and the relative permeability of the toroid is 300

(b) The force on the conductor when $I_2 = 2$ mA and the permeability of the toroid is infinite. Neglect fringing in the gap in both cases.

8.52 A section of an electromagnet with a plate below it carrying a load is shown in Figure 8.44. The electromagnet has a contact area of 200 cm^2 per pole, and the middle pole has a winding of 1000 turns with $I = 3$ A. Calculate the maximum mass that can be lifted. Assume that the reluctance of the electromagnet and the plate is negligible.

8.53 Figure 8.45 shows the cross section of an electromechanical system in which the plunger moves freely between two nonmagnetic sleeves. Assuming that all legs have the same cross-sectional area S, show that

$$\mathbf{F} = -\frac{2N^2I^2\mu_o S}{(a+2x)^2}\mathbf{a}_x$$