9.9 Which of the following statements is not true of a phasor?

(a) It may be a scalar or a vector.
(b) It is a time-dependent quantity.
(c) A phasor V_s may be represented as $V_o \angle\theta$ or $V_o e^{j\omega}$ where $V_o = |V_s|$.
(d) It is a complex quantity.

9.10 If $\mathbf{E}_s = 10\, e^{j4x}\, \mathbf{a}_y$, which of these is not a correct representation of **E**?

(a) $\mathrm{Re}(\mathbf{E}_s e^{j\omega t})$
(b) $\mathrm{Re}(\mathbf{E}_s e^{-j\omega t})$
(c) $\mathrm{Im}(\mathbf{E}_s e^{j\omega t})$
(d) $10 \cos(\omega t + j4x)\, \mathbf{a}_y$
(e) $10 \sin(\omega t + 4x)\, \mathbf{a}_y$

Answers: 9.1b, 9.2b, d, 9.3a, 9.4c, 9.5a, 9.6c, 9.7a,c, 9.8b, 9.9b, 9.10d.

PROBLEMS

Sections 9.2 and 9.3—Faraday's Law and Electromotive Forces

9.1 A conducting circular loop of radius 20 cm lies in the $z = 0$ plane in a magnetic field $\mathbf{B} = 10 \cos 377t\, \mathbf{a}_z$ mWb/m². Calculate the induced voltage in the loop.

9.2 A rod of length ℓ rotates about the z-axis with an angular velocity ω. If $\mathbf{B} = B_o \mathbf{a}_z$, calculate the voltage induced on the conductor.

9.3 The circuit in Figure 9.16 exists in a magnetic field $\mathbf{B} = 40 \cos(30\pi t - 2y)\mathbf{a}_z$ Wb/m². Assume that the wires connecting the resistors have negligible resistances. Find the current in the circuit.

9.4 A circuit conducting loop lies in the x-y plane as shown in Figure 9.17. The loop has a radius of 0.2 m and resistance $\mathbf{R} = 4\Omega$. If $\mathbf{B} = 40 \sin 10^4\, t\mathbf{a}_z$ mWb/m², find the currrent.

9.5 Two conducting bars slide over two stationary rails, as illustrated in Figure 9.18. If $\mathbf{B} = 0.2\mathbf{a}_z$ Wb/m², determine the induced emf in the loop thus formed.

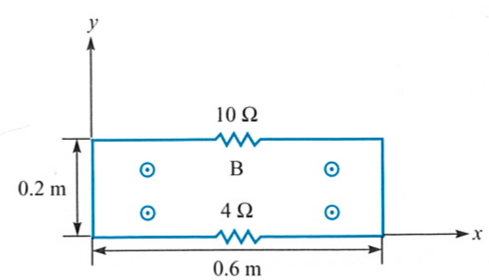

FIGURE 9.16 For Problem 9.3.

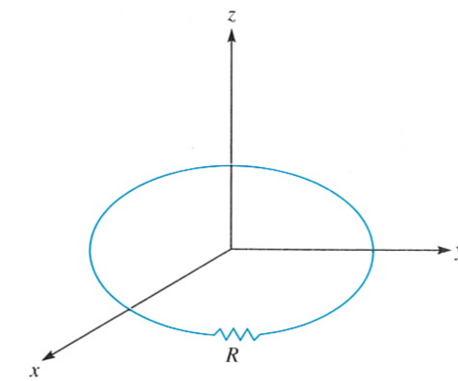

FIGURE 9.17 For Problem 9.4.

9.6 A coil comprises 50 turns of wire wrapped around a square frame with each side measuring 20 cm. If the coil is centered at the origin on plane $z = 0$ such that its sides are parallel to the x- and y-axes and is subject to $\mathbf{B} = 2 \cos y \cos 10^3\, t\, \mathbf{a}_z$ Wb/m², calculate the induced emf on the coil.

9.7 A square loop of side a recedes with a uniform velocity $u_o\mathbf{a}_y$ from an infinitely long filament carrying current I along $\mathbf{a}_z$ as shown in Figure 9.19. Assuming that $\rho = \rho_o$ at time $t = 0$, show that the emf induced in the loop at $t > 0$ is

$$V_{\text{emf}} = \frac{u_o a^2 \mu_o I}{2\pi\rho(\rho + a)}$$

9.8 A conducting rod moves with a constant velocity of 3 $\mathbf{a}_z$ m/s parallel to a long straight wire carrying a current of 15 A as in Figure 9.20. Calculate the emf induced in the rod and state which end is at the higher potential.

9.9 A conducting bar is connected via flexible leads to a pair of rails in a magnetic field $\mathbf{B} = 6 \cos 10t\, \mathbf{a}_x$ mWb/m² as in Figure 9.21. If the z-axis is the equilibrium position of the bar and its velocity is $2 \cos 10t\, \mathbf{a}_y$ m/s, find the voltage induced in it.

9.10 A car travels at 120 km/hr. If the earth's magnetic field is 4.3×10^{-5} Wb/m², find the induced voltage in the car bumper of length 1.6 m. Assume that the angle between the earth's magnetic field and the normal to the car is 65°.

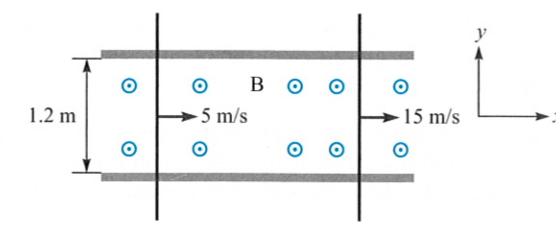

FIGURE 9.18 For Problem 9.5.

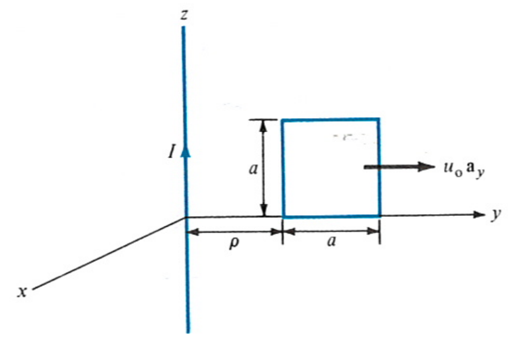

FIGURE 9.19 For Problem 9.7.

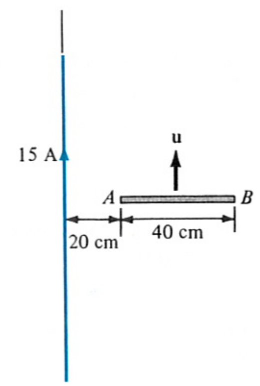

FIGURE 9.20 For Problem 9.8.

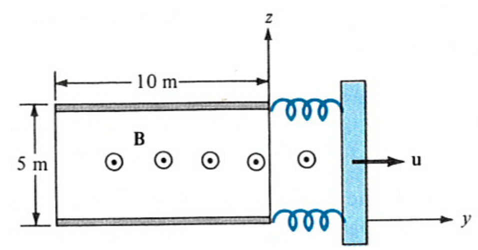

FIGURE 9.21 For Problem 9.9.

9.11 As portrayed in Figure 9.22, a bar magnet is thrust toward the center of a coil of 10 turns and resistance 15 Ω. If the magnetic flux through the coil changes from 0.45 Wb to 0.64 Wb in 0.02 s, find the magnitude and direction (as viewed from the side near the magnet) of the induced current.

9.12 The cross section of a homopolar generator disk is shown in Figure 9.23. The disk has inner radius $\rho_1 = 2$ cm and outer radius $\rho_2 = 10$ cm and rotates in a uniform magnetic field 15 mWb/m^2 at a speed of 60 rad/s. Calculate the induced voltage.

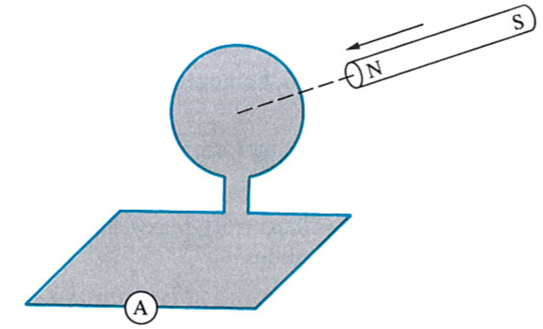

FIGURE 9.22 For Problem 9.11.

Section 9.4—Displacement Current

9.13 A 50 V voltage generator at 20 MHz is connected to the plates of an air dielectric parallel-plate capacitor with a plate area of 2.8 cm^2 and a separation distance of 0.2 mm. Find the maximum value of displacement current density and displacement current.

9.14 The ratio J/J_d (conduction current density to displacement current density) is very important at high frequencies. Calculate the ratio at 1 GHz for:

(a) distilled water ($\mu = \mu_o$, $\varepsilon = 81\varepsilon_o$, $\sigma = 2 \times 10^{-3}$ S/m)

(b) seawater ($\mu = \mu_o$, $\varepsilon = 81\varepsilon_o$, $\sigma = 25$ S/m)

(c) limestone ($\mu = \mu_o$, $\varepsilon = 5\varepsilon_o$, $\sigma = 2 \times 10^{-4}$ S/m)

9.15 Assume that dry soil has $\sigma = 10^{-4}$ S/m, $\varepsilon = 3\varepsilon_o$, and $\mu = \mu_o$. Determine the frequency at which the ratio of the magnitudes of the conduction current density and the displacement current density is unity.

9.16 If $\boldsymbol{D} = D_o \cos(t + \beta z)\boldsymbol{a}_y$ C/m^2 in free space, find $\boldsymbol{B}$.

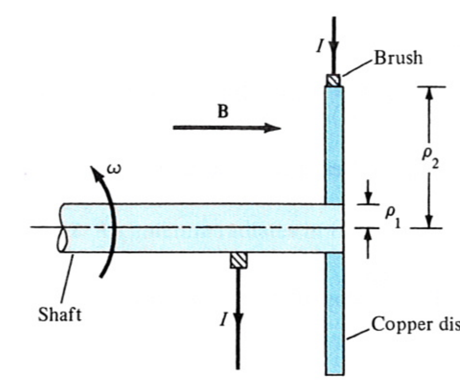

FIGURE 9.23 For Problem 9.12.

Section 9.5—Maxwell's Equations

9.17 (a) Write Maxwell's equations for a linear, homogeneous medium in terms of $\mathbf{E}_s$ and $\mathbf{H}_s$, assuming only the time factor $e^{-j\omega t}$.

(b) In Cartesian coordinates, write the point form of Maxwell's equations in Table 9.2 as eight scalar equations.

9.18 Show that in a source-free region ($\mathbf{J} = 0, \rho_v = 0$), Maxwell's equations can be reduced to two. Identify the two all-embracing equations.

9.19 Express Maxwell's equations completely in terms of **D** and **B**. Assume that the medium is homogeneous, linear, and isotropic.

9.20 Show that fields

$$\mathbf{E} = \mathrm{E}_o \cos x \cos t\mathbf{a}_y \quad \text{and} \quad \mathbf{H} = \frac{E_o}{\mu_o} \sin x \sin t\mathbf{a}_z$$

do not satisfy all of Maxwell's equations.

9.21 In a linear homogeneous and isotropic conductor, show that the charge density ρ_v satisfies

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$$

9.22 Assuming a source-free region, derive the diffusion equation

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t}$$

9.23 In a certain region,

$$\mathbf{J} = (2y\mathbf{a}_x + xz\mathbf{a}_y + z^3\mathbf{a}_z) \sin 10^4 t \text{ A/m}$$

find ρ_v if $\rho_v(x, y, 0, t) = 0$.

9.24 In a material ($\mu = \mu_0$, $\varepsilon = 4\,\varepsilon_0$), $\mathbf{H} = 0.4 \sin(10\pi x/3) \cos(2\pi \times 10^9 t - \beta z)\mathbf{a}_y$ A/m. Find β and **E**.

9.25 If $\mathbf{D} = D_o \cos(\omega t + \beta z)\mathbf{a}_y$ C/m² in free space, find **B**.

9.26 In free space,

$$\mathbf{E} = \frac{50}{\rho} \cos(10^8 t - kz)a_\rho \text{V/m}$$

Find k, $\mathbf{J}_d$, and **H**.

9.27 In a certain region for which $\sigma = 0$, $\mu = 2\mu_o$, and $\varepsilon = 10\varepsilon_o$

$$\mathbf{J} = 60 \sin(10^9 t - \beta_z)\mathbf{a}_x \text{ mA/m}^2$$

(a) Find **D** and **H**.

(b) Determine β.

9.28 In free space,

$$\mathbf{E} = \mathbf{E}_o \sin \alpha x \cos(\omega t - \beta y)\mathbf{a}_z$$

find the corresponding **H** field.

9.29 Check whether the following fields are genuine EM fields (i.e., they satisfy Maxwell's equations). Assume that the fields exist in charge-free regions.

(a) $\mathbf{A} = 40 \sin(\omega t + 10x)\mathbf{a}_z$

(b) $\mathbf{B} = \dfrac{10}{\rho} \cos(\omega t - 2\rho)\mathbf{a}_\phi$

(c) $\mathbf{C} = \left(3\rho^2 \cot \phi\, \mathbf{a}_\rho + \dfrac{\cos \phi}{\rho}\mathbf{a}_\phi\right) \sin \omega t$

(d) $\mathbf{D} = \dfrac{1}{r} \sin \theta \sin(\omega t - 5r)\mathbf{a}_\theta$

9.30 Given the total electromagnetic energy

$$W = \frac{1}{2}\int (\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B})\, dv$$

show from Maxwell's equations that

$$\frac{\partial W}{\partial t} = -\oint_S (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{S} - \int_v \mathbf{E} \cdot \mathbf{J}\, dv$$

9.31 In free space,

$$\mathbf{H} = \rho(\sin \phi\, \mathbf{a}_\rho + 2 \cos \phi\, \mathbf{a}_\phi) \cos(4 \times 10^6 t) \text{ A/m}$$

find $\mathbf{J}_d$ and **E**.

9.32 An antenna radiates in free space and

$$\mathbf{H} = \frac{12 \sin \theta}{r} \cos(2\pi \times 10^8 t - \beta r)\mathbf{a}_\theta \text{ mA/m}$$

Find the corresponding **E** in terms of β.

9.33 The electric field in air is given by $\mathbf{E} = \rho t e^{-\rho - t}\mathbf{a}_\phi$ V/m; find **B** and **J**.

Section 9.6—Time-Varying Potentials

9.34 In free space ($\rho_v = 0, \mathbf{J} = 0$). Show that

$$\mathbf{A} = \frac{\mu_0}{4\pi r}(\cos\theta\, \mathbf{a}_r - \sin\theta\, \mathbf{a}_\theta)e^{j\omega(t-r/c)}$$

satisfies the wave equation in eq. (9.52). Find the corresponding V. Take c as the speed of light in free space.

9.35 Show that another form of Faraday's law is

$$\mathbf{E} = -\frac{\partial A}{\partial t}$$

where **A** is the magnetic vector potential.

9.36 In free space, the retarded potentials are given by

$$V = x(z - ct)\text{V}, \quad \mathbf{A} = x(z/c - t)\mathbf{a}_z \text{ Wb/m}$$

where $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$

(a) Prove that $\nabla \cdot \mathbf{A} = \mu_o \varepsilon_o \frac{\partial V}{\partial t}$.

(b) Determine **E**.

Section 9.7—Time-Harmonic Fields

9.37 Evaluate the following complex numbers and express your answers in polar form:

(a) $(4\angle 30° - 10\angle 50°)^{1/2}$

(b) $\dfrac{1 + j2}{6 + j8 - 7\angle 15°}$

(c) $\dfrac{(3 + j4)^2}{12 - j7 + (-6 + j10)^*}$

(d) $\dfrac{(3.6\angle -200°)^{1/2}}{(2.4\angle 45°)^2(-5 + j8)^*}$

9.38 Express the following time-harmonic fields as phasors.

(a) $\mathbf{A} = 5\sin(2t + \pi/3)\mathbf{a}_x + 3\cos(2t + 30°)\mathbf{a}_y$

(b) $\mathbf{B} = \dfrac{100}{\rho}\sin(\omega t - 2\pi z)\mathbf{a}_\rho$

(c) $\mathbf{C} = \dfrac{\cos\theta}{r}\sin(\omega t - 3r)\mathbf{a}_\theta$

(d) $\mathbf{D} = 10\cos(k_1 x)\cos(\omega t - k_2 z)\mathbf{a}_y$

9.39 Express the following phasors as real-time quantities.

(a) $\mathbf{A}_s = \dfrac{50}{j}\mathbf{a}_x - j10\mathbf{a}_y$

(b) $\mathbf{B}_s = \dfrac{j20}{\rho}e^{-j(\pi/6)z}\,\mathbf{a}_\phi$

(c) $\mathbf{C}_s = \dfrac{30}{r^2}(1 + j)e^{-j3\phi}\cos\theta\mathbf{a}_\phi$

9.40 In a source-free vacuum region,

$$\mathbf{H} = \frac{1}{\rho}\cos(\omega t - 3z)\mathbf{a}_\phi \text{ A/m}$$

(a) Express **H** in phasor form.
(b) Find the associated **E** field.
(c) Determine ω.

9.41 Given $\mathbf{A} = 4\sin\omega t\,\mathbf{a}_x + 3\cos\omega t\,\mathbf{a}_y$ and $\mathbf{B}_s = j10ze^{-jz}\mathbf{a}_x$, express **A** in phasor form and $\mathbf{B}_s$ in instantaneous form.

9.42 Show that in a linear homogeneous, isotropic source-free region, both $\mathbf{E}_s$ and $\mathbf{H}_s$ must satisfy the wave equation

$$\nabla^2\mathbf{A}_s + \gamma^2\mathbf{A}_s = 0$$

where $\gamma^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma$ and $\mathbf{A}_s = \mathbf{E}_s$ or $\mathbf{H}_s$.