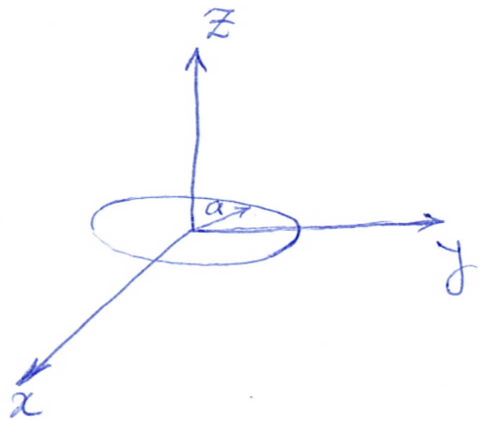
9.1

$a = 20$ cm

loop at $z=0$ plane

$\underline{B} = 10\cos(377t)\,\underline{a}_z$ mWb/m^2

The induced voltage in the loop = ?



$$V = -\frac{\partial \lambda}{\partial t} = -N\frac{\partial \psi}{\partial t}$$

$$N = 1 \Rightarrow V = -\frac{\partial \lambda}{\partial t} = -\frac{\partial \psi}{\partial t}$$

$$\psi = \iint_S \underline{B}\cdot d\underline{s} \qquad \Leftarrow \underline{B} \text{ is independent of } x \text{ and } y$$

$$= \underline{B}\cdot\underline{S} \qquad \Leftarrow \underline{S} = \pi a^2\,\underline{a}_z$$

$$= 10\cos(377t)\times 10^{-3}\times\pi\times(0.2)^2$$

$$= 0.4\pi\cos(377t)\times 10^{-3}$$

$$\Rightarrow V = -\frac{\partial \psi}{\partial t}$$

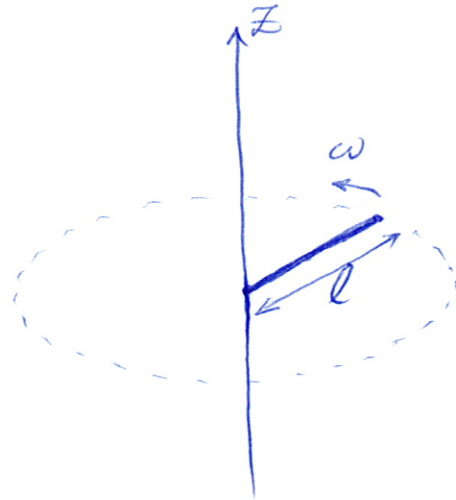
$$= -0.4\pi\times 377(-\sin(377t))\times 10^{-3}$$

$$= 0.4738\sin(377t) \quad V$$

9.2

$\underline{B} = B_0 \underline{a}_z$

The induced voltage = ?



$$V_{emf} = \int_L \underline{E}_m \cdot d\underline{\ell} = \int_L (\underline{u} \times \underline{B}) \cdot d\underline{\ell}$$

$$\underline{u} = \frac{\rho \, d\varphi \, \underline{a}_\varphi}{dt} = \rho \underbrace{\frac{d\varphi}{dt}}_{\omega} \underline{a}_\varphi = \rho \omega \underline{a}_\varphi$$

$$\underline{u} \times \underline{B} = (\rho \omega \underline{a}_\varphi) \times (B_0 \underline{a}_z) = B_0 \omega \rho \underline{a}_\rho$$

$$d\underline{\ell} = d\rho \, \underline{a}_\rho$$

$$V_{emf} = \int_L (B_0 \omega \rho \underline{a}_\rho) \cdot (d\rho \underline{a}_\rho) = \int_L B_0 \omega \rho \, d\rho$$

$$= \int_0^\ell B_0 \omega \rho \, d\rho = B_0 \omega \left. \frac{\rho^2}{2} \right|_0^\ell$$

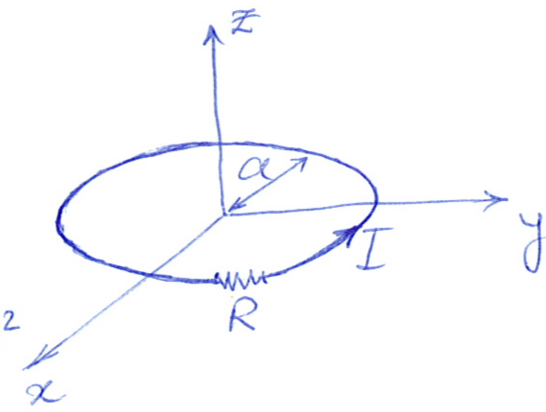
$$= \frac{1}{2} B_0 \omega \ell^2$$

9.4 loop in x-y plane

$a = 0.2\,m$

$R = 4\,\Omega$

$\underline{B} = 40 \sin(10^4 t)\,\underline{a}_z \; mWb/m^2$



$I = ?$

$$V = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$

$$N = 1 \Rightarrow V = -\frac{d\psi}{dt}$$

$$\psi = \iint_S \underline{B}\cdot d\underline{s} = \underline{B}\cdot\underline{S} = (40\sin(10^4 t)\underline{a}_z)\cdot(\pi a^2 \underline{a}_z)$$

$$= 40\sin(10^4 t) \times \pi \times (0.2)^2 \; mWb$$

$$= 1.6\pi \times 10^{-3} \sin(10^4 t) \;\; Wb$$

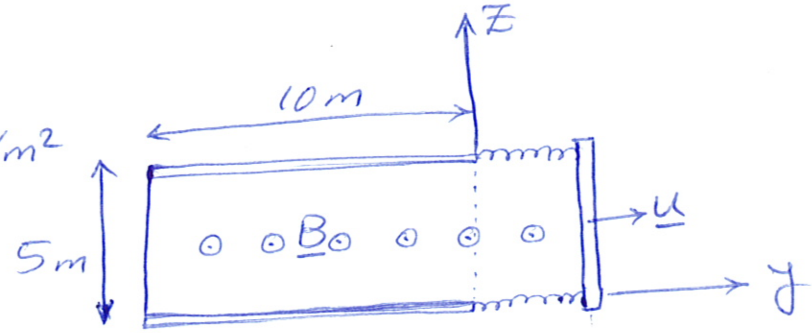
$$V = -\frac{d\psi}{dt} = -1.6\pi \times 10^{-3} \times 10^4 \cos(10^4 t)$$

$$= -16\pi \cos(10^4 t)$$

$$I = \frac{V}{R} = \frac{-16\pi\cos(10^4 t)}{4} = -4\pi\cos(10^4 t)$$

$$= -12.5664 \cos(10^4 t) \;\; A$$

9.9



$\underline{B} = 6\cos(10t)\underline{a}_x \ mWb/m^2$

$\underline{u} = 2\cos(10t)\underline{a}_y \ m/s$

$$V_{emf} = -\frac{\partial \lambda}{\partial t} = -N\frac{\partial \psi}{\partial t} = -\frac{\partial \psi}{\partial t}$$

$$\psi = \iint_S \underline{B}\cdot d\underline{s} = \int_{z=0}^{5}\int_{y=-10}^{y} (6\cos(10t)\underline{a}_x)\cdot(dy\,dz\,\underline{a}_x)$$

$$= \int_{z=0}^{5}\int_{y=-10}^{y} 6\cos(10t)\,dy\,dz$$

$$= 6\cos(10t)\int_{z=0}^{5}\int_{y=-10}^{y} dy\,dz = 6\times 5\times(y+10)\cos(10t)$$

$$= 30(y+10)\cos(10t) \ mWb$$

$$V_{emf} = -\frac{\partial \psi}{\partial t} = -\left[30\frac{dy}{dt}\cos(10t) + 30(y+10)(-10\sin(10t))\right]$$

$$= -30\frac{dy}{dt}\cos(10t) + 300(y+10)\sin(10t) \ mV$$

$$\frac{dy}{dt} = u = 2\cos(10t) \Rightarrow y = \int 2\cos(10t)\,dt$$

$$= \frac{2}{10}\sin(10t)$$

9.9

$$V_{emf} = -30 \times 2\cos^2(10t) + 300\left[\frac{2}{10}\sin(10t) + 10\right]\sin(10t) \quad mV$$

$$= \left[\underbrace{-60\cos^2(10t) + 60\sin^2(10t)}_{-60\cos(2\times 10t)} + 3000\sin(10t)\right] \times 10^{-3} \quad V$$

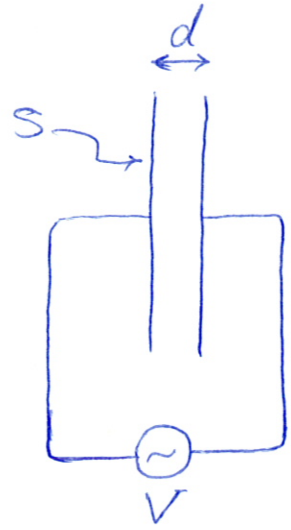
$$= -0.06\cos(20t) + 3\sin(10t) \quad V$$

9.13

$d = 0.2\ mm$

$S = 2.8\ cm^2$

$V = 50V$ at $20MHz$



$|\underline{J}_d|_{max} = ?$

$(I_d)_{max} = ?$

$$\underline{J}_d = \frac{\partial \underline{D}}{\partial t} \equiv J_{ds} = j\omega D_s = j\omega\varepsilon E_s = j\omega\varepsilon \frac{V_s}{d}$$

$$\Rightarrow |\underline{J}_d|_{max} = |J_{ds}| = \omega\varepsilon\frac{V_s}{d}$$

$$= (2\pi \times 20 \times 10^6) \times \frac{10^{-9}}{36\pi} \times \frac{50}{0.2 \times 10^{-3}}$$

$$= \frac{10^4}{36} = 277.7778\ A/m^2$$

$$\Rightarrow (I_d)_{max} = |\underline{J}_d|_{max} \times S = \frac{10^4}{36} \times 2.8 \times 10^{-4}$$

$$= \frac{2.8}{36} = 77.7778\ mA$$

9.15

Dry soil: $\sigma = 10^{-4}$ S/m

$\varepsilon = 3\varepsilon_0$

$\mu = \mu_0$

$$\frac{|\underline{J}|}{|\underline{J}_d|} = 1 \Rightarrow f = ?$$

$$\underline{J} = \sigma \underline{E}$$

$$\underline{J}_d = \frac{\partial D}{\partial t} = \omega \varepsilon \underline{E}$$

$$\Rightarrow \frac{J}{J_d} = \frac{\sigma}{\omega \varepsilon} = 1 \quad , \omega = 2\pi f$$

$$\Rightarrow \sigma = 2\pi f \varepsilon \Rightarrow f = \frac{\sigma}{2\pi \varepsilon}$$

$$\Rightarrow f = \frac{10^{-4}}{2\pi \times 3 \frac{10^{-9}}{36\pi}} = 6 \times 10^5 = 600\,KHz$$

9.23

$$\underline{J} = (2y\,\underline{a}_x + xz\,\underline{a}_y + z^3\,\underline{a}_z)\sin(10^4 t) \quad A/m$$

$$\rho_v(x,y,0,t) = 0 \Rightarrow \rho_v = ?$$

$$\nabla \cdot \underline{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \underline{J} = \left[\frac{\partial}{\partial x}(2y) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(z^3)\right]\sin(10^4 t)$$

$$= (0 + 0 + 3z^2)\sin(10^4 t)$$

$$= -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \rho_v = -\int 3z^2 \sin(10^4 t)\,dt$$

$$= \frac{3z^2}{10^4}\cos(10^4 t) + C_0$$

$$\rho_v(x,y,0,t) = 0 + C_0 = 0 \quad \Rightarrow C_0 = 0$$

$$\Rightarrow \rho_v = 3z^2 \times 10^{-4}\cos(10^4 t) \quad C/m^3$$

9.25

$$\underline{D} = D_0 \cos(\omega t + \beta z)\, \underline{a_y} \quad C/m^2$$

free space

$$\underline{B} = ?$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \Rightarrow \nabla \times (\underbrace{\varepsilon_0 \underline{E}}_{\underline{D}}) = -\varepsilon_0 \frac{\partial \underline{B}}{\partial t}$$

$$\nabla \times \underline{D} = \begin{vmatrix} \underline{a_x} & \underline{a_y} & \underline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & D_0 \cos(\omega t + \beta z) & 0 \end{vmatrix} = D_0 \beta \sin(\omega t + \beta z)\, \underline{a_x}$$

$$\Rightarrow \underline{B} = \left(-\int \frac{D_0 \beta}{\varepsilon_0} \sin(\omega t + \beta z)\, dt\right) \underline{a_x}$$

$$= \frac{D_0 \beta}{\varepsilon_0 \omega} \cos(\omega t + \beta z)\, \underline{a_x}$$

9.26 free space

$$\underline{E} = \frac{50}{\rho}\cos(10^8 t - kz)\,\underline{a}_\rho \quad V/m$$

$$\begin{cases} k = ? \\ \underline{J}_d = ? \\ \underline{H} = ? \end{cases}$$

$$\underline{J}_d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$= \frac{50\varepsilon_0}{\rho}\left(-10^8 \sin(10^8 t - kz)\right)\underline{a}_\rho$$

$$= -\frac{50\times 10^8 \varepsilon_0}{\rho}\sin(10^8 t - kz)\,\underline{a}_\rho \quad A/m^2$$

$$= -\frac{5\times 10^9 \times \frac{10^{-9}}{36\pi}}{\rho}\sin(10^8 t - kz)\,\underline{a}_\rho$$

$$= -\frac{5}{36\pi\rho}\sin(10^8 t - kz)\,\underline{a}_\rho \quad A/m^2$$

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\nabla \times \underline{E} = \frac{1}{\rho}\begin{vmatrix} \underline{a}_\rho & \rho\underline{a}_\varphi & \underline{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ \frac{50}{\rho}\cos(10^8 t - kz) & 0 & 0 \end{vmatrix} = \frac{-\rho\underline{a}_\varphi}{\rho}\left[-\frac{50}{\rho}(-k)(-\sin(\dots))\right]$$

$$= \frac{50k}{\rho}\sin(10^8 t - kz)\,\underline{a}_\varphi$$

9.26

$$\underline{H} = -\frac{1}{\mu_0}\int \frac{50k}{\rho}\sin(10^8 t - kz)\,\underline{a}_\varphi\,dt$$

$$= \frac{50k}{10^8 \mu_0 \rho}\cos(10^8 t - kz)\,\underline{a}_\varphi \quad A/m$$

$$= \frac{50k}{10^8 \times 4\pi \times 10^{-7} \rho}\cos(10^8 t - kz)\,\underline{a}_\varphi$$

$$= \frac{5k}{4\pi\rho}\cos(10^8 t - kz)\,\underline{a}_\varphi \quad A/m$$

$k = ?$

$$\nabla \times \underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t} = 0 + \frac{\partial \underline{D}}{\partial t} = \underline{J}_d$$

$$\nabla \times \underline{H} = \frac{1}{\rho}\begin{vmatrix} \underline{a}_\rho & \rho\underline{a}_\varphi & \underline{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \underbrace{\frac{5k\rho}{4\pi\rho}\cos(10^8 t - kz)}_{\rho H_\varphi} & 0 \end{vmatrix} = \frac{1}{\rho}\underline{a}_\rho\left[-\frac{5k}{4\pi}(-k)(-\sin(\cdots))\right]$$

$$= \frac{-5k^2}{4\pi\rho}\sin(10^8 t - kz)\,\underline{a}_\rho$$

$$\underline{J}_d = \frac{-5}{36\pi\rho}\sin(10^8 t - kz)\,\underline{a}_\rho$$

$$\nabla \times \underline{H} = \underline{J}_d \Rightarrow k^2 = \frac{1}{9}$$

$$\Rightarrow k = \frac{1}{3}$$

9.32 Antenna in free space

$$\underline{H} = \frac{12 \sin\theta}{r} \cos(2\pi \times 10^8 t - \beta r)\, \underline{a}_\theta \quad mA/m$$

$$\underline{E} = ?$$

$$\nabla \times \underline{H} = \underline{J} + \underline{J}_d$$

$$\underline{J} = \sigma \underline{E} \quad (\sigma = 0) \Rightarrow \underline{J} = \underline{0}$$

$$\underline{J}_d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times \underline{H} = \frac{-1}{r\sin\theta} \underbrace{\frac{\partial H_\theta}{\partial \varphi}}_{=0} \underline{a}_r + \frac{1}{r} \frac{\partial (r H_\theta)}{\partial r} \underline{a}_\varphi$$

$$= 0 + \frac{1}{r} \times 12 \sin\theta \times (-\beta)(-\sin(2\pi \times 10^8 t - \beta r))\, \underline{a}_\varphi$$

$$= \frac{12 \beta \sin\theta}{r} \sin(2\pi \times 10^8 t - \beta r)\, \underline{a}_\varphi \quad mA/m^2$$

$$\underline{E} = \frac{1}{\varepsilon_0} \int \nabla \times \underline{H}\, dt$$

$$= \frac{12 \beta \sin\theta}{\varepsilon_0 r} \int \sin(2\pi \times 10^8 t - \beta r) \times 10^{-3}\, \underline{a}_\varphi\, dt$$

$$= -\frac{12 \beta \sin\theta \times 10^{-3}}{2\pi \times 10^8 \varepsilon_0 r} \cos(2\pi \times 10^8 t - \beta r)\, \underline{a}_\varphi$$

$$= -\frac{12 \beta \sin\theta \times 10^{-3}}{2\pi \times 10^8 \times \frac{10^{-9}}{36\pi}\, r} \cos(2\pi \times 10^8 t - \beta r)\, \underline{a}_\varphi$$

$$= -\frac{2.16 \beta \sin\theta}{r} \cos(2\pi \times 10^8 t - \beta r)\, \underline{a}_\varphi \quad V/m$$

9.34 Free space

$\rho_v = 0$

$\underline{J} = \underline{0}$

c : speed of light in free space

$$\underline{A} = \frac{\mu_o}{4\pi r}(\cos\theta\, \underline{a}_r - \sin\theta\, \underline{a}_\theta) e^{jw(t-\frac{r}{c})}$$

$\underline{A}$ satisfies $\left(\nabla^2 \underline{A} - \mu\varepsilon \frac{\partial^2 \underline{A}}{\partial t^2} = -\mu \underline{J}\right)$?

$V = ?$

$$\left.\begin{array}{l}\underline{J} = \underline{0} \\ \text{free space}\end{array}\right\} \Rightarrow \nabla^2 \underline{A} = \mu_0 \varepsilon_0 \frac{\partial^2 \underline{A}}{\partial t^2}$$

$$\frac{\partial^2 \underline{A}}{\partial t^2} = (j\omega)(j\omega)\, \underline{A} = -\omega^2 \underline{A}$$

$\Rightarrow$ We have to show that $\nabla^2 \underline{A} = -\mu_0 \varepsilon_0 \omega^2 \underline{A}$

We know that $\underline{a}_z = \cos\theta\, \underline{a}_r - \sin\theta\, \underline{a}_\theta$

$$\Rightarrow \underline{A} = \frac{\mu_0}{4\pi r} e^{j\omega t} e^{-j\frac{\omega}{c}r}\, \underline{a}_z$$

$$\nabla^2 \underline{A} = \nabla^2 A_x \underline{a}_x + \nabla^2 A_y \underline{a}_y + \nabla^2 A_z \underline{a}_z$$

$$= 0 + 0 + \nabla^2 A_z \underline{a}_z$$

9.34

$$\nabla^2 A_z = \frac{\mu_0}{4\pi} e^{j\omega t} \times \nabla^2\left(\frac{e^{-j\frac{\omega}{c}r}}{r}\right)$$

$$\nabla^2\left(\frac{e^{-j\frac{\omega}{c}r}}{r}\right) = \frac{1}{r^2} \times \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(\frac{e^{-j\frac{\omega}{c}r}}{r}\right)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{-j\frac{\omega}{c}r - 1}{r^2}\right)e^{-j\frac{\omega}{c}r}\right]$$

$$= \frac{1}{r^2}\left[\left(\cancel{-j\frac{\omega}{c}} - j\frac{\omega}{c}\left(-j\frac{\omega}{c}r\right) \cancel{+j\frac{\omega}{c}}\right)e^{-j\frac{\omega}{c}r}\right]$$

$$= -\frac{\omega^2}{c^2}\frac{e^{-j\frac{\omega}{c}r}}{r}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

$$\Rightarrow \nabla^2 \underline{A} = -\frac{\omega^2}{c^2}\frac{\mu_0}{4\pi} e^{j\omega t}\frac{e^{-j\frac{\omega}{c}r}}{r}\underline{a}_z$$

$$= -\frac{\omega^2}{c^2}\underline{A}$$

$$= -\omega^2 \mu_0 \varepsilon_0 \underline{A}$$

$$\Rightarrow \nabla^2 \underline{A} = -\mu_0 \varepsilon_0 \omega^2 \underline{A}$$

9.34

$V = ?$

$$\nabla \cdot \underline{A} = -\mu \varepsilon \frac{\partial V}{\partial t} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$$

$$\Rightarrow V = -\frac{1}{\mu_0 \varepsilon_0} \int \nabla \cdot \underline{A} \, dt$$

$$\nabla \cdot \underline{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(A_\theta \sin\theta)$$

$$\underline{A} = \frac{\mu_0}{4\pi} e^{j\omega t} \left[\frac{\cos\theta}{r} e^{-j\frac{\omega}{c} r} \underline{a_r} - \frac{\sin\theta}{r} e^{-j\frac{\omega}{c} r} \underline{a_\theta} \right]$$

$$\nabla \cdot \underline{A} = \frac{\mu_0}{4\pi} e^{j\omega t} \Big[\frac{1}{r^2} (1 - j\frac{\omega}{c} r) \cos\theta \, e^{-j\frac{\omega}{c} r}$$

$$+ \frac{1}{r \sin\theta} \left(\frac{-2 \sin\theta \cos\theta}{r} e^{-j\frac{\omega}{c} r} \right) \Big]$$

$$= \frac{\mu_0}{4\pi} e^{j\omega t} \left(-j\frac{\omega}{cr} - \frac{1}{r^2} \right) \cos\theta \, e^{-j\frac{\omega}{c} r}$$

$$\Rightarrow V = -\frac{1}{\mu_0 \varepsilon_0} \int \nabla \cdot \underline{A} \, dt$$

$$= -\frac{1}{\mu_0 \varepsilon_0} \times \frac{\mu_0}{4\pi} \cos\theta \, e^{-j\frac{\omega}{c} r} \left(-\frac{1}{r^2} - j\frac{\omega}{cr} \right) \int e^{j\omega t} \, dt$$

9.34

$$V = \frac{\cos\theta}{4\pi\varepsilon_0 (j\omega) r}\left(j\frac{\omega}{c} + \frac{1}{r}\right) e^{j\omega(t - \frac{r}{c})}$$

9.40 source-free vacuum region

$$\underline{H} = \frac{1}{\rho}\cos(\omega t - 3z)\,\underline{a}_\varphi \quad A/m$$

a) $\underline{H}$ in phasor form = ?

$$\underline{H} = Re\left[\underline{H}_s\, e^{j\omega t}\right]$$

$$\underline{H}_s = \frac{1}{\rho}\, e^{-j3z}\,\underline{a}_\varphi$$

b) $\underline{E}$ = ?

$$\nabla \times \underline{H} = \underline{J} + \underline{J}_d = \underline{0} + \underline{J}_d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\underline{E} = \frac{1}{\varepsilon_0}\int \nabla \times \underline{H}\, dt$$

$$\nabla \times \underline{H} = -\frac{\partial H_\varphi}{\partial z}\underline{a}_\rho + \frac{1}{\rho}\frac{\partial(\rho H_\varphi)}{\partial \rho}\underline{a}_z$$

$$= -\frac{-3}{\rho}(-\sin(\omega t - 3z))\underline{a}_\rho + \frac{1}{\rho}\times 0\,\underline{a}_z$$

$$= -\frac{3}{\rho}\sin(\omega t - 3z)\,\underline{a}_\rho$$

$$\underline{E} = -\frac{3}{\varepsilon_0 \rho}\int \sin(\omega t - 3z)\,dt\,\underline{a}_\rho$$

$$= \frac{3}{\omega \varepsilon_0 \rho}\cos(\omega t - 3z)\,\underline{a}_\rho$$

9.40

c) $\omega = ?$

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\underline{H} = -\frac{1}{\mu_0}\int \nabla \times \underline{E}\, dt$$

$$\nabla \times \underline{E} = \frac{\partial E_\rho}{\partial z}\underline{a}_\varphi - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \varphi}\underline{a}_z$$

$$= \frac{3}{\omega \varepsilon_0 \rho}(-3)(-\sin(\omega t - 3z))\underline{a}_\varphi - 0$$

$$= \frac{9}{\omega \varepsilon_0 \rho}\sin(\omega t - 3z)\underline{a}_\varphi$$

$$\underline{H} = -\frac{1}{\mu_0}\frac{9}{\omega \varepsilon_0 \rho}\int \sin(\omega t - 3z)\,dt\,\underline{a}_\varphi$$

$$= \frac{9}{\omega^2 \mu_0 \varepsilon_0 \rho}\cos(\omega t - 3z)\underline{a}_\varphi$$

And this should be equal to $\underline{H} = \frac{1}{\rho}\cos(\omega t - 3z)\underline{a}_\varphi$

$$\Rightarrow \frac{9}{\omega^2 \mu_0 \varepsilon_0} = 1 \quad \Rightarrow \omega = \sqrt{\frac{9}{\mu_0 \varepsilon_0}} = \frac{3}{\sqrt{\mu_0 \varepsilon_0}} = \frac{3}{\sqrt{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}}}$$

$$\Rightarrow \omega = 9 \times 10^8 \text{ rad/s}$$