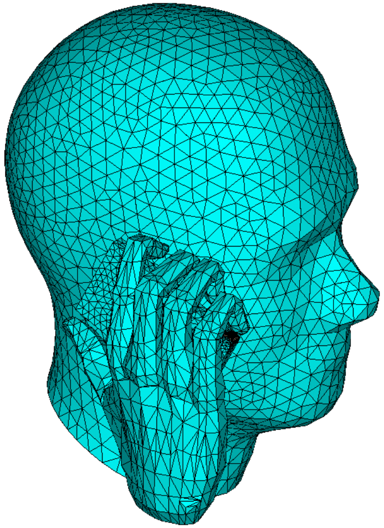
Lecture 1: Vector Algebra

scalars and vectors, unit vectors, subtraction and addition, position and distance vectors, vectors multiplications, Chapter 1, pages 3-15

Why Vectors?



Bakr et al., 2012

http://readious.blogspot.ca/2012/12/how-to-protect-yourself-from-harmful.html

Electromagnetic fields with different directions and magnitudes exist inside our bodies and everywhere in space

To determine the total field at any point, we apply vector operations

Vectors and Scalars

Scalars are quantities that are fully defined by their magnitude only. No need to mention a direction for these quantities for example, mass, time, distance, and population

Vectors are quantities that have both magnitude and direction such as velocity, force, and field intensity

In electromagnetics-related journals, conference proceedings, and books, vectors are indicated by bold face (**A, b,** or ***A***) or with arrows ($\vec{\mathrm{A}}$) while scalars are indicated by regular fact (a, or *a*)

A field is a function that specifies a quantity everywhere in a predefined region

Magnitudes and Unit Vectors (Cartesian Coordinates)

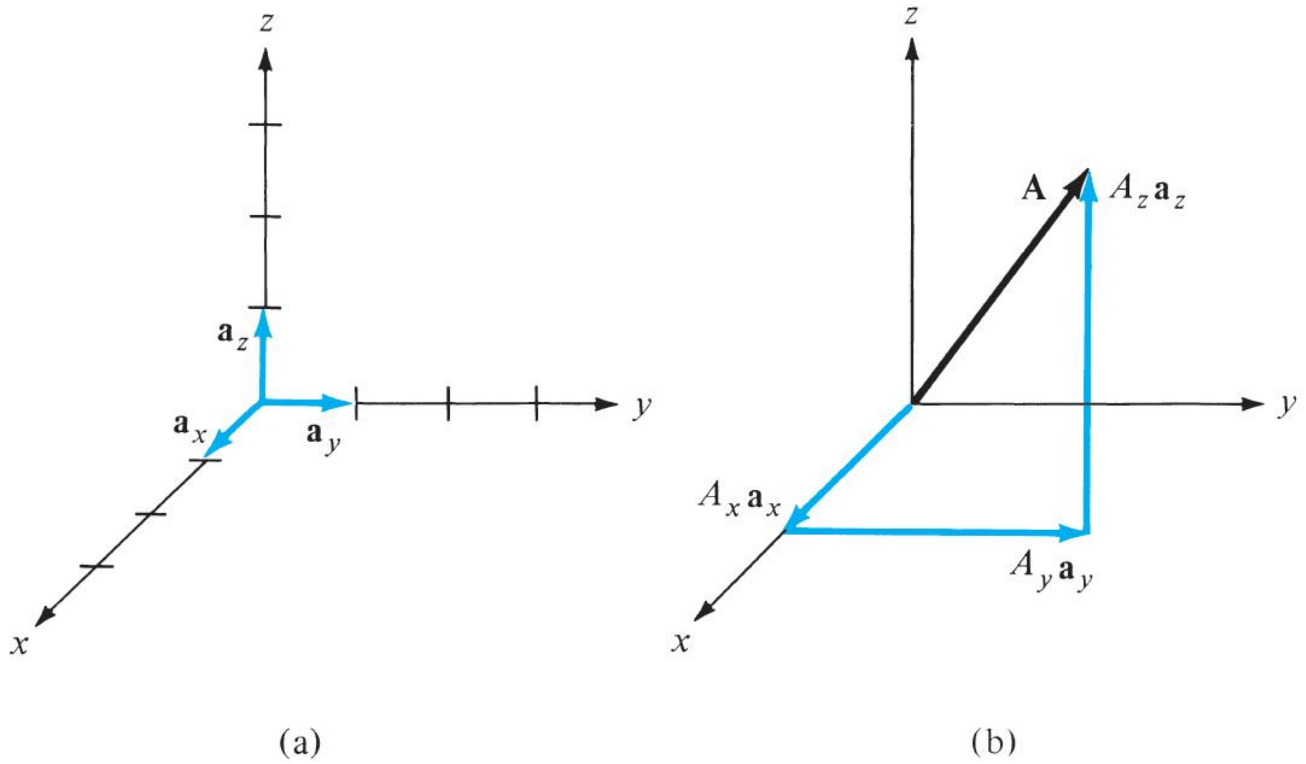
The vector magnitude of a vector **A** is the strength of the vector and is denoted by $|\mathbf{A}|$

A unit vector in the direction of ***A*** is a vector pointing in the same direction with a unity strength and is given by $\mathbf{a}_A=\mathbf{A}/|\mathbf{A}|$

In Cartesian coordinates, a vector is given by 3 components $\mathbf{A}=A_x\,\mathbf{a}_x+A_y\,\mathbf{a}_y+A_z\,\mathbf{a}_z$

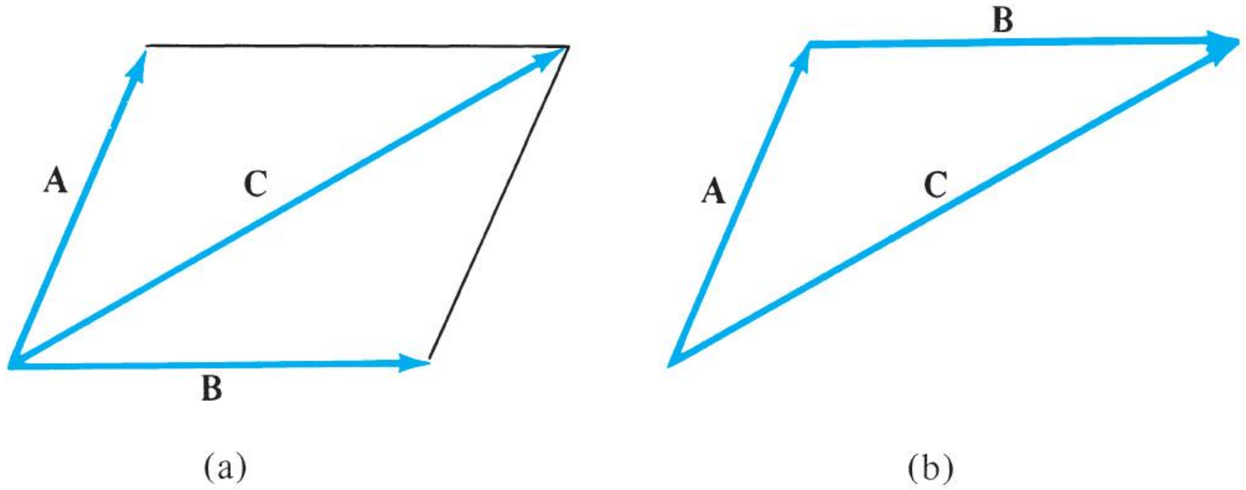
Its magnitude is given by $\mathbf{A}=\sqrt{A_x^2+A_y^2+A_z^2}$

Magnitudes (Cont'd)



Notice that a vector does not define a line in space. It just defines a direction. A point of reference is needed to determine a line

Vector Addition

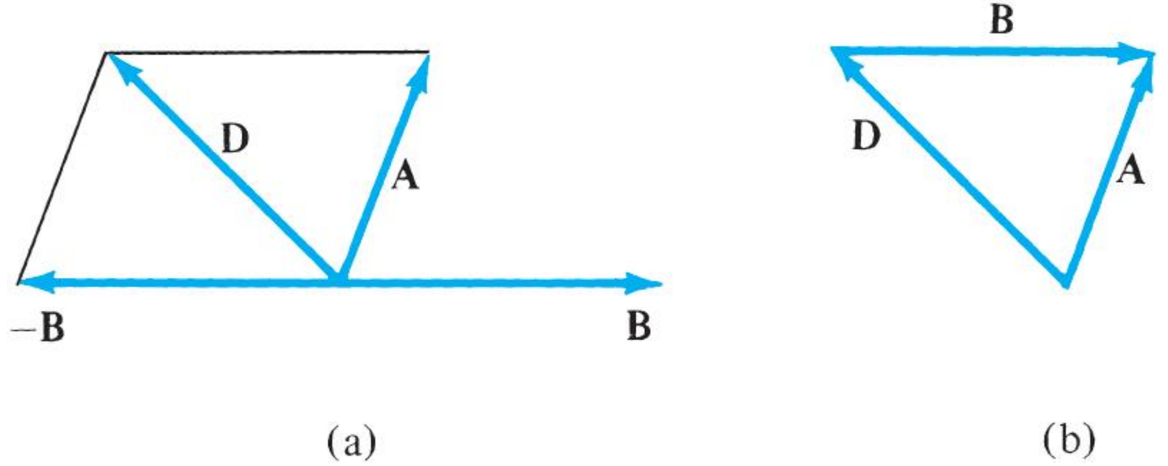


The sum of vectors **C**=**A**+**B** implies adding the corresponding components
C=$(A_x+B_x)\mathbf{a}_x+(A_y+B_y)\mathbf{a}_y+(A_z+B_z)\mathbf{a}_z$

Pictorially, this implies completing the parallelogram

Vector summation is commutative, associative, and distributive with respect to scalars

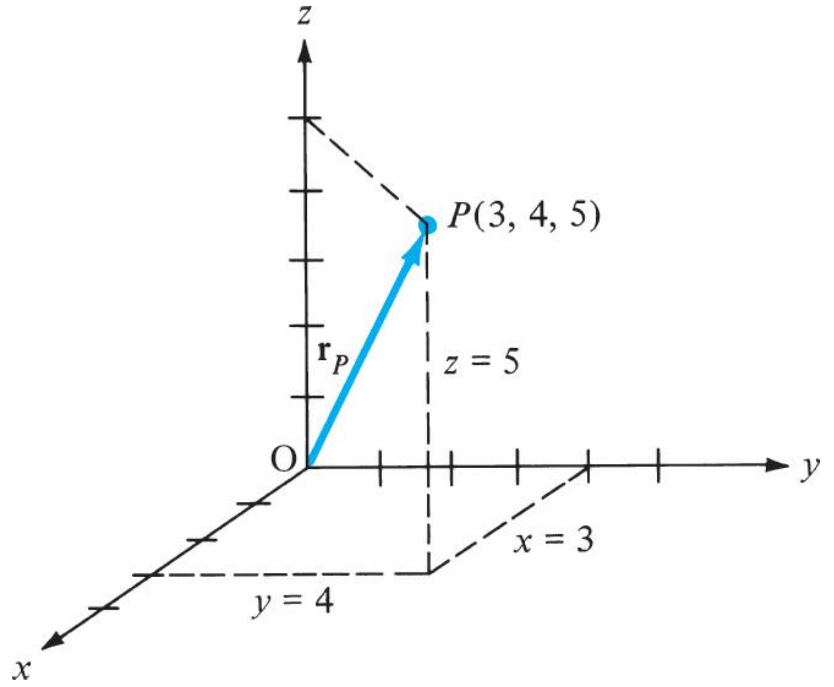
Vector Subtraction



The vector **D**=**A**−**B**=**A**+(-**B**) is obtained by subtracting the corresponding components **D**=$(A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z$

This implies that **A**=**D**+**B**

Position and Distance Vectors

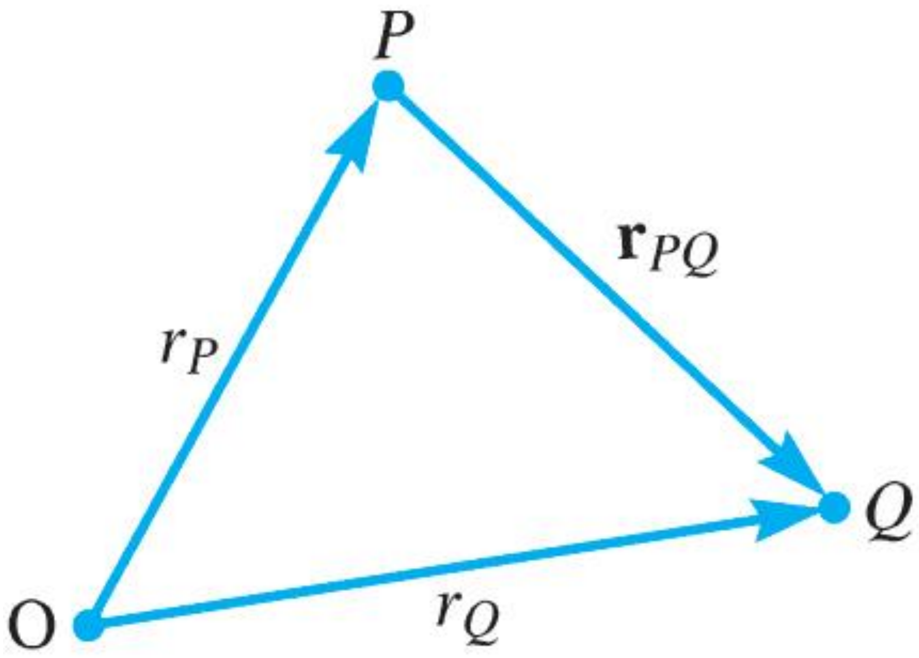


Position vector of a point is the directed line from the origin to that point with its components as the coordinates of the point

$\mathbf{r}_P = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

In figure $x=3$, $y=4$, $z=5$

Distance Vector

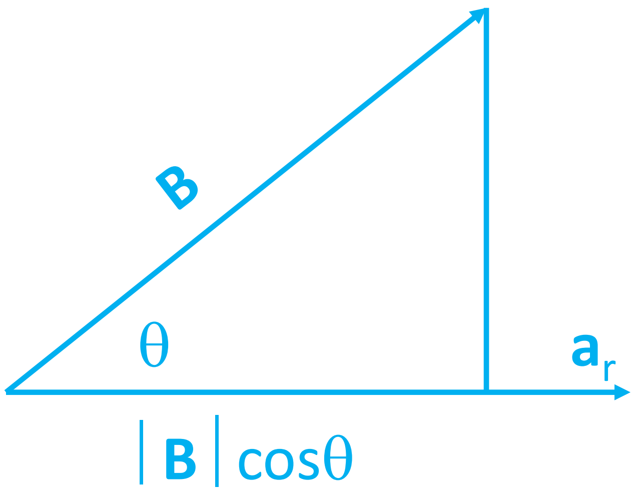


The distance vector pointing from point *P* to point *Q* is the vector
$\mathbf{r}_{pQ}=\mathbf{r}_{Q}-\mathbf{r}_{P}=(x_{Q}-x_{P})\mathbf{a}_{\mathrm{x}}+(y_{Q}-y_{P})\mathbf{a}_{\mathrm{y}}+(z_{Q}-z_{P})\mathbf{a}_{\mathrm{z}}$

The dot (scalar) product

This product between two vectors results in a scalar

If θ is the smallest angle between the two vector, the scalar product is defined by $\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta$



Inner product with a unit vector gives the component in the direction of the unit vector!

Properties of inner product

$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$

Commutative $\mathbf{A} \bullet \mathbf{B} = \mathbf{B} \bullet \mathbf{A}$, Distributive $\mathbf{A} \bullet (\mathbf{B} + \mathbf{C}) = \mathbf{A} \bullet \mathbf{B} + \mathbf{A} \bullet \mathbf{C}$

$$\mathbf{A} \bullet \mathbf{A} = A_x A_x + A_y A_y + A_z A_z = A_x^2 + A_y^2 + A_z^2 = \left|\mathbf{A}\right|^2$$

The dot product of two orthogonal vectors (in 2D and 3D) is zero

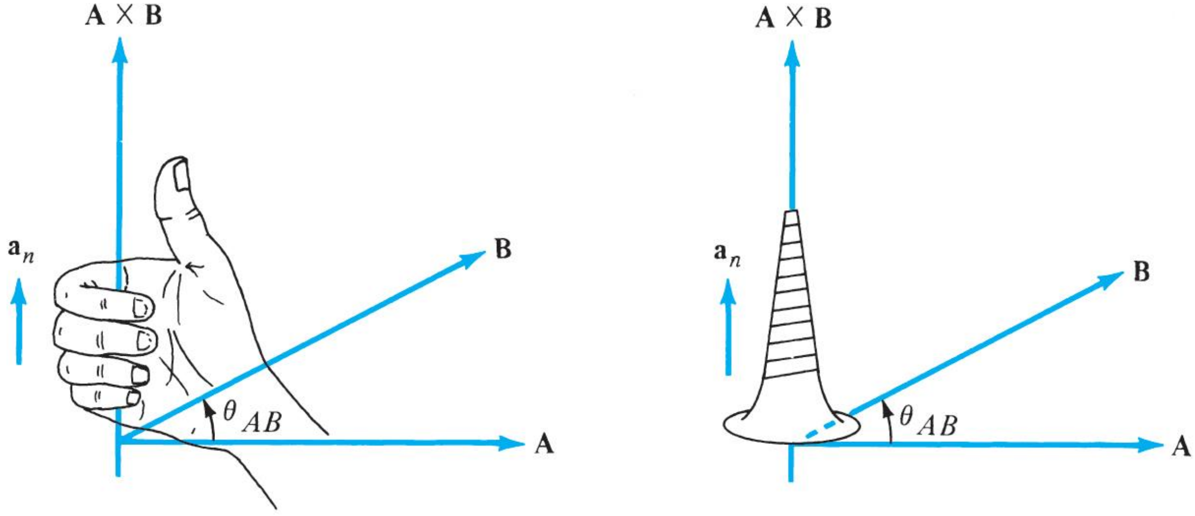
$$\mathbf{a}_x \cdot \mathbf{a}_y = 0, \ \mathbf{a}_x \cdot \mathbf{a}_z = 0, \ \mathbf{a}_y \cdot \mathbf{a}_z = 0,$$

$$\mathbf{a}_x \cdot \mathbf{a}_x = 1, \ \mathbf{a}_y \cdot \mathbf{a}_y = 1, \ \mathbf{a}_z \cdot \mathbf{a}_z = 1$$

Cross Product

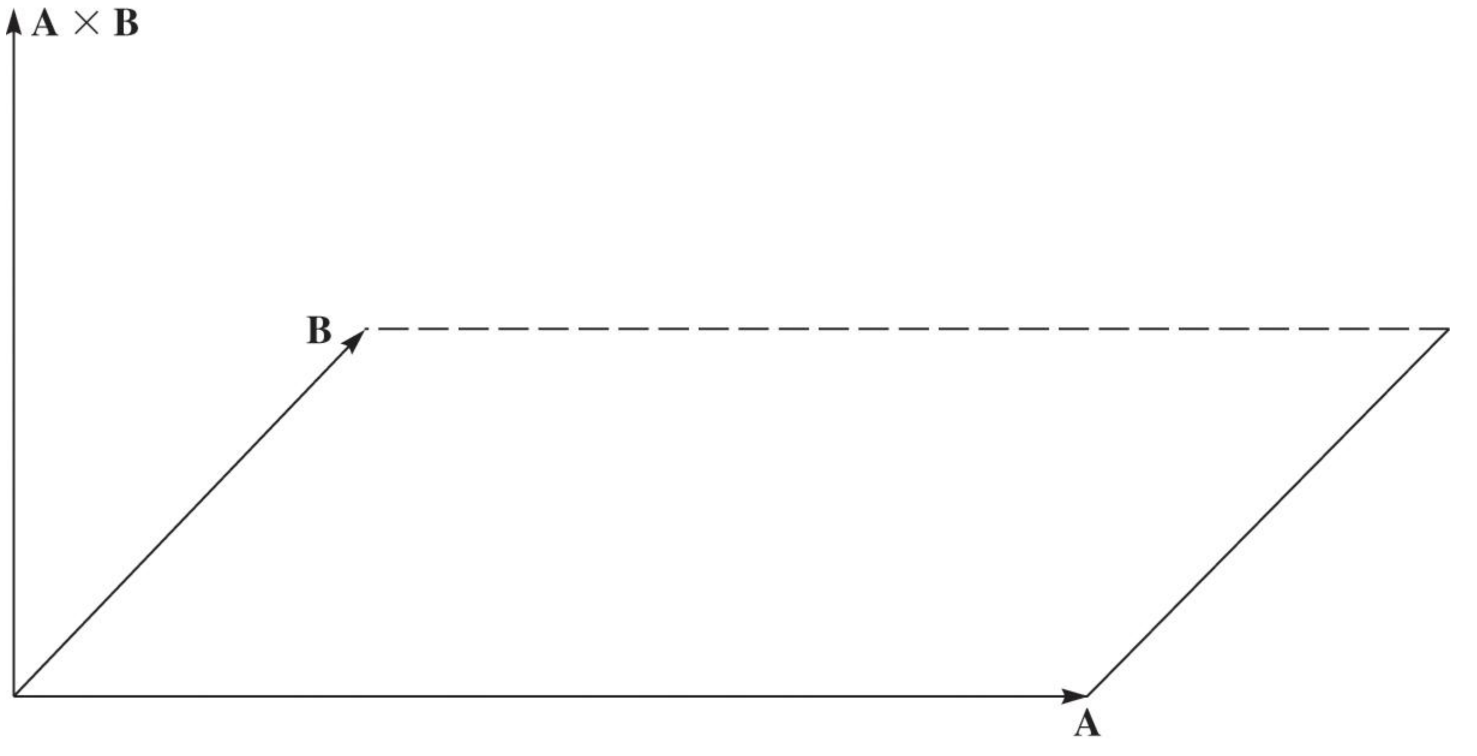
The cross product of two vectors gives another vector normal to their plane

The cross product is defined by $\mathbf{A}\times\mathbf{B}=|\mathbf{A}|\,|\mathbf{B}|\sin\theta\,\mathbf{a}_n$, where θ is the smallest angle between the two vectors and $\mathbf{a}_n$ is a unit vector normal to their plane in the direction of progress of a right hand screw rotating from vector **A** into vector **B**



(a) (b)

Properties of Cross Product



The magnitude of the cross product is the area of the parallelogram created by both vectors

Properties (Cont'd)

Cross product in Cartesian coordinates is given by the following determinant

$$\mathbf{A}\times\mathbf{B}=\begin{bmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

$$=\left(A_yB_z-A_zB_y\right)\mathbf{a}_x-\left(A_xB_z-A_zB_x\right)\mathbf{a}_y+\left(A_xB_y-A_yB_x\right)\mathbf{a}_z$$

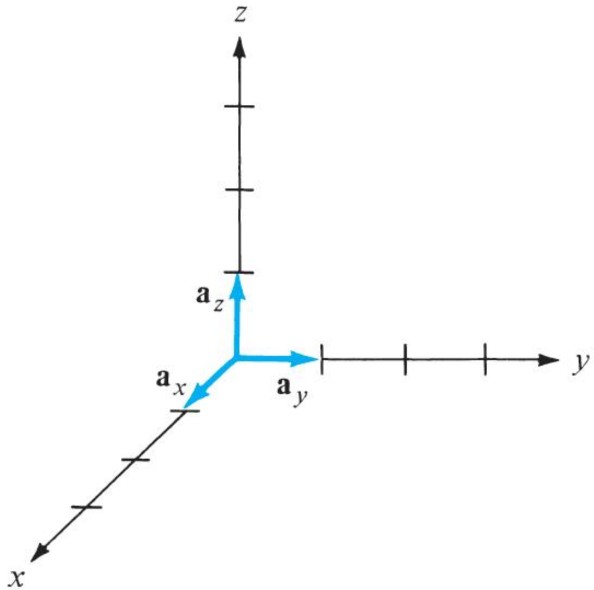
Cross product is anticommutative $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$

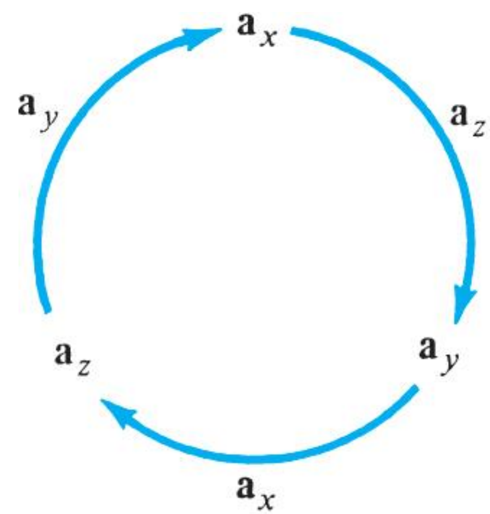
Cross product is not associative $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})\neq(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$

Cross product is distributive $\mathbf{A}\times(\mathbf{B}+\mathbf{C})=\mathbf{A}\times\mathbf{B}+\mathbf{A}\times\mathbf{C}$

$\mathbf{A}\times\mathbf{A}=\mathbf{0}$

Properties (Cont'd)





For a right-handed Cartesian system, we have

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z, \quad \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x, \quad \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y,$$

$$\mathbf{a}_x \times \mathbf{a}_x = \mathbf{0}, \quad \mathbf{a}_y \times \mathbf{a}_y = \mathbf{0}, \quad \mathbf{a}_z \times \mathbf{a}_z = \mathbf{0}$$