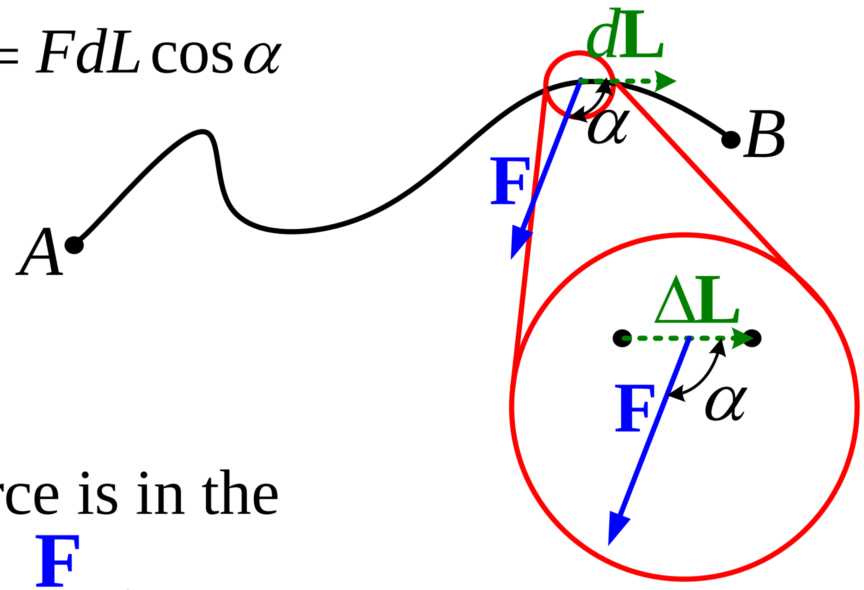
Lecture 10: Electrostatics

Electric potential, relations between E and V,
Chapter 4, pages 137-146

Work

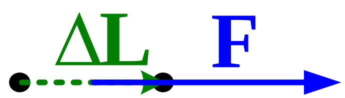
differential work $dW = \mathbf{F} \cdot d\mathbf{L} = F dL \cos\alpha$



work done from A to B

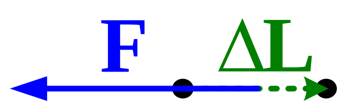
$$W = \int_A^B \mathbf{F} \cdot d\mathbf{L}, \ \text{J}$$

work is positive if the applied force is in the direction of displacement



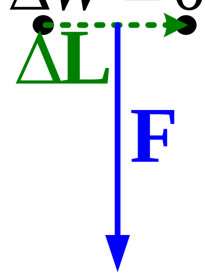
$$\Delta W = F \Delta L > 0$$

work is negative if the applied force is opposite to the direction of displacement



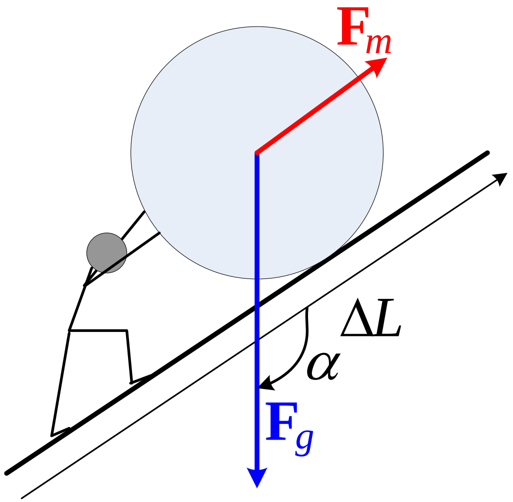
$$\Delta W = -F \Delta L < 0$$

$$\Delta W = 0$$



work is zero if force is orthogonal

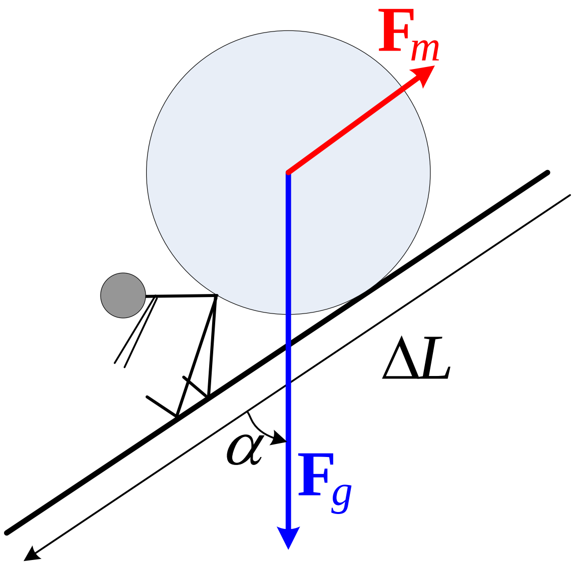
Gravity Example (uniform motion)



$$W_m = F_m \Delta L > 0, \text{ man does work (expends energy)}$$

$$W_g = F_g \Delta L \cos\alpha < 0, \text{ rock gains energy}$$

$$W_m = -W_g$$
$$F_m = -F_g \cos\alpha$$

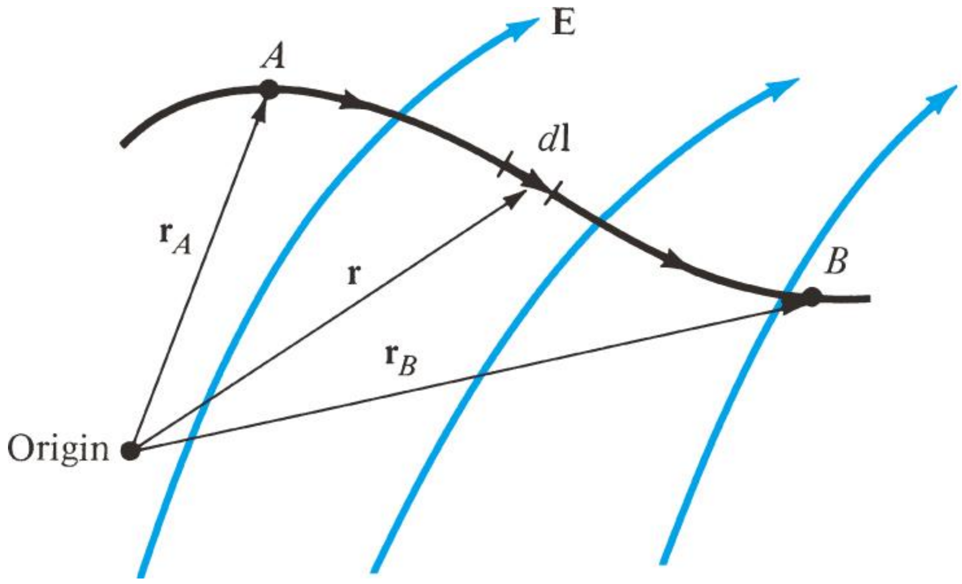


$$W_g = F_g \Delta L \cos\alpha > 0, \text{ gravity does work (rock loses energy)}$$

$$W_m = -F_m \Delta L < 0$$

man provides enough friction to prevent acceleration, gains energy (heat)

Work Against Electric Field



work done against electric field to move a charge Q from point A to point B is

$$W = -Q\int_A^B \mathbf{E}\cdot d\mathbf{L}$$

why –ve sign?

Electrostatic Potential Difference

potential difference V_{AB} is the work done by the electric field in moving a unit test charge from point B to point A

$$V_{AB} = V_B - V_A = \int_B^A \mathbf{E} \cdot d\mathbf{L},\ \text{V}$$

OR: potential difference V_{AB} is the work done by external forces in moving a unit test charge from point A to point B

$$V_{AB} = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{L},\ \text{V}$$

Electrostatic Potential

the potential at a point A is the work done by the electric field in moving a unit test charge from point A to a reference point A_0 where the potential is assumed equal to zero

OR: the potential at a point A is the work done by an external force in moving a unit test charge from a reference point A_0, where the potential is assumed equal to zero, to the point A.

$$V_A = \int_{A}^{A_0} \mathbf{E} \cdot d\mathbf{L} = -\int_{A_0}^{A} \mathbf{E} \cdot d\mathbf{L}, \text{ V}$$

the electrostatic potential is the potential energy of a unit test charge

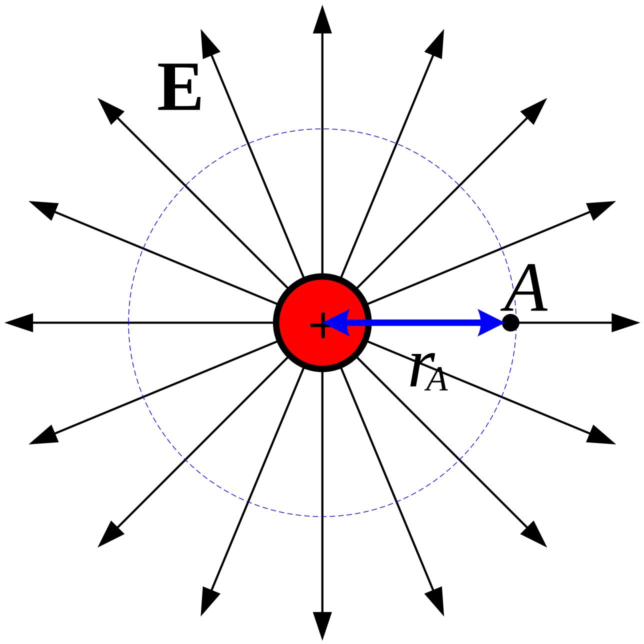
Potential of a Point Charge

assume potential is zero at infinity

integrate along any radial line

$$V_A = \int_A^{\infty} \mathbf{E} \cdot d\mathbf{L} = \int_{r_A}^{\infty} \frac{Q}{4\pi\varepsilon r^2} \mathbf{a}_R \cdot \underbrace{\mathbf{a}_R dr}_{d\mathbf{L}}$$

$$\Rightarrow V_A = -\frac{Q}{4\pi\varepsilon} \frac{1}{r}\bigg|_{r_A}^{\infty} \quad \Rightarrow V_A = \frac{Q}{4\pi\varepsilon} \frac{1}{r_A}$$



the potential is the same for all points on the sphere of radius r_A

the sphere is an equipotential surface

notice the $1/r$ dependence on distance!

Potential (Cont'd)

if the points charge is at $\boldsymbol{r}'$, the potential at a point $\boldsymbol{r}$ is given by

$$V(\boldsymbol{r}) = \frac{Q}{4\pi\varepsilon\left|\boldsymbol{r}-\boldsymbol{r}'\right|}$$

for a number of discrete charges, we have

$$V(\boldsymbol{r}) = \sum_{i=1}^{N} \frac{Q_i}{4\pi\varepsilon\left|\boldsymbol{r}-\boldsymbol{r}_i'\right|}$$

remember that all these potentials are relative to a point in ∞!

Potential (Cont'd)

for continuous charge distributions, we have

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon}\int \frac{\rho_L(\boldsymbol{r}')dl'}{4\pi\varepsilon|\boldsymbol{r}-\boldsymbol{r}'|}$$ line charge

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon}\int \frac{\rho_s(\boldsymbol{r}')ds'}{4\pi\varepsilon|\boldsymbol{r}-\boldsymbol{r}'|}$$ surface charge

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon}\int \frac{\rho_v(\boldsymbol{r}')dv'}{4\pi\varepsilon|\boldsymbol{r}-\boldsymbol{r}'|}$$ volume charge

Potential of a Line Charge

$$V_{AB} = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{L} = -\int_{\rho_A}^{\rho_B} \frac{\rho_L}{2\pi\rho} \mathbf{a}_\rho \cdot \mathbf{a}_\rho d\rho$$

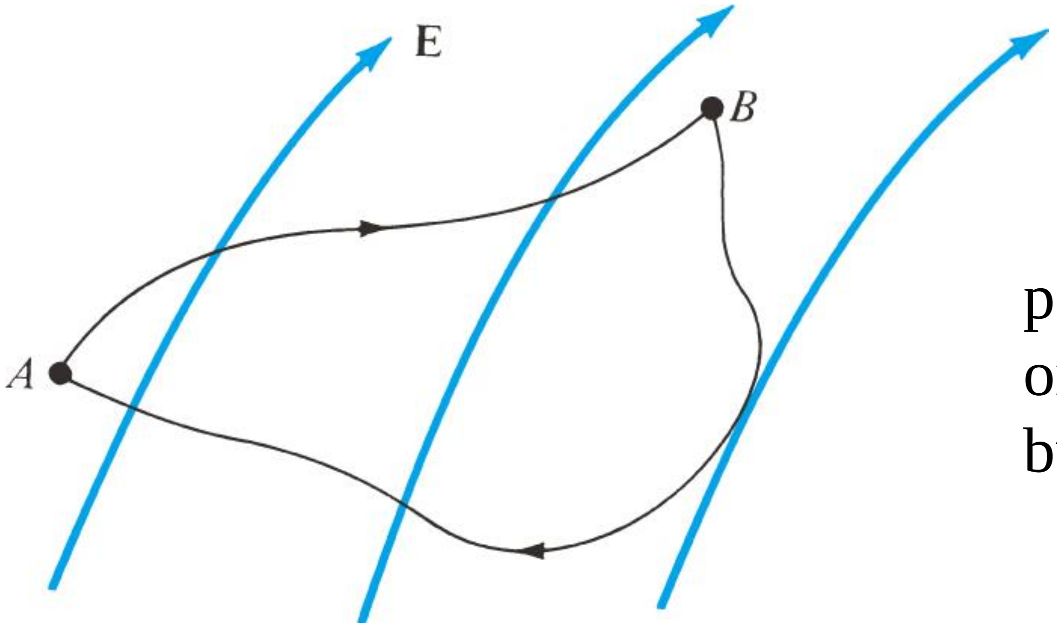
$$\Rightarrow V_{AB} = \frac{\rho_l}{2\pi\varepsilon} \cdot \ln\left(\frac{\rho_A}{\rho_B}\right)$$

Can we choose ρ_B or ρ_A at infinity or at zero?

if we choose ρ_B=1.0 m, and $V(\rho_B)$=0, we get

$$\Rightarrow V_{AB} = -V_A = \frac{\rho_l}{2\pi\varepsilon} \cdot \ln(\rho_A) \Rightarrow V_A = \frac{\rho_l}{2\pi\varepsilon} \cdot \ln\left(\frac{1}{\rho_A}\right)$$

Relation between E and V



potential does not depend on contour of integration but on position of points

$$V_{AB} = -\int_{C1} \boldsymbol{E}.d\mathbf{L} = -\int_{C2} \boldsymbol{E}.d\mathbf{L} \Rightarrow \oint \boldsymbol{E}.d\mathbf{L} = 0$$ ESF is conservative!

using Stoke's Theorem, $\oint \mathbf{E}.d\mathbf{L} = \iint_S (\nabla \times \mathbf{E}).d\boldsymbol{s} = 0 \;\Rightarrow (\nabla \times \mathbf{E}) = \mathbf{0}$

it follows that $\mathbf{E} = -\nabla V$ (why?)

Relation between E and V (Cont'd)

the differential potential difference over a differential length $d\mathbf{L}$ is given by

$$dV = -\mathbf{E}.d\mathbf{L} = -E_x dx - E_y dy - E_z dz$$

using first-order Taylor expansion, we have

$$dV = \frac{\partial V}{\partial x}\, dx + \frac{\partial V}{\partial y}\, dy + \frac{\partial V}{\partial y}\, dz$$

equating, we have

$$E_x = -\frac{\partial V}{\partial x},\ \ E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$