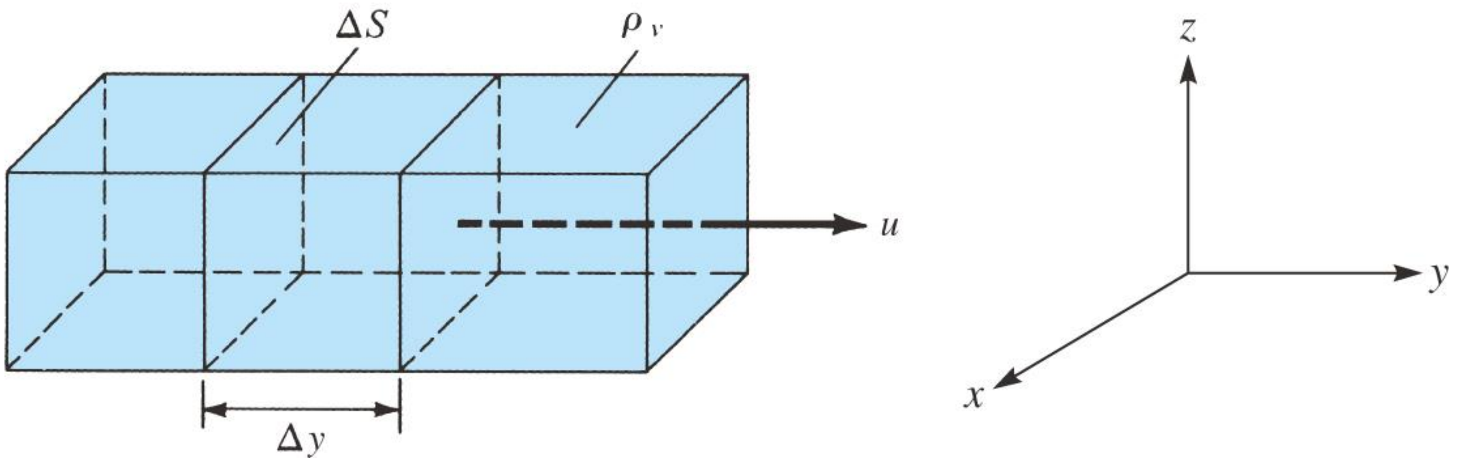
Lecture 12: Electrostatics

Convection and Conduction currents, Conductors, Chapter 5, pages 173-182

Current Density

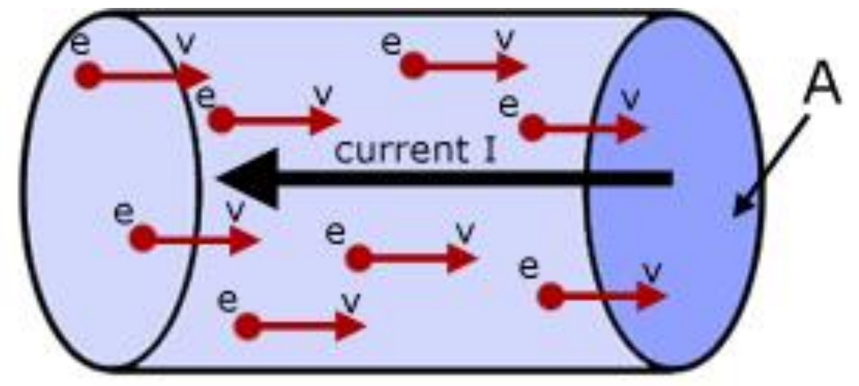


$$J = \frac{\Delta I}{\Delta S} \Rightarrow \Delta I = \mathbf{J}.\Delta\mathbf{S} \Rightarrow I = \iint \mathbf{J}.\Delta\mathbf{S}$$

if a charge with a volumetric density ρ_v moves with a velocity **u**, we get a current

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \Delta S \frac{\Delta y}{\Delta t} = \rho_v \Delta S u_y \Rightarrow \mathbf{J} = \rho_v \mathbf{u}$$

Materials in Field



applied electric fields causes free charges to drift with a drift velocity that is proportional to the field intensity

$$\Rightarrow \mathbf{J} = \sigma \mathbf{E}$$

electric conductivity in S/m is a property of matter that describes the availability of free charges and how easily they drift with applied electric field

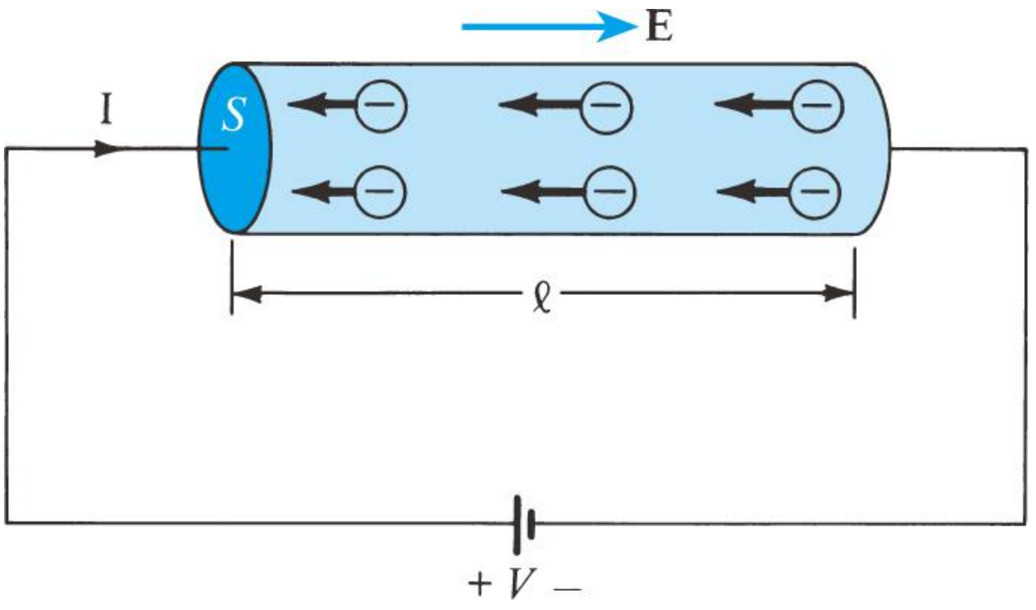
Resistance of a Conductor

$$E = \frac{V}{l},\ J = \frac{I}{\Delta S}$$

$$\frac{I}{\Delta S} = \sigma \frac{V}{l}$$

$$\Downarrow$$

$$R = \frac{V}{I} = \frac{l}{\sigma S} = \frac{\rho_c l}{S}$$



ρ_c is the resistivity which is the inverse of the conductivity

Resistance of a Conductor (Cont'd)

in general, $R = \dfrac{V}{I} = \dfrac{\int \mathbf{E}.\mathrm{d}\mathbf{L}}{\iint \sigma \mathbf{E}.\mathrm{d}\mathbf{S}}$ **(why there is no negative sign!)**

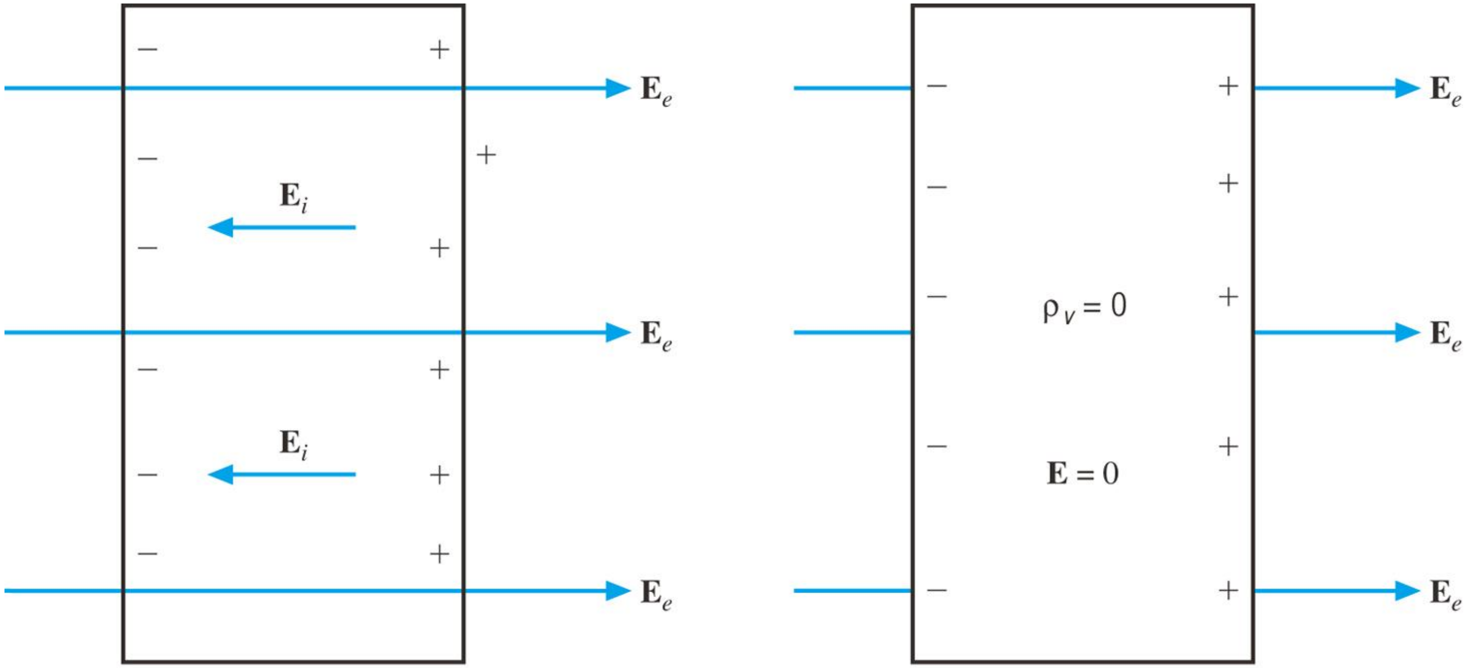
power dissipated in a conductor is defined as the rate of change of energy or force times velocity

$$P = \iiint_V \rho_v dv\, \mathbf{E.u} = \iiint_V \mathbf{E.J}\ dv \text{ (watts) Joule's law}$$

power density $= \mathbf{E.J} = \sigma |\boldsymbol{E}|^2$ watts/m^3

$$P = \iiint_V \mathbf{E.J}\ dv = \iiint_V J\, ds\, E dl = \int E dl \iint_S J\, ds = VI$$

Perfect Conductor



there is no electric field inside a perfect conductor (why)

the perfect electric conductor is an equipotential surface (why)