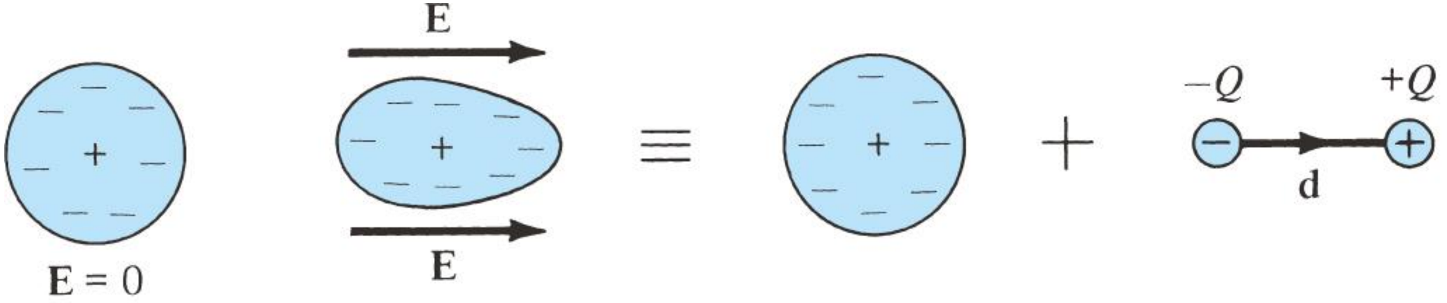
Lecture 13: Electrostatics

Polarization in dielectrics, dielectric constant and strength, linear, isotropic, and homogenous medium, Chapter 5 183-191

Polarization in Dielectrics

dielectrics have negligible DC conductivity and their charges are assumed immobile, i.e., they are *bound* to a fixed location

polarization involves the orientation of microscopic *bound* dipoles so that their moments align more or less with the external **E** field



materials are either nonpolar or polar

Polarization in Dielectrics

polarization vector is the dipole moment per unit volume

$$\mathbf{P} = \lim_{\Delta v \to 0} \frac{\sum_i \mathbf{p}_i}{\Delta v}, \quad \text{C/m}^2 \qquad d\mathbf{p} = \mathbf{P}\,dv$$

assume n (m^{-3}) bound dipoles (atoms, molecules, sub-domains) per unit volume of averaged distance and orientation given by $\mathbf{d}_{av}$ and dipole charge q_d

$$\Rightarrow \mathbf{P} = n\, q_d\, \mathbf{d}_{av}$$

$\mathbf{p}_{av}$

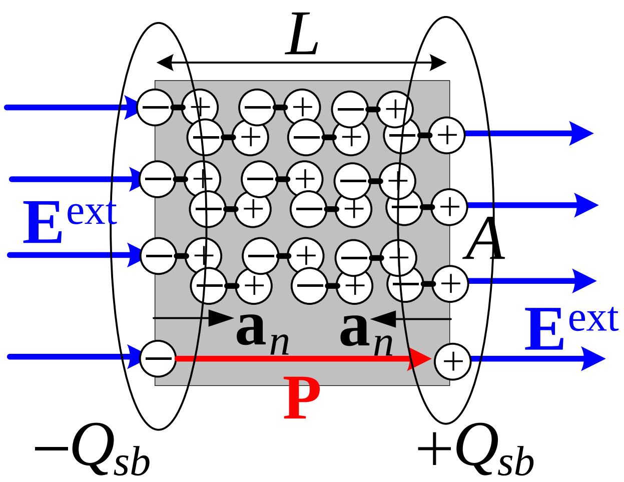
number density of bound dipoles

Polarization Vector and Bound Surface Charges

$\mathbf{E}^{ext}$ is orthogonal to surface planes

unit normals $\mathbf{a}_n$ point inwards

uncompensated bound charge exists on surface only $\rightarrow \rho_{sb} = Q_{sb}/A$



it can be shown that $\mathbf{P} = -\rho_{sb}\mathbf{a}_n$

$\mathbf{P}$ is along $\mathbf{E}^{ext}$ (isotropic material) and points from $-\rho_{sb}$ to $+\rho_{sb}$

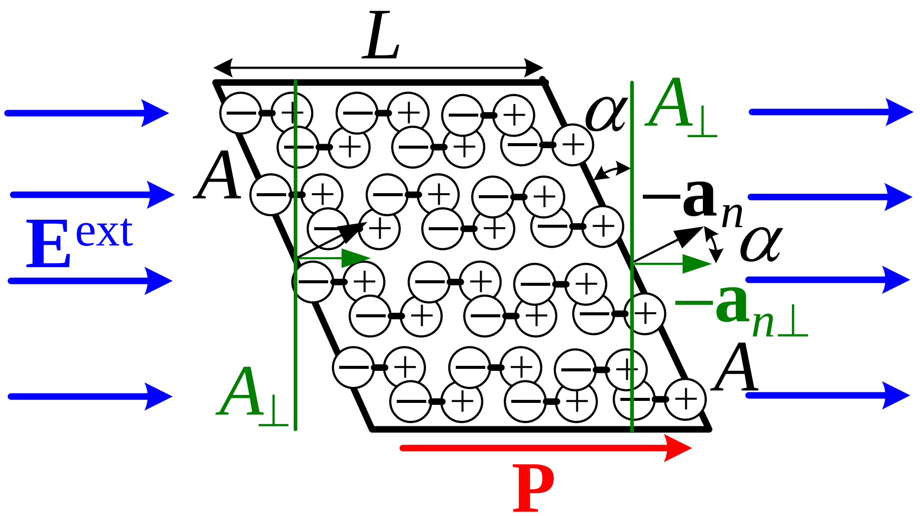
$$\mathbf{P} = P_n\mathbf{a}_n,\ \text{where } P_n = -\rho_{sb}$$

Polarization Vector (Cont'd)

in general

$$P_n = \mathbf{P} \cdot \mathbf{a}_n = -\rho_{sb}$$

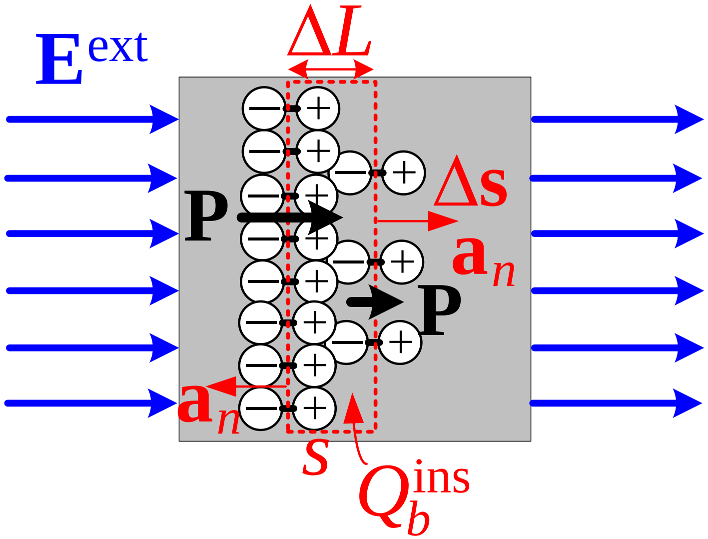
in free space $\mathbf{P} = 0$



Bound Volume Charge

make use of relation to surface charge inside volume

$$|P_n| = |\rho_{sb}^{\text{ins}}|$$



flux of **P** is negative if bound charge inside surface s is positive

$$\Rightarrow \mathbf{P}\cdot\Delta\mathbf{s} = -\rho_{sb}^{\text{ins}}\Delta s$$

take integral over s

$$\oiint_s \mathbf{P}\cdot d\mathbf{s} = -Q_b^{\text{ins}}$$

$$\Rightarrow \oiint_{s[v]} \mathbf{P}\cdot d\mathbf{s} = -\iiint_v \rho_{vb}\,dv$$

$$\Rightarrow \nabla\cdot\mathbf{P} = -\rho_{vb}$$

Compare with $\nabla\cdot\mathbf{D} = \rho_v$

Susceptibility and Permittivity

free charge (in conductors) and bound charge (in dielectrics)

vacuum flux density relates to the total charge density

$$\nabla \cdot (\varepsilon_0 \mathbf{E}) = \rho_{vf} + \rho_{vb}$$

D (flux density) vector is related to free charges only

$$\nabla \cdot \mathbf{D} = \rho_{vf} \Rightarrow \nabla \cdot (\varepsilon_0 \mathbf{E}) = \nabla \cdot \mathbf{D} - \nabla \cdot \mathbf{P} \Rightarrow \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

polarization **P** and **E**-field in isotropic materials are related by the susceptibility

$$\mathbf{P} = \chi_e (\varepsilon_0 \mathbf{E}) \quad \Rightarrow \mathbf{D} = \varepsilon_0 (1 + \chi_e) \mathbf{E}$$

relative permittivity $\varepsilon_r = 1 + \chi_e \Rightarrow \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} = \varepsilon \mathbf{E}$

Anisotropicity and Nonlinearity

anisotropic dielectrics (permittivity tensor)

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \Leftrightarrow \mathbf{D} = \overleftrightarrow{\boldsymbol{\varepsilon}}\mathbf{E}$$

nonlinear dielectrics [$\chi_e = f(\mathbf{E})$, ferroelectric materials]

Dielectric Breakdown

if the field magnitude exceeds certain critical value (the dielectric strength of the material), the polarization forces become too strong, the electrons break free from the atoms, accelerate through the material, thereby heating it, damaging its molecular structure, and ultimately causing even more electrons to break free. This avalanche process leads to arcing/sparking, which destroys the material (dielectric breakdown). The dielectric strength E_{ds} is the strongest **E**-field that the material can sustain without breakdown.

glass	25-40 MV/m
mica	200 MV/m
oil	12 MV/m
air	3 MV/m