# Lecture 14: Electrostatics

continuity equation, relaxation time, boundary conditions, Chapter 5, pages 192-206

# Continuity Equation

$$I_{out} = \oint \mathbf{J}.\mathrm{d}\mathbf{S} = -\frac{\partial Q}{\partial t} = -\frac{\partial}{\partial t}\iiint_{V} \rho_v \, dv$$

$$\iiint_{V} \nabla \bullet \mathbf{J} dv = -\frac{\partial}{\partial t}\iiint_{V} \rho_v \, dv$$

$$\Downarrow$$

$$\nabla \bullet \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

if the positive charge density is decreasing at a point with time then current is flowing out (diverging) from this point

# Relaxation Time

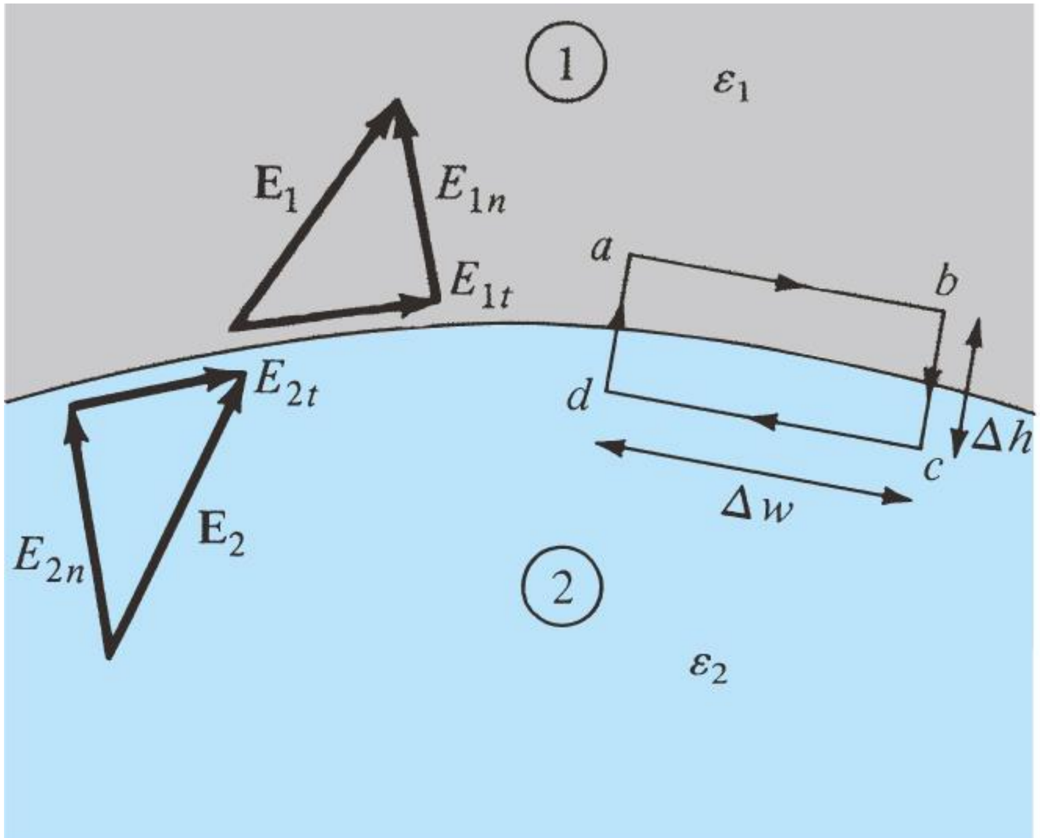
if a volume charge is initially placed inside a material with a finite conductivity, the charge disperses towards the boundaries with time

starting with the continuity equation $\nabla \bullet \mathbf{J} = -\dfrac{\partial \rho_v}{\partial t}$, using $\mathbf{J}=\sigma\mathbf{E}$, and noting that $\nabla \bullet \mathbf{E} = \rho_v / \varepsilon$, we get

$$\frac{\partial \rho_v}{\partial t} = -\frac{\sigma}{\varepsilon}\rho_v \Rightarrow \rho_v = \rho_{vo} e^{-t/T_r},\ T_r = \frac{\varepsilon}{\sigma}$$

for a perfect conductor $T_r$=0, and for an ideal dielectric $T_r=\infty$

# Tangential Electric Fields




as the electric field is conservative, we have

$$\oint \mathbf{E}.d\mathbf{L} = 0$$

as $\Delta h \to 0$, we have

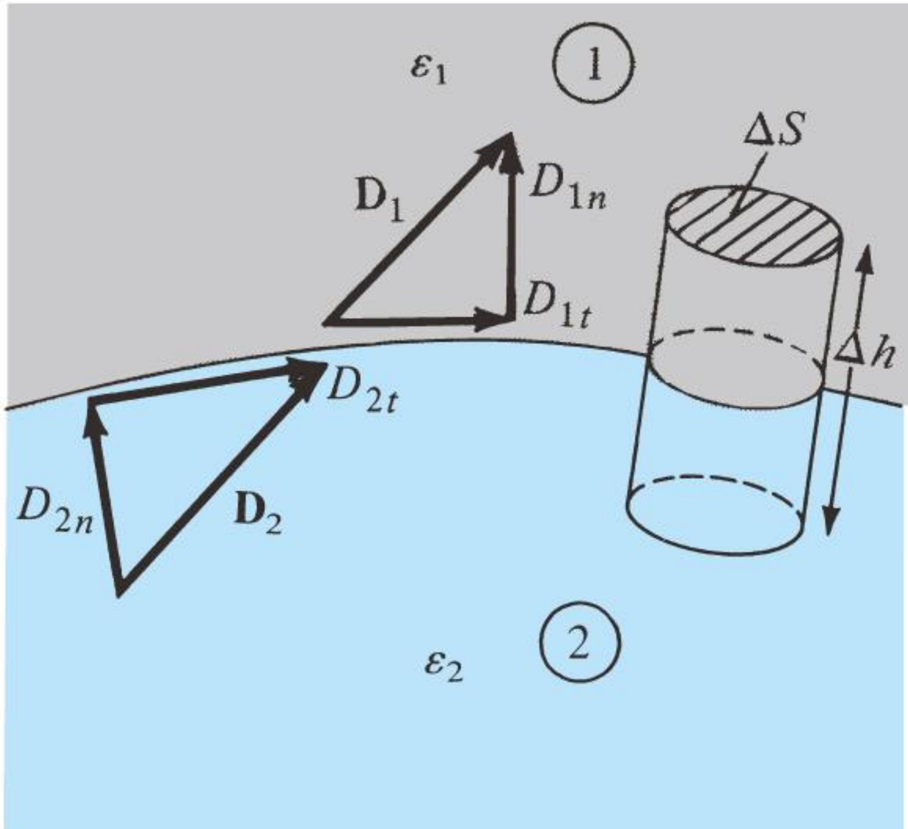
$$E_{1T}\Delta w - E_{2T}\Delta w = 0$$

$$\Downarrow$$

$$E_{1T} = E_{2T}$$

tangential component of electric field is continuous across the interface!

# Normal E Component




applying Gauss' law as $\Delta h \rightarrow 0$

$$D_{1n}\Delta s - D_{2n}\Delta s = \rho_s \Delta s$$

$$\Downarrow$$

$$D_{1n} - D_{2n} = \rho_s$$

if $\rho_s$=0

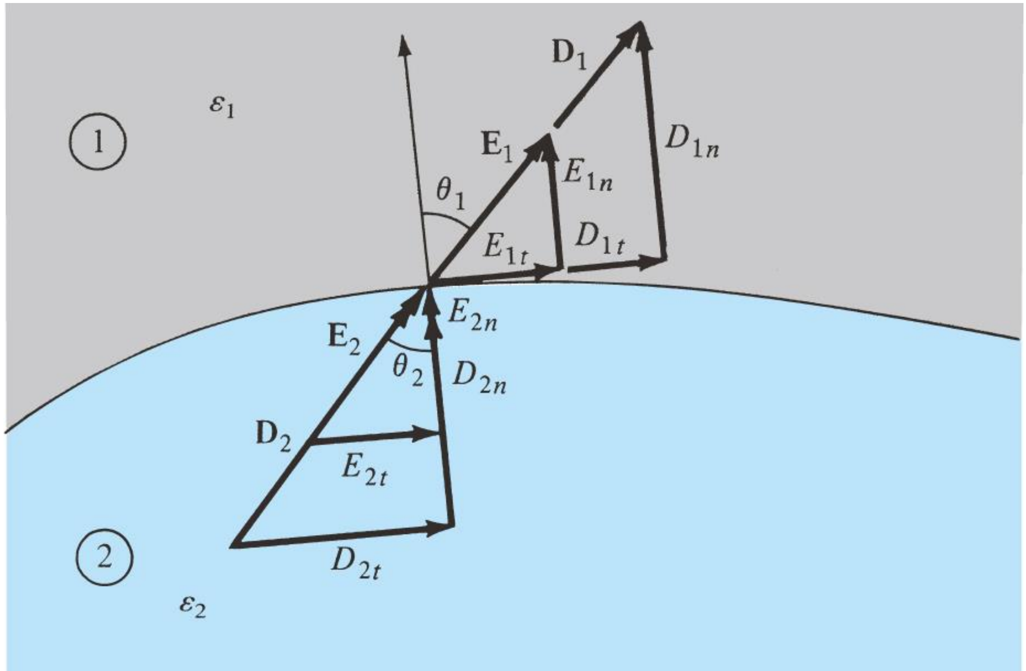
$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\Downarrow$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{2n}$$

normal component of electric field is discontinuous across the interface!

# Interface of Two Dielectrics




$$E_{1T} = E_{2T}$$

$$E_1 \sin(\theta_1) = E_2 \sin(\theta_2)$$

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\varepsilon_1 E_1 \cos(\theta_1) = \varepsilon_2 E_2 \cos(\theta_2)$$

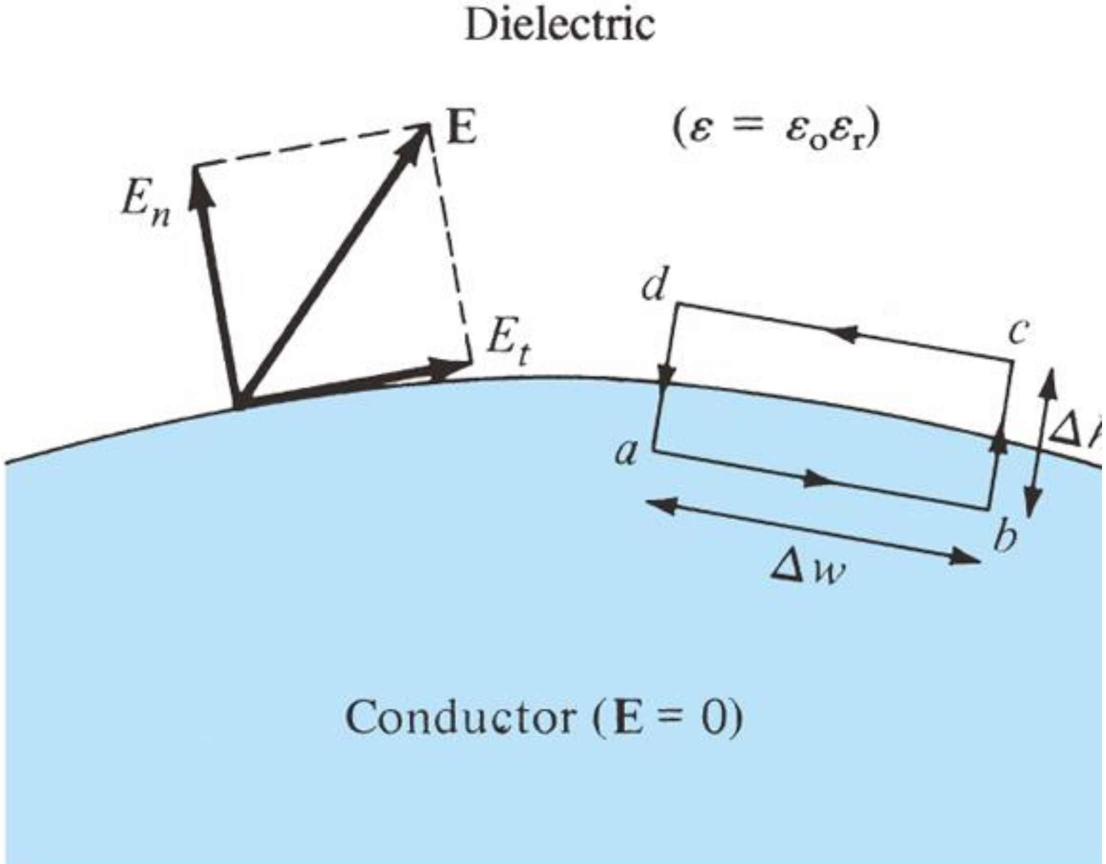
dividing, we get

$$\varepsilon_2 \tan(\theta_1) = \varepsilon_1 \tan(\theta_2)$$

$$\Downarrow$$

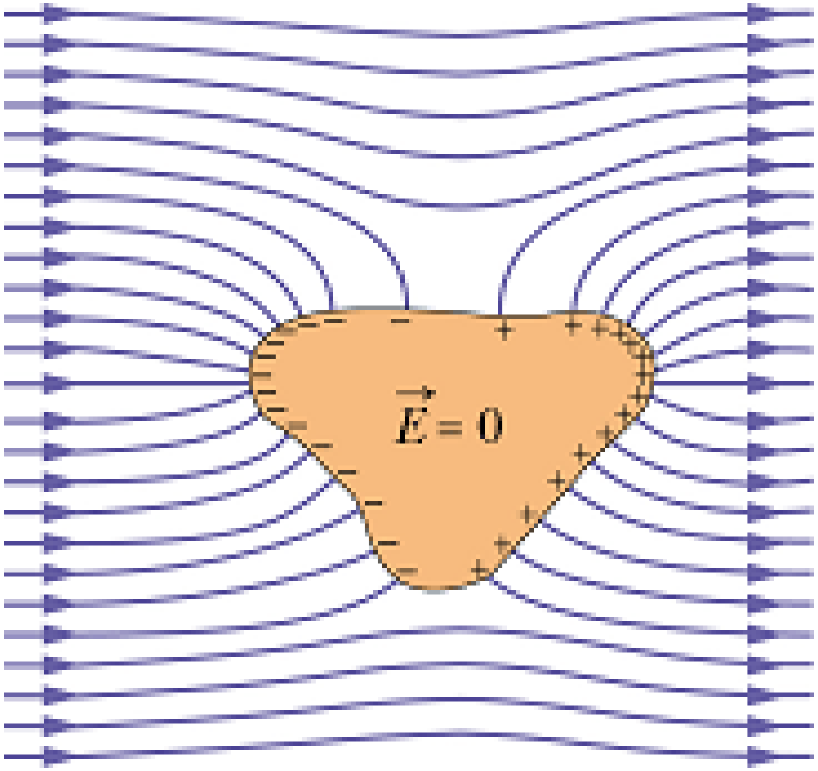
$$\frac{\tan(\theta_1)}{\tan(\theta_2)} = \frac{\varepsilon_1}{\varepsilon_2}$$

# Dielectric-Conductor Interface




because there is no field inside the conductor, $E_t$=0 at the dielectric-conductor interface

# Fields and Perfect Conductors

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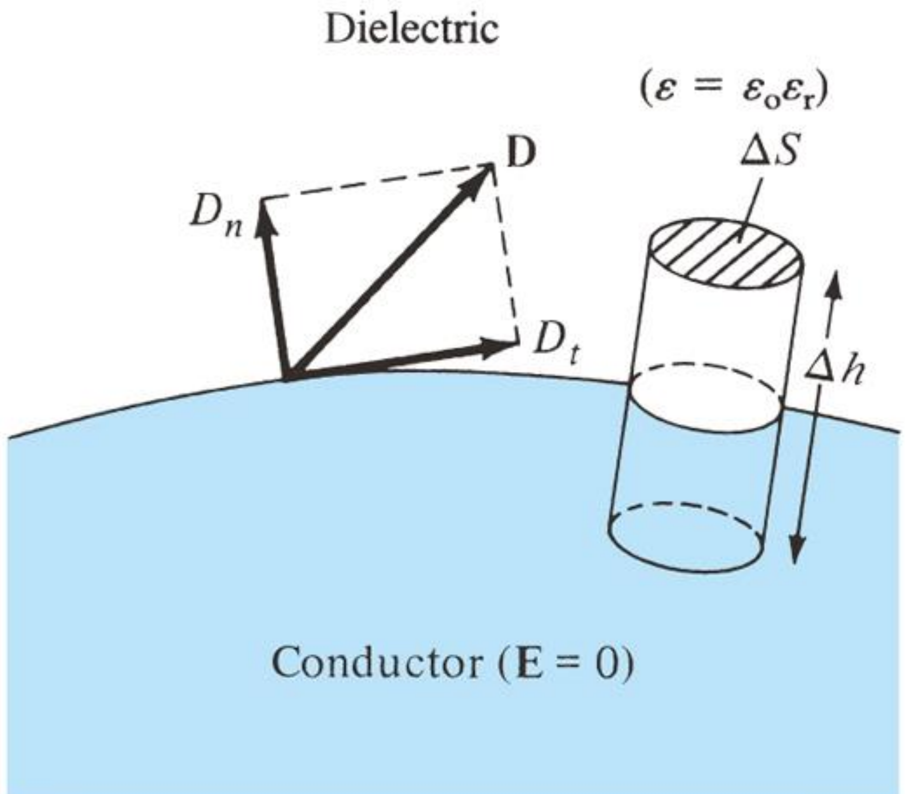
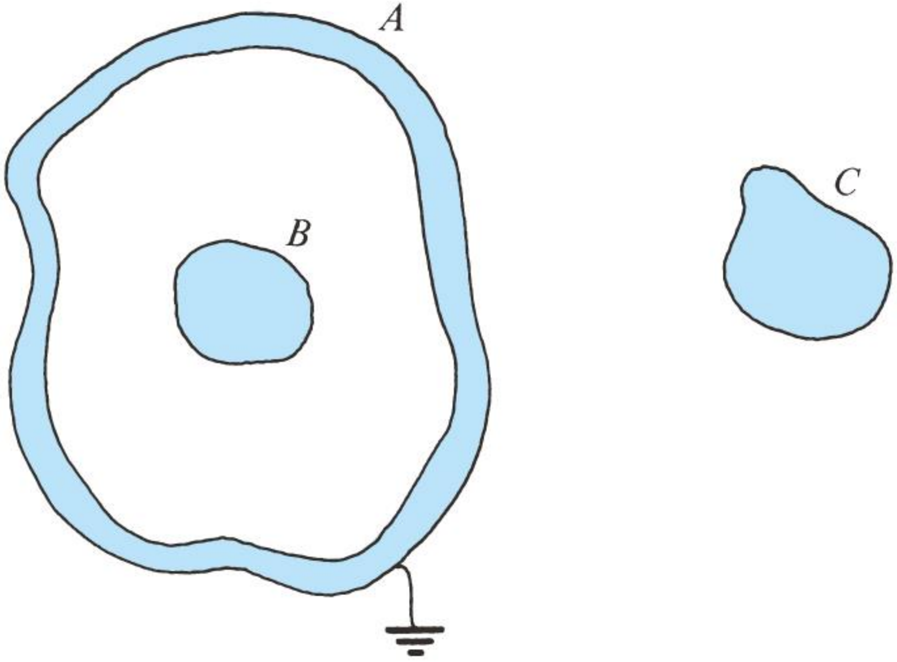
field lines meets a perfect conductor at a right angle

# Dielectric-Conductor Interface (Cont'd)




because there is no field inside the conductor, $D_n = \rho_s$

# Electrostatic Shielding




by surrounding a region by a grounded conductor, this region becomes electromagnetically isolated from any fields or charges outside the shield